15-150

Principles of Functional Programming

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Course Webpage

http://www.cs.cmu.edu/~15150/

Policies: http://www.cs.cmu.edu/~15150/policy.html

Lectures: http://www.cs.cmu.edu/~15150/lect.html

Course Philosophy

Computation is Functional.

Programming is an explanatory linguistic process.

Computation is Functional

values : types

expressions

Functions map values to values

Imperative vs. Functional

Command

↓

- executed
- has an effect

$x := 5$

(state)

Expression

↓

- evaluated
- no effect

$3 + 4$

(value)

Programming as Explanation

Problem statement

↓

high expectation to explain precisely & concisely

- invariants
- specifications
- proofs of correctness
- code

Analyze, Decompose & Fit, Prove

Parallelism

/\
< 1, 0, 0, 1, 1 > ➔ 3,
< 1, 0, 1, 1, 0 > ➔ 3,
< 1, 1, 1, 0, 1 > ➔ 4,
< 0, 1, 1, 0, 0 > ➔ 2,
\/
↓
12

Parallelism

```
sum : int sequence  →  int
type row  =  int sequence
type room = row sequence

fun count  (class : room) : int =
        sum (map sum class)
```

Parallelism

- Work:
  - Sequential Computation
  - Total sequential time;

    number of operations

- Span:
  - Parallel Computation
  - How long would it take if one could have as many processors as one wants;

    length of longest critical path

Defining ML (Effect-Free Fragment)

- Types t
- Expressions e
- Values v (subset of expressions)

Examples:

$$(3+4) * 2$$

$$\xRightarrow{1} \quad 7 * 2$$

$$\xRightarrow{1} \quad 14$$

$$(3+4) * (2+1)$$

$$\xRightarrow{3} \quad 21$$

`"the " ^ "walrus"`

$\xRightarrow{1}$ `"the walrus"`

The expression

`"the " ^ "walrus"`

reduces to the value

`"the walrus"` .

It has type `string`.

"the walrus" + 1

$\Longrightarrow$??

The expression

"the walrus" + 1

does not have a type and it does not reduce to a value.

Types

A ***type*** is a **prediction** about the kind of value an expression will have if it winds up reducing to a value.

An expression is ***well-typed*** if it has at least one type, and ***ill-typed*** otherwise.

(We may also say that an expression ***type-checks***, meaning that it is well-typed.)

First, type-check an expression.

If the expression is well-typed,
then evaluate the expression.

(The ML compiler does that.)

Expressions

Every well-formed ML expression `e`

- has a type `t`, written as $\underline{\texttt{e : t}}$
- may have a value `v`, written as $\underline{\texttt{e} \hookrightarrow \texttt{v}}$.
- may have an effect (not for our effect-free fragment)

Example: $(3+4)*2 \; : \; \text{int}$

$(3+4)*2 \hookrightarrow 14$

Type `int`

Values …, `~1, 0, 1`, …,

that is, every integer n.

Expressions e_1 `+` e_2, e_1 `-` e_2, e_1 `*` e_2,

e_1 `div` e_2, e_1 `mod` e_2, *etc.*

Example: ~4 * 3

Typing Rules

- n : `int`
- e_1 `+` e_2 : `int`

  if e_1 : `int` and e_2 : `int`

similar for other operations.

Example:

(3 + 4) * 2 : int

Why?

3+4 : int and 2 : int

Why?

3 : int and 4 : int

Evaluation Rules

- $e_1 + e_2 \xrightarrow{1} e_1' + e_2$ if $e_1 \xrightarrow{1} e_1'$
- $n_1 + e_2 \xrightarrow{1} n_1 + e_2'$ if $e_2 \xrightarrow{1} e_2'$
- $n_1 + n_2 \xrightarrow{1} n$,

  with n the sum of the integer values n_1 and n_2.

Example of a well-typed expression with no value

5 div 0 : int

5 div 0 : int

because 5 : int
& 0 : int

and because div expects
two ints and returns an int.

However, 5 div 0
does not reduce to a value.

Notation Recap

$e : t$ "e has type t"

$e \Rightarrow e'$ "e reduces to e'"

$e \hookrightarrow v$ "e evaluates to v"

Extensional Equivalence

$$\cong$$

An equivalence relation on expressions
(of the same type).

Extensional Equivalence

- Expressions are *extensionally equivalent* *if they have the same type and one of the following is true:*

  both expressions reduce to the same value,
  or both expressions raise the same exception,
  or both expressions loop forever.

- Functions are *extensionally equivalent* if they map equivalent arguments to equivalent results.

- In proofs, we use $\cong$ as shorthand for "is equivalent to".

- Examples: $21 + 21 \cong 42 \cong 6 * 7$
  $[2, 7, 6] \cong [1+1, 2+5, 3+3]$
  $(\text{fn } x \Rightarrow x + x) \cong (\text{fn } y \Rightarrow 2 * y)$

- Functional programs are *referentially transparent*, meaning:
  - The *value* of an expression depends only on the *values* of its sub-expressions.
  - The *type* of an expression depends only on the *types* of its sub-expressions.

Types in ML

Basic types:

int, real, bool, char, string

Constructed types:

product types
function types
user-defined types

Products, Expressions

Types $t_1 * t_2$ for any type t_1 and t_2.

Values $(v_1,\ v_2)$ for values v_1 and v_2.

Expressions $(e_1,\ e_2),\quad \underbrace{\#1\ e,\quad \#2\ e}_{\text{DO NOT USE!}^{*}}$

Examples: (3+4, true)

(1.0, ~15.6)

(8, 5, false, ~2)

* You will learn how to extract components using pattern matching

Typing Rules

- $(e_1,\ e_2)\ :\ t_1\ *\ t_2$

  if $e_1 : t_1$

  and $e_2 : t_2$

Example: $(3+4,\ \text{true}) : \text{int} * \text{bool}$

Evaluation Rules

- $(e_1,\ e_2) \xrightarrow{1} (e_1',\ e_2) \quad \text{if } e_1 \xrightarrow{1} e_1'$
- $(v_1,\ e_2) \xrightarrow{1} (v_1,\ e_2') \quad \text{if } e_2 \xrightarrow{1} e_2'$

Functions

In math, one talks about a function f mapping between spaces X and Y,

$$f : X \to Y$$

In SML, we will do the same, with X and Y being types.

Issue: Computationally, a function may not always return a value. That complicates checking equivalence.

Definition: A function f is ***total*** if f(x) returns a value for all values x in X.

(Totality is a key difference between math and computation.)

Sample Function Code

```
(* square : int -> int
   REQUIRES: true
   ENSURES:  square(x) evaluates to x * x
*)

fun square (x:int) : int = x * x



(* Testcases: *)

val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

Sample Function Code

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(* square : int -> int
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function type

```
fun square (x:int) : int = x * x
```

keyword

function name

argument name & type

result type

body of function

```
(* Testcases: *)

val 0 = square 0
val 49 = square 7
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```

Five-Step Methodology

① `(* square : int -> int` function type

② `REQUIRES: true`

③ `ENSURES:  square(x) evaluates to x * x`

`*)`

④

`fun`	`square`	`(x:int)`	`: int`	`= x * x`
keyword	function name	argument name & type	result type	body of function

⑤ `(* Testcases: *)`

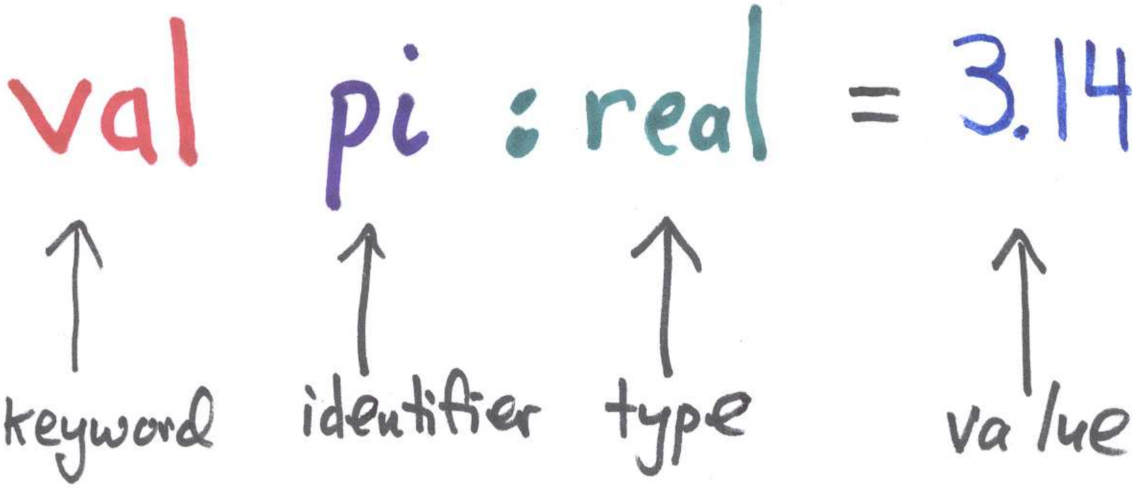
```
val 0 = square 0
val 49 = square 7
val 81 = square (~9)
```

Declarations

Environments

Scope

Declaration



```
val pi : real = 3.14
```

Introduces binding of 3.14 to pi
(sometimes written [3.14/pi])

Lexically statically scoped.

```
val x : int = 8-5        [3/x]
val y : int = x+1        [4/y]
val x : int = 10         [10/x]
val z : int = x+1        [11/z]
```

second binding of x *shadows* first binding.

First binding has been *shadowed*.

Local Declarations

`let ... in ... end`

```
let
    val m : int = 3
    val n : int = m*m
in
    m+n
end
```

This is an expression.

What type does it have? int

What value? 12

Local Declarations

```
val k : int = 4
```

```
let
    val k : real = 3.0
in
    k * k
end
```

↪ 9.0 : real

Type?

Value?

```
k
```

← Type?

Value?

↪ 4 : int

Concrete Type Def

```
type float = real

type point = float*float

val p : point = (1.0,2.6)
```

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds a closure to the identifier `square`.

Closures

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```
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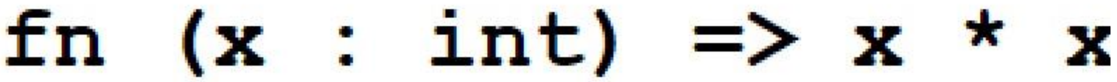
binds a closure to the identifier **`square`**.

The closure consists of two parts:

- A lambda expression (anonymous function value):



```
fn (x : int) => x * x
```

keyword

argument
name & type

body of function

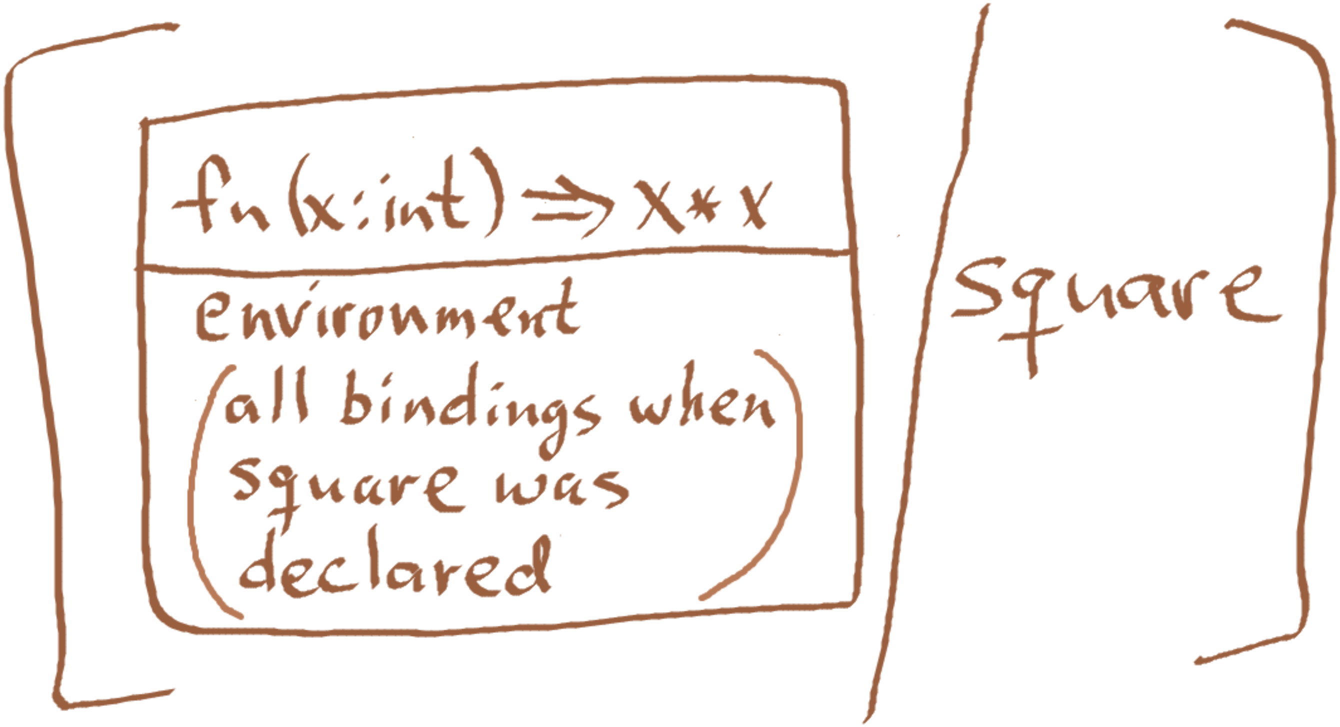
- An environment (all prior value bindings).

Closures

Function declarations also create value bindings:

```
fun square (x:int) : int = x * x
```

binds a closure to the identifier **`square`**.



Course Tasks

Assignments	35%
Labs	10%
Midterm 1	15%
Midterm 2	15%
Final	25%

Roughly one assignment per week, one lab per week.

Collaboration

Be sure to read the
course and university webpages
regarding academic integrity.