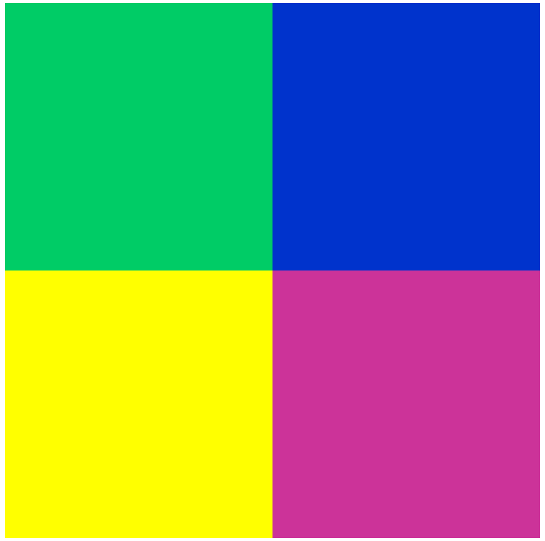

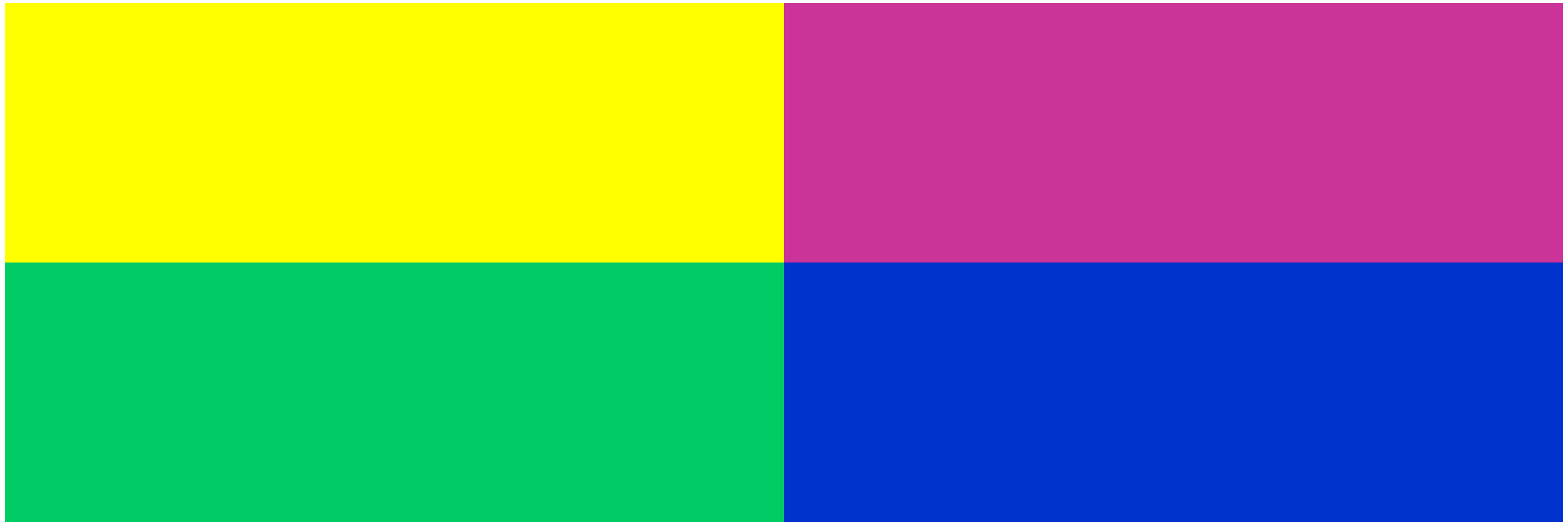

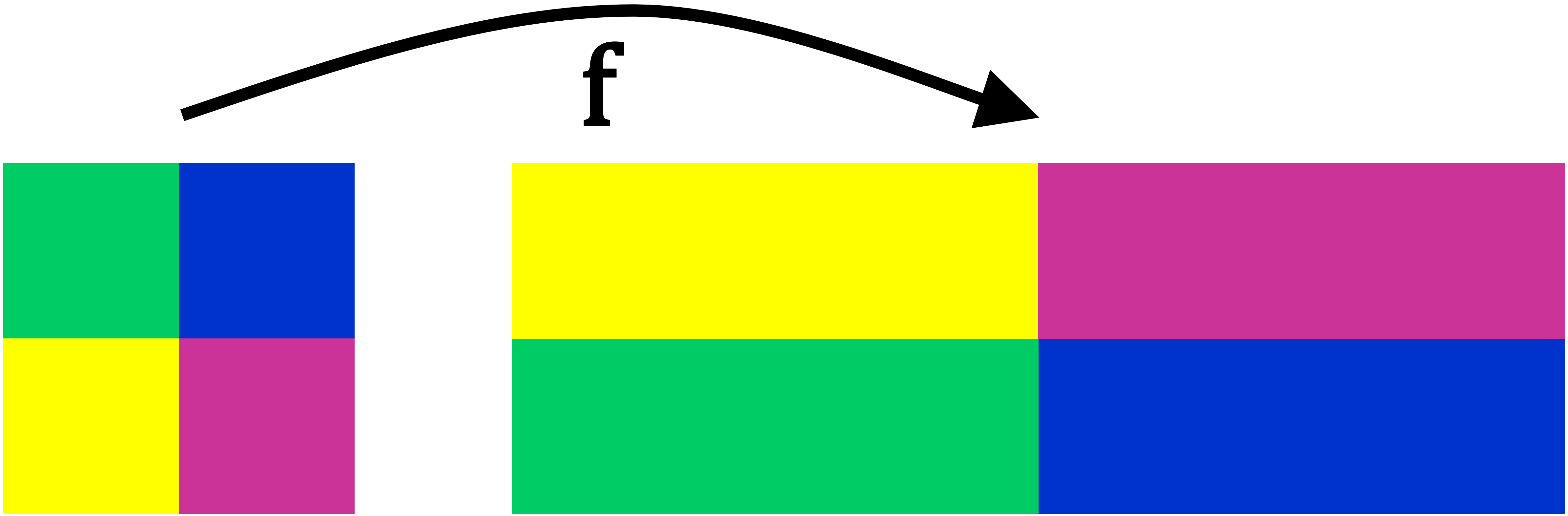
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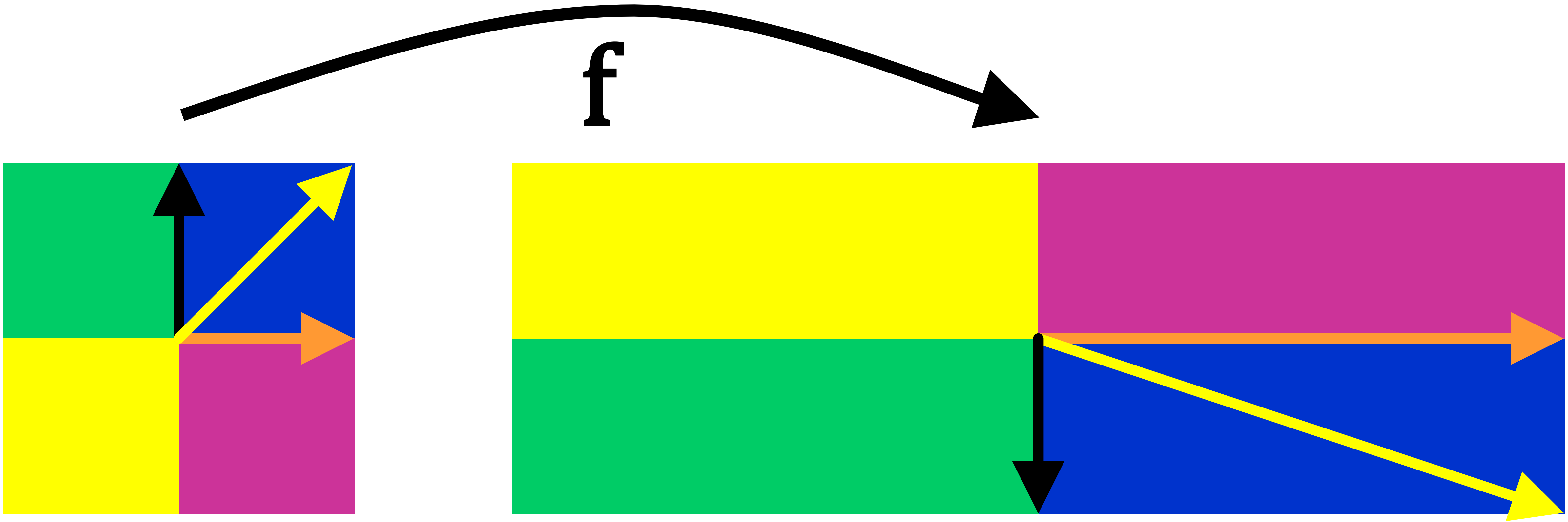
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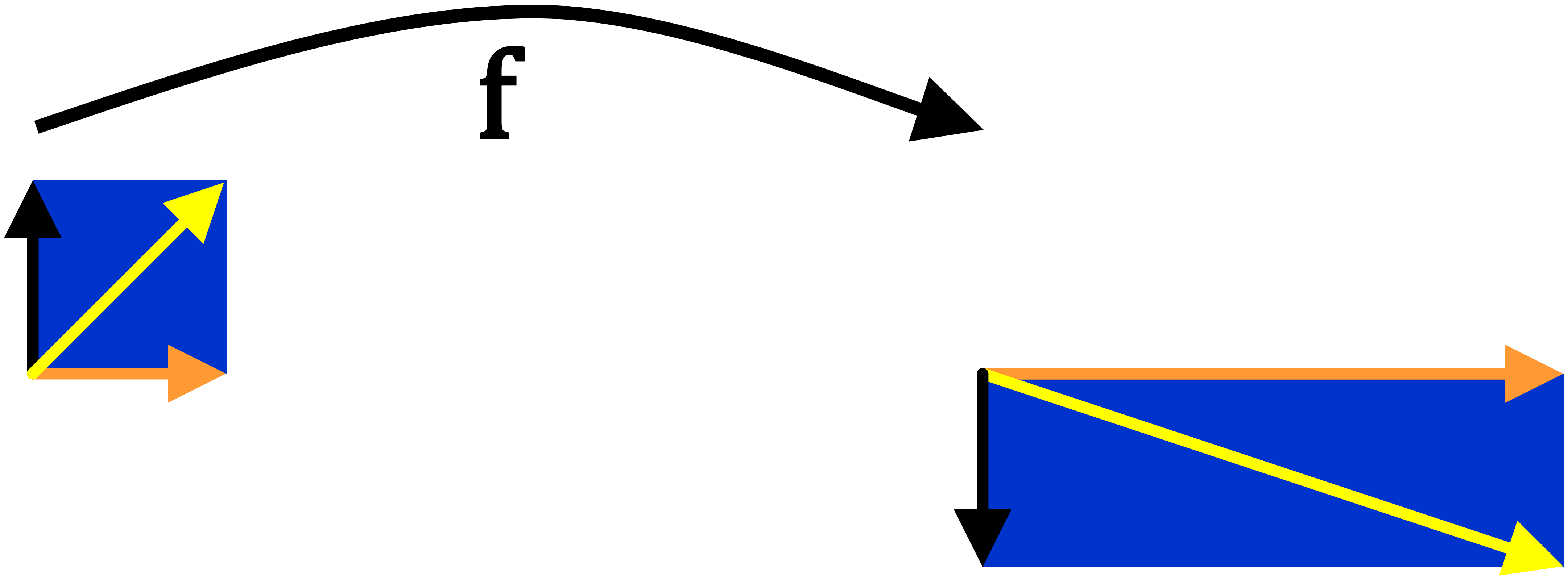
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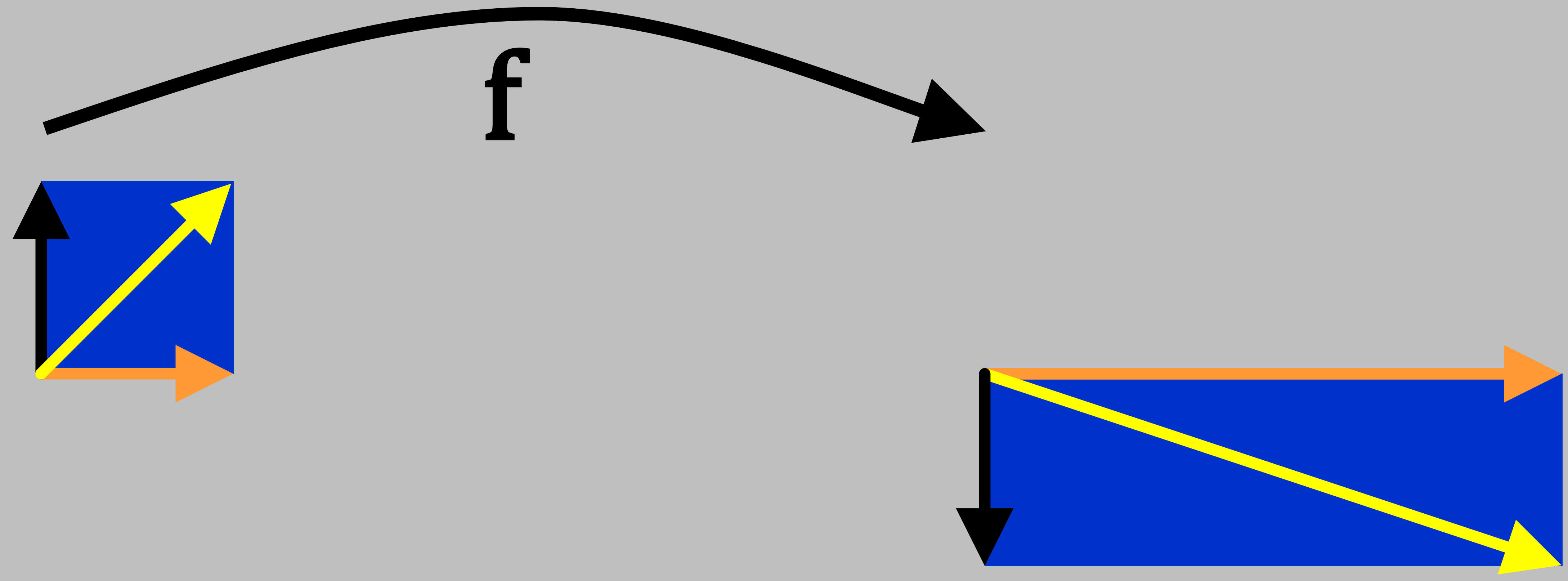
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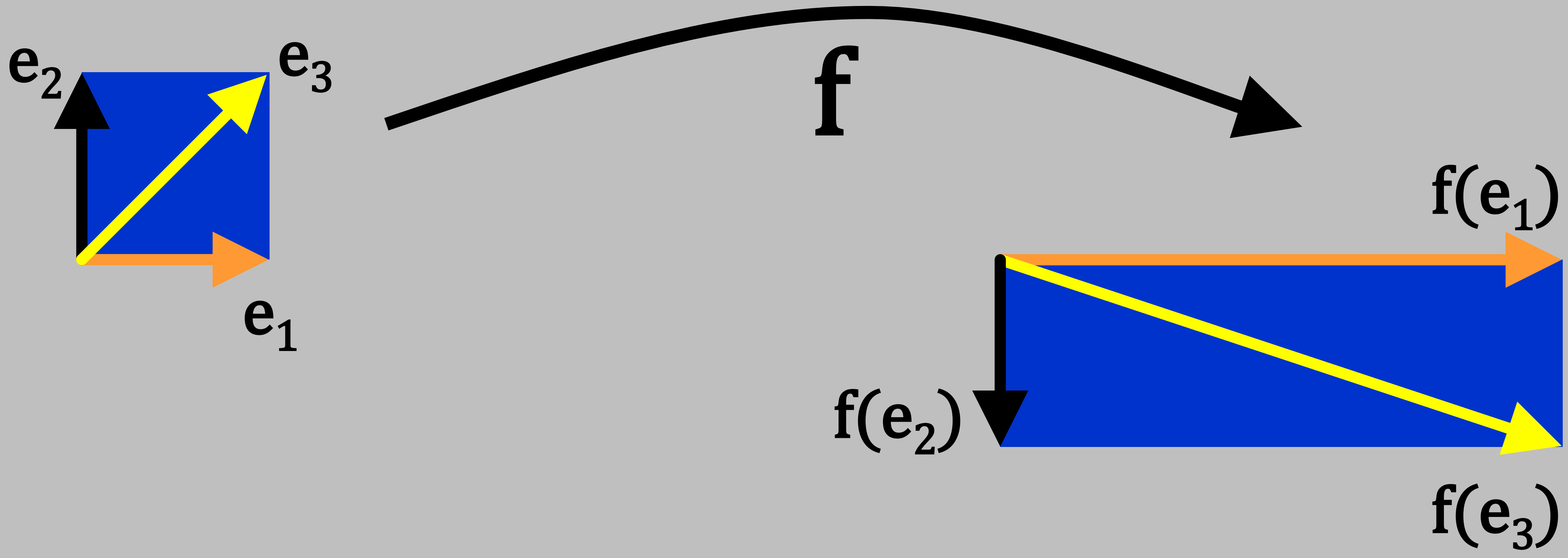
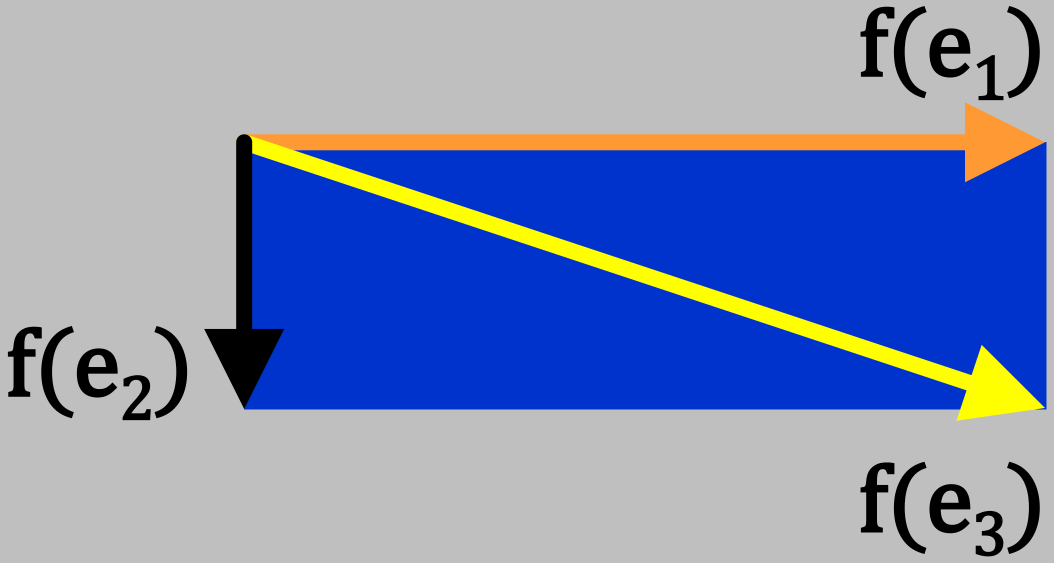
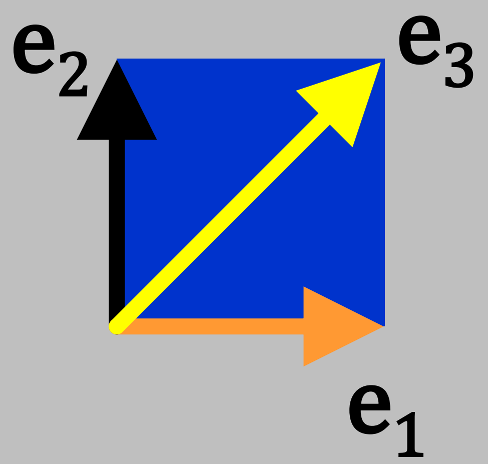

e2
e3
e1
f
f(e1)
f(e2)
f(e3)

$$f(e_1) = 3*e_1 + 0*e_2$$
$$f(e_2) = 0*e_1 + (-1)*e_2$$





$f(e_1) = 3^*e_1 + 0^*e_2$
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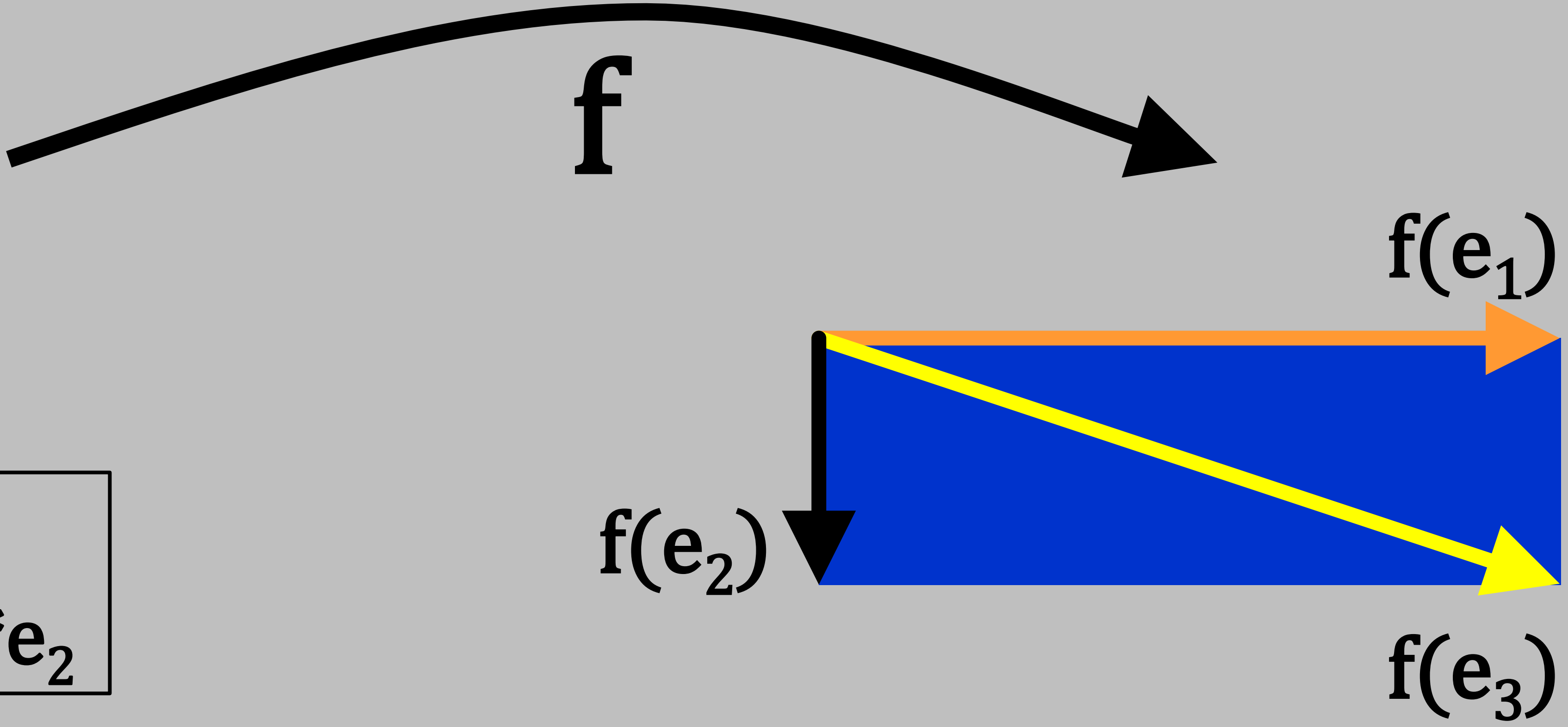
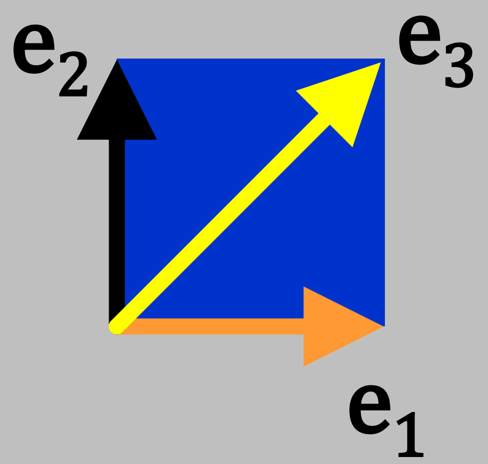
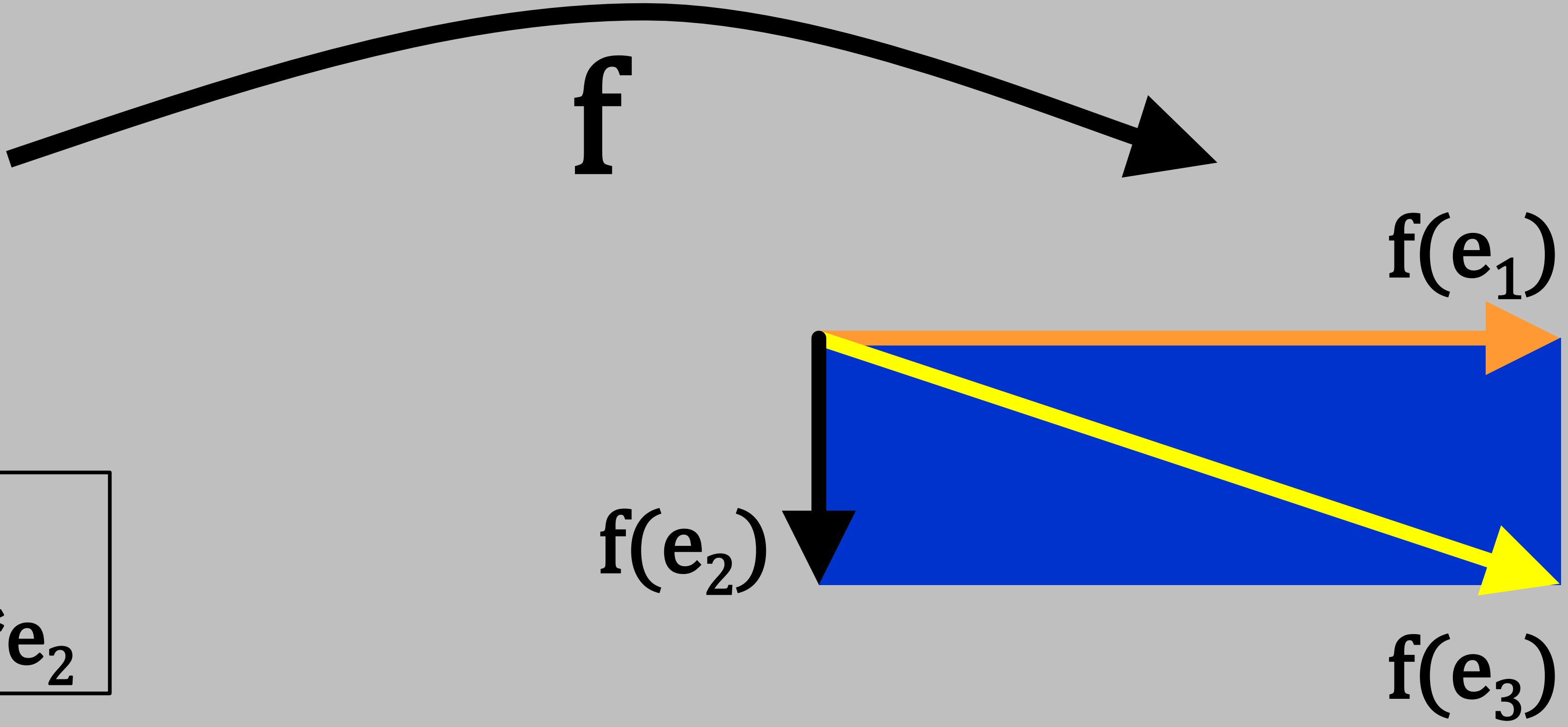


Let $\mathfrak{B} = \{e_1, e_2\}$ be a basis for $\mathbb{R}^2$.
Use for domain & range. Get matrix:





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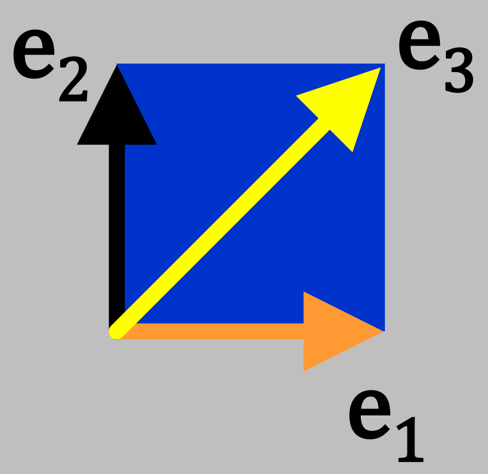

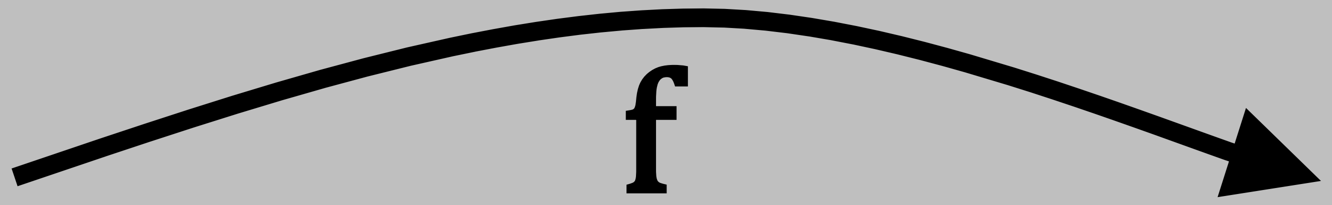

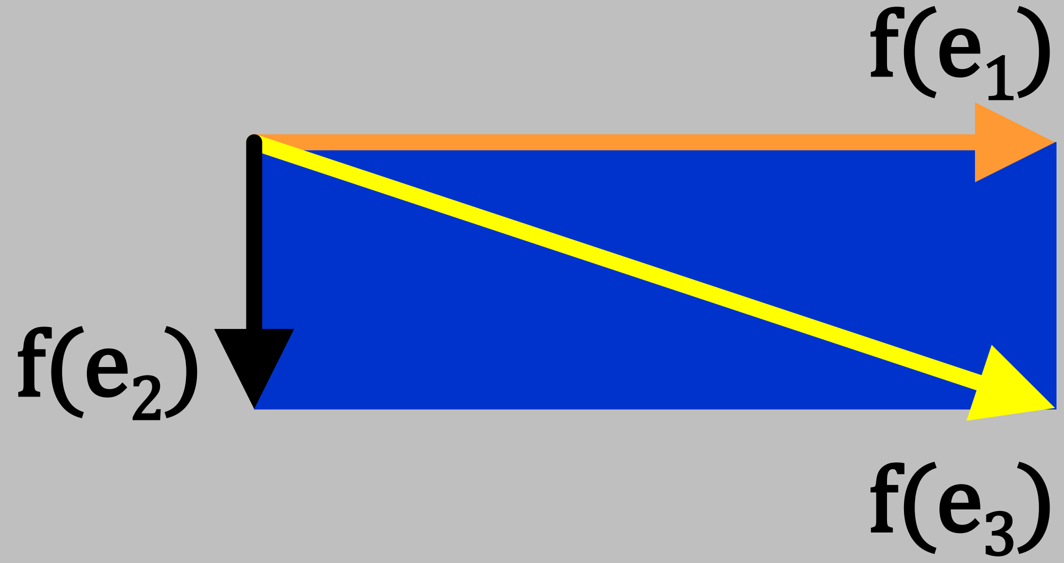

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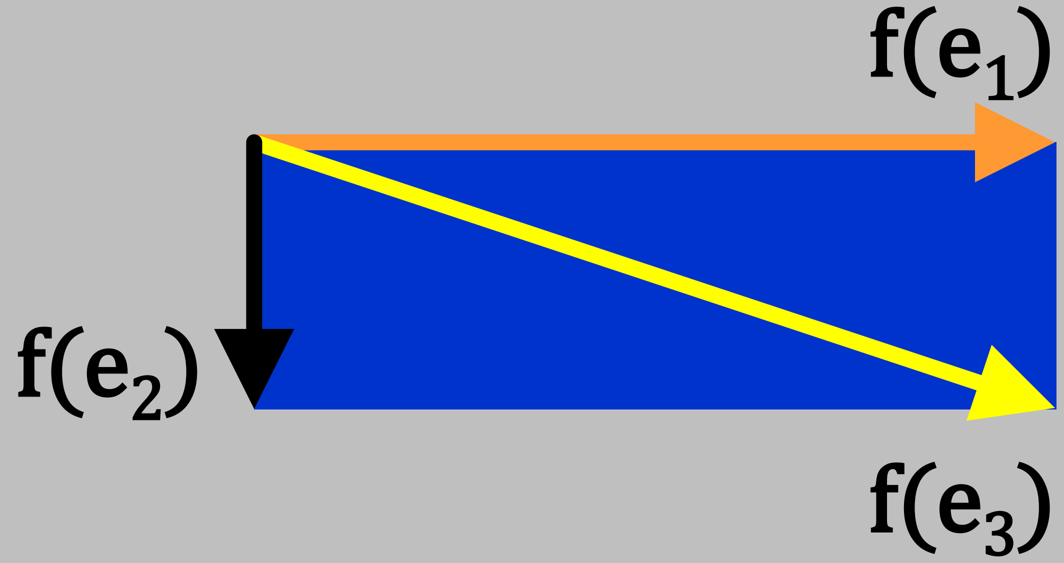
Use for domain & range. Get matrix:

$$A = {}^{\mathfrak{B}}[f]_{\mathfrak{B}} = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$$

Matrix column #i expresses $\mathbf{f}(\mathbf{e}_i)$ in terms of $\{e_1, e_2\}$.

Matrix entry A_{ij} comes from $\mathbf{f}(\mathbf{e}_i) = \cdots + A_{ij}*\mathbf{e}_j + \cdots$.



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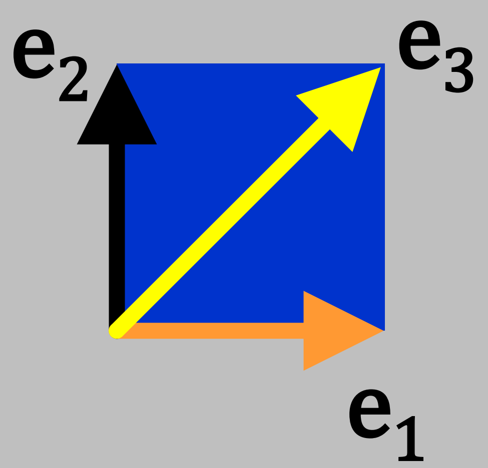

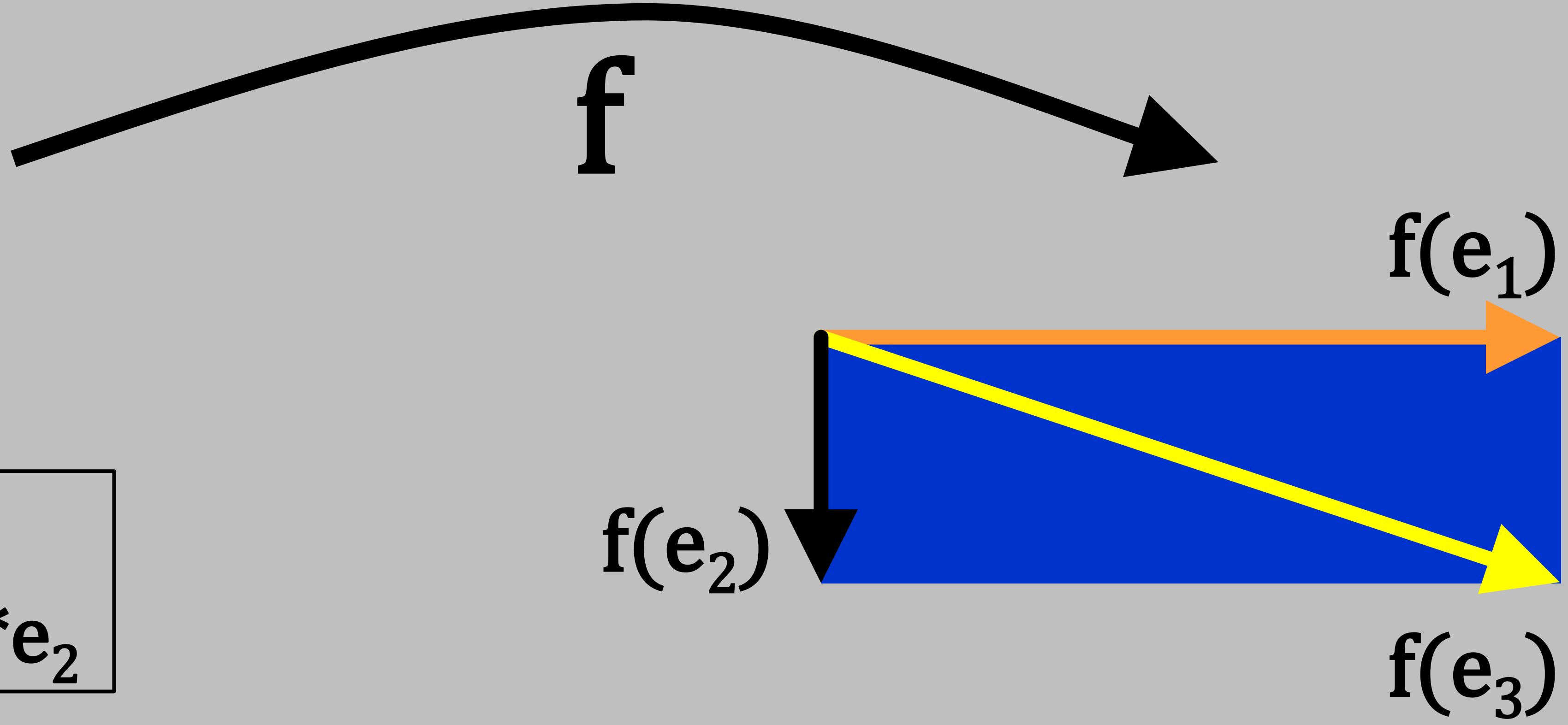

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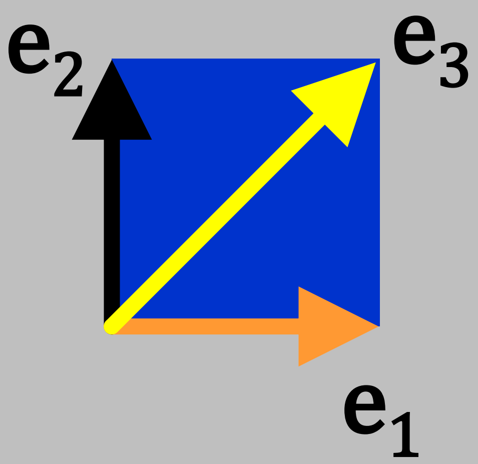

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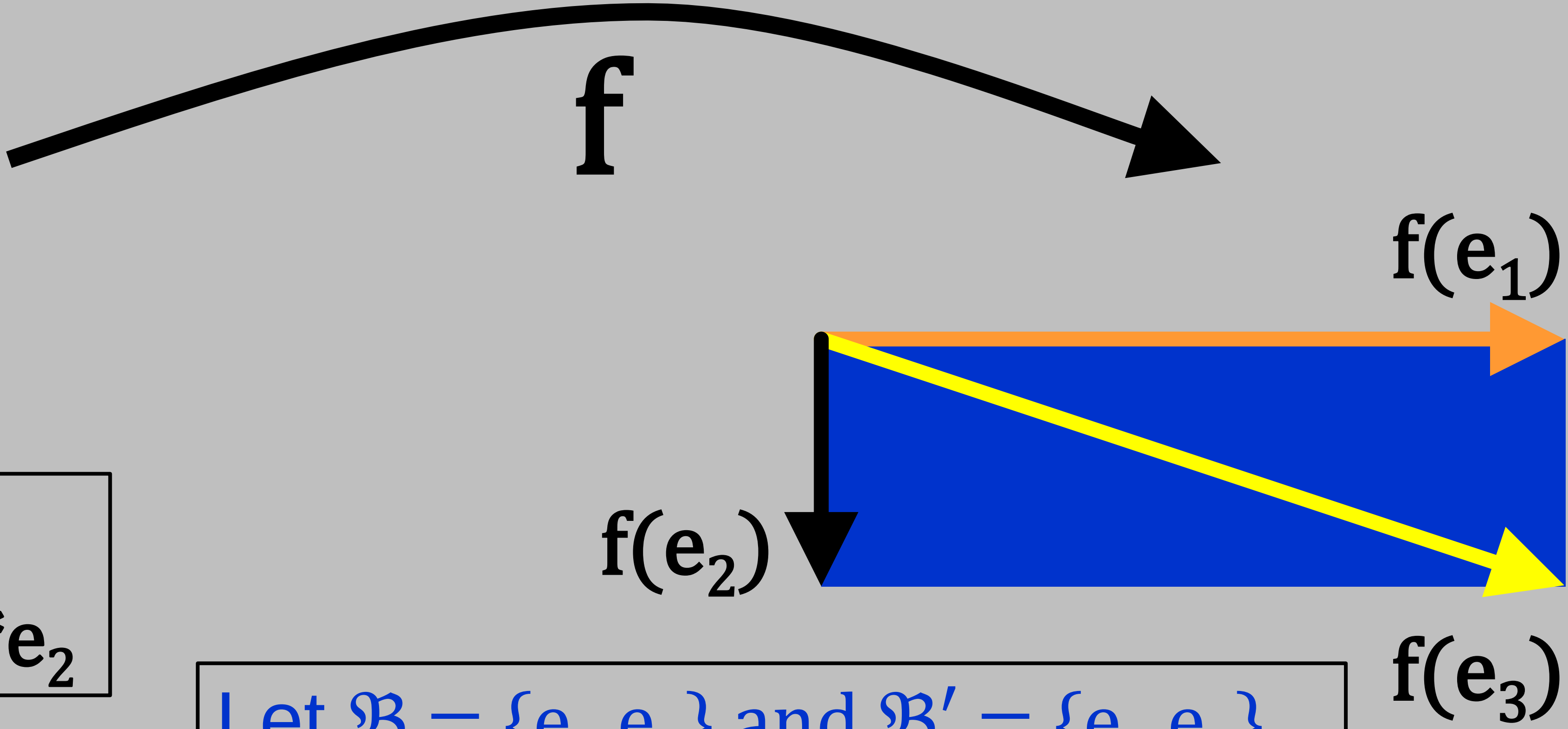

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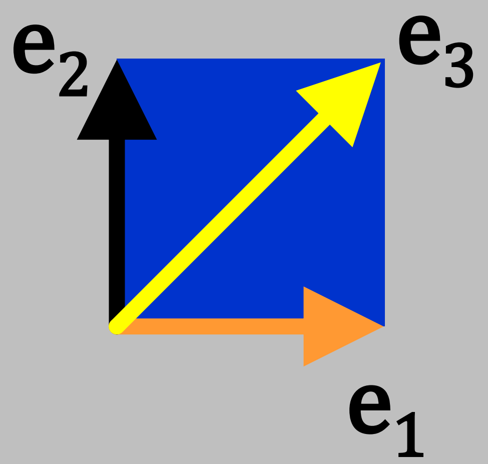

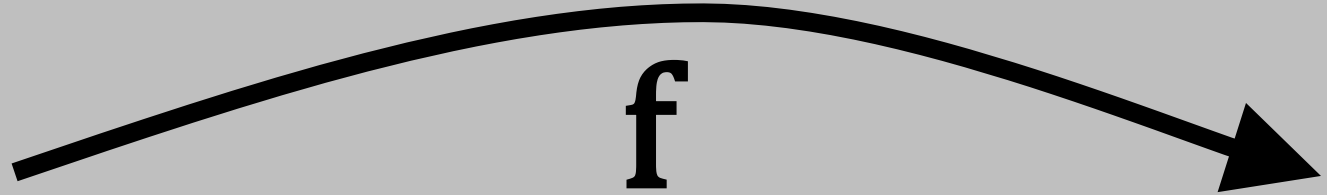

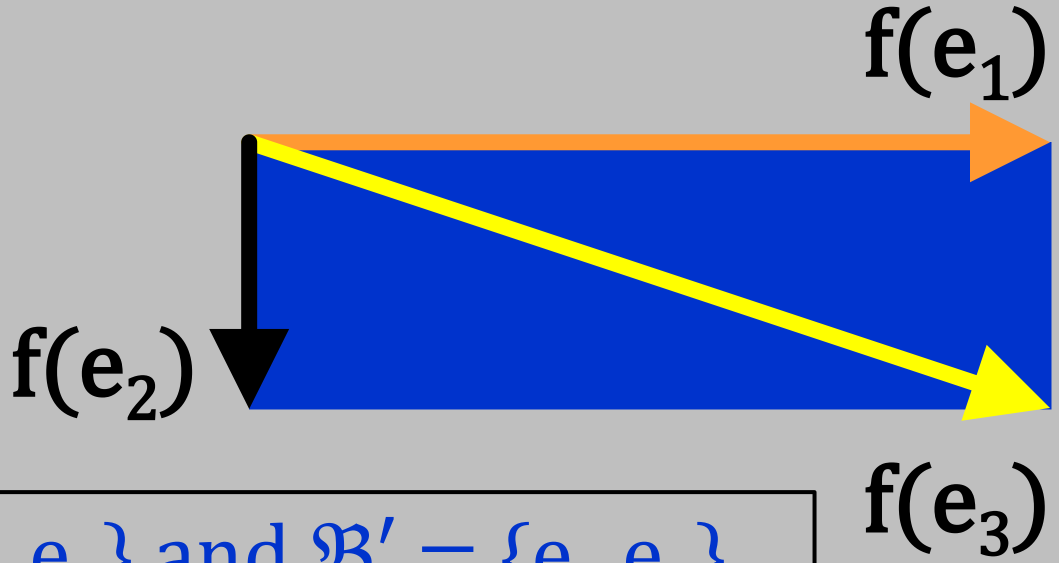

$f(e_1) = 3*e_1 + 0*e_2$
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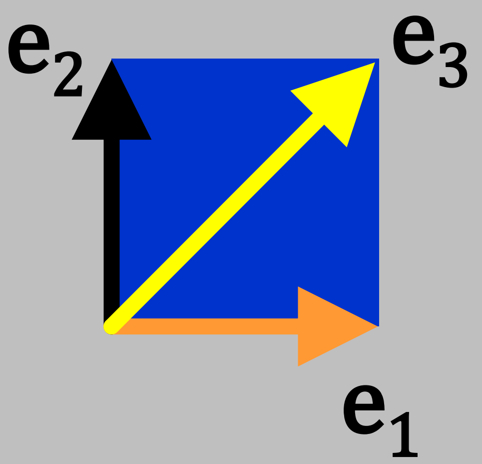
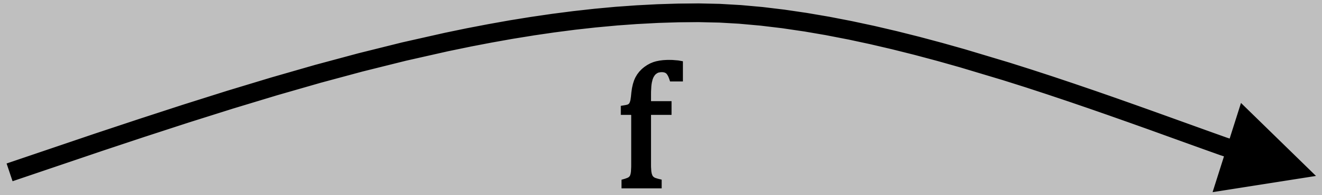
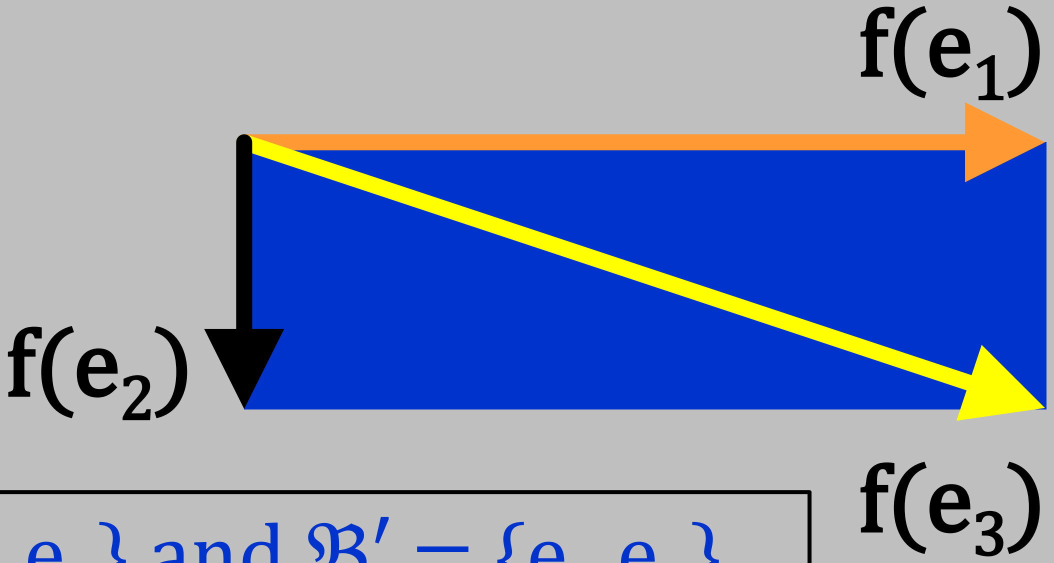
Use $\mathfrak{B}'$ for domain & $\mathfrak{B}$ for range:

$$ {}^{\mathfrak{B}}[f]_{\mathfrak{B}'} = \begin{pmatrix} 3 & 3 \\ 0 & -1 \end{pmatrix} $$

Since $f(e_3) = 3*e_1 + (-1)*e_2$.







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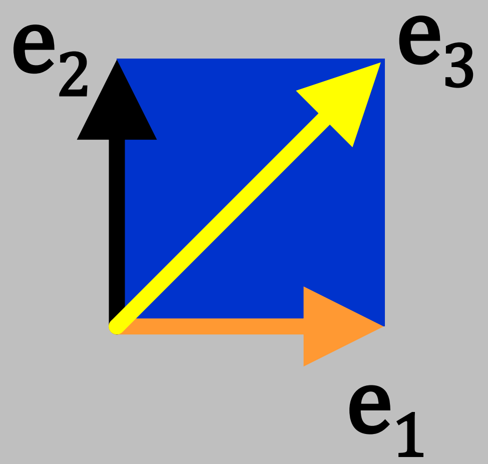

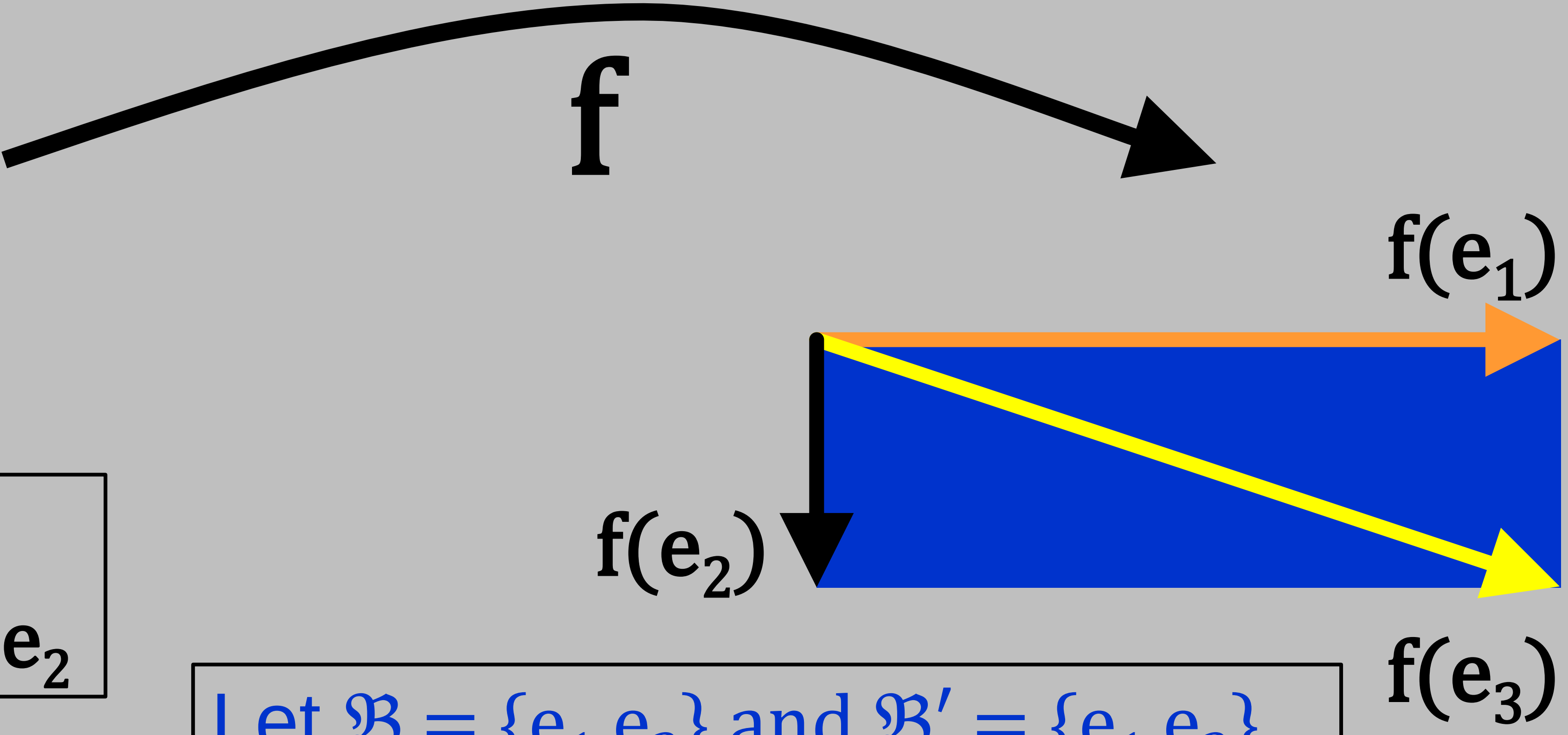

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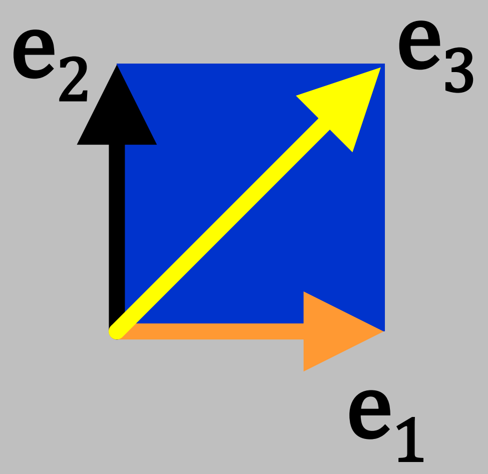

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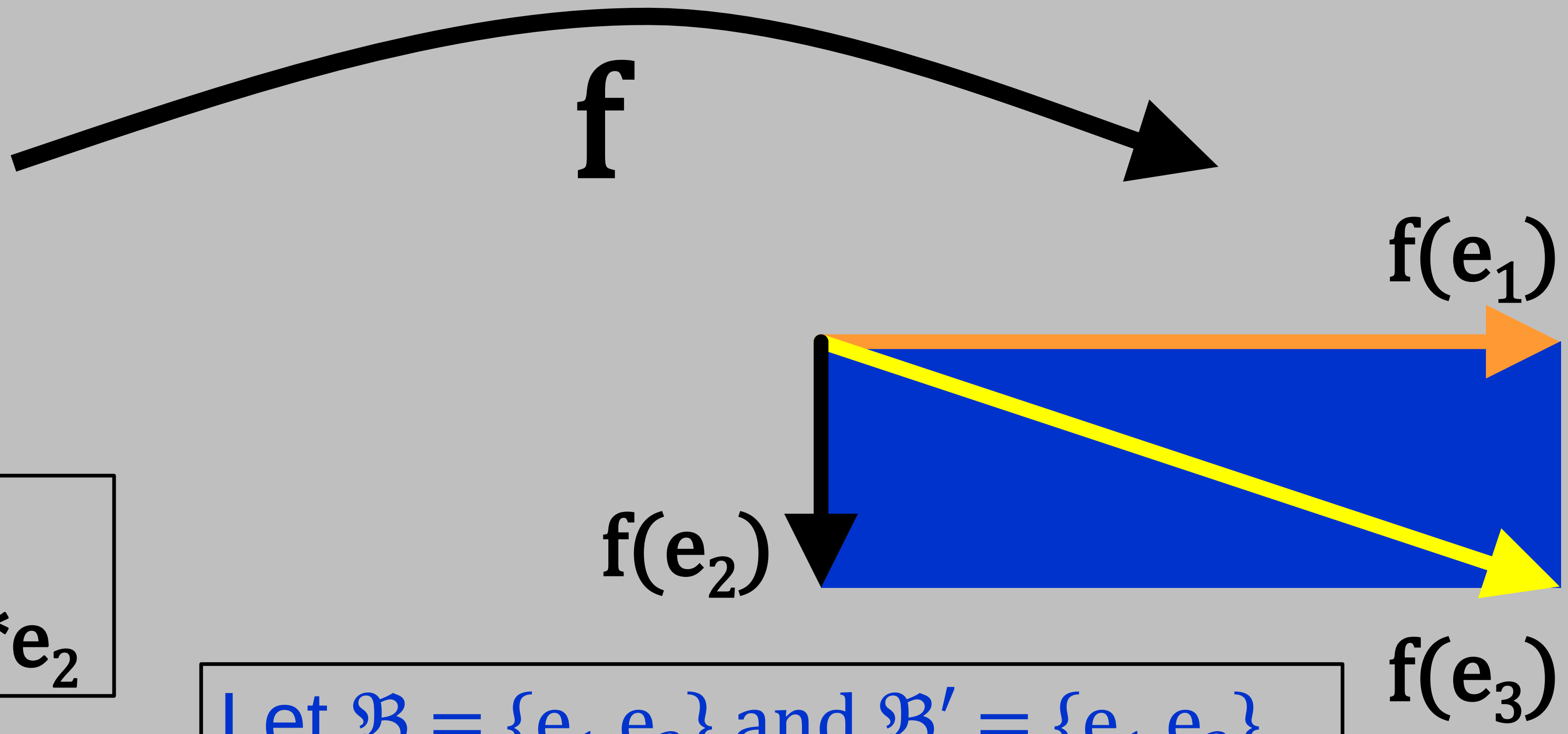

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columns are $\mathfrak{B}$ vectors expressed in $\mathfrak{B}'$ coordinates

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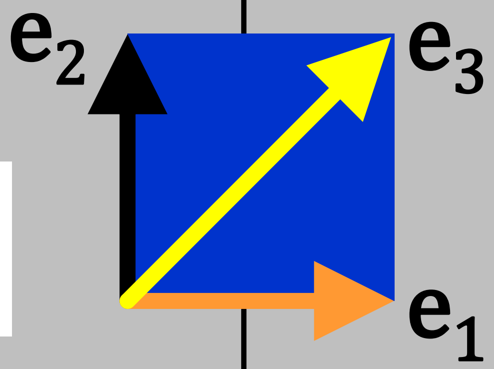

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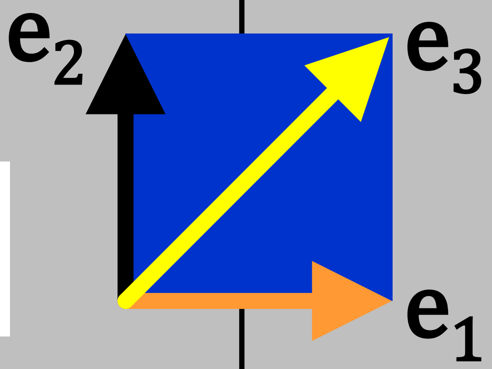

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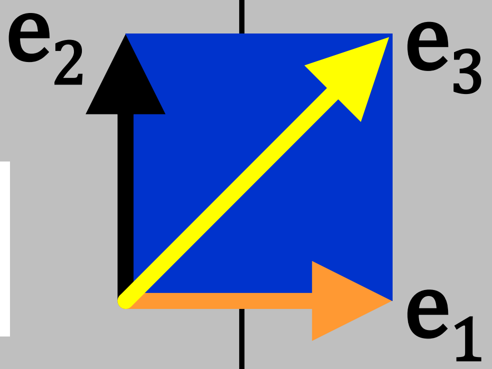

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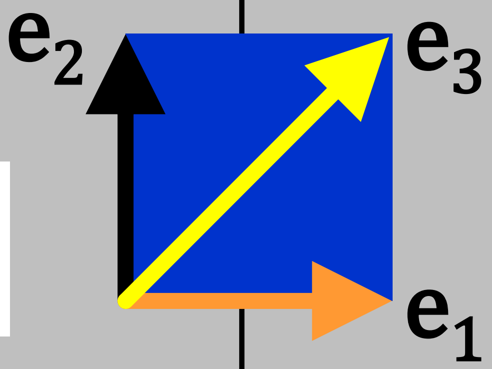

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(there exist other bases and matrices)

Expressing f in terms of $\mathfrak{B}'$ for range and domain

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$$

$$
= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}
$$

$$
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$$

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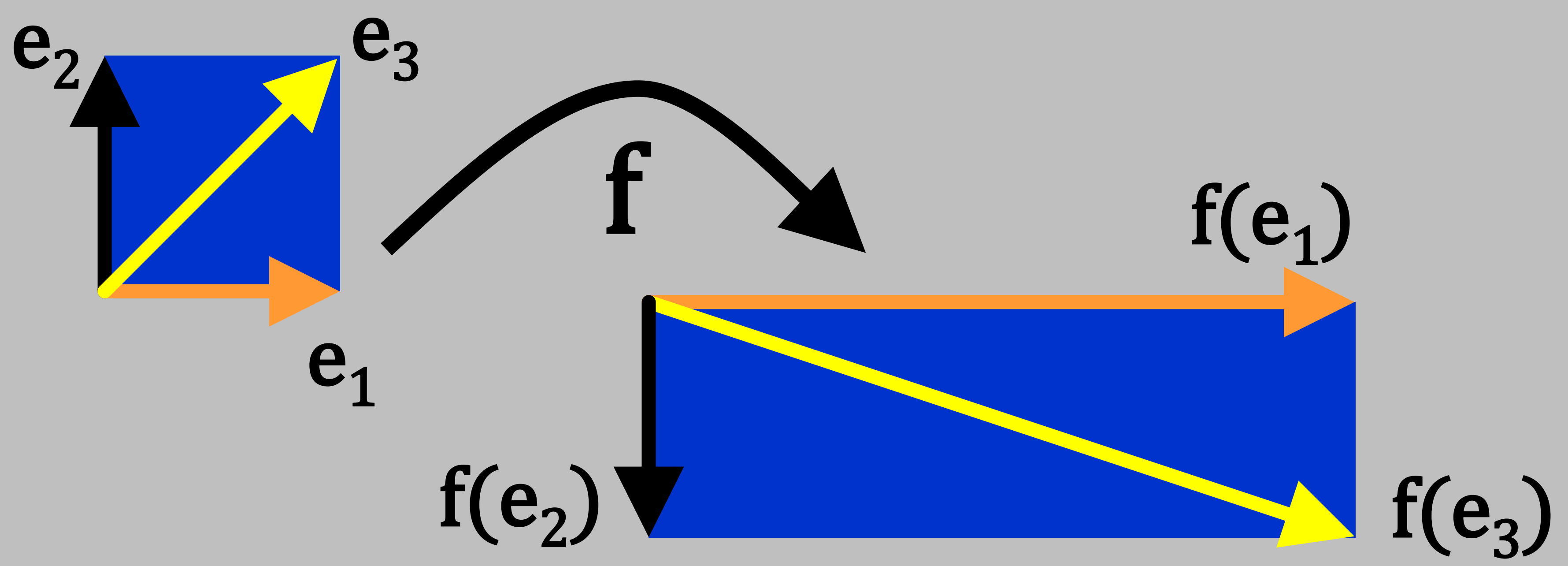

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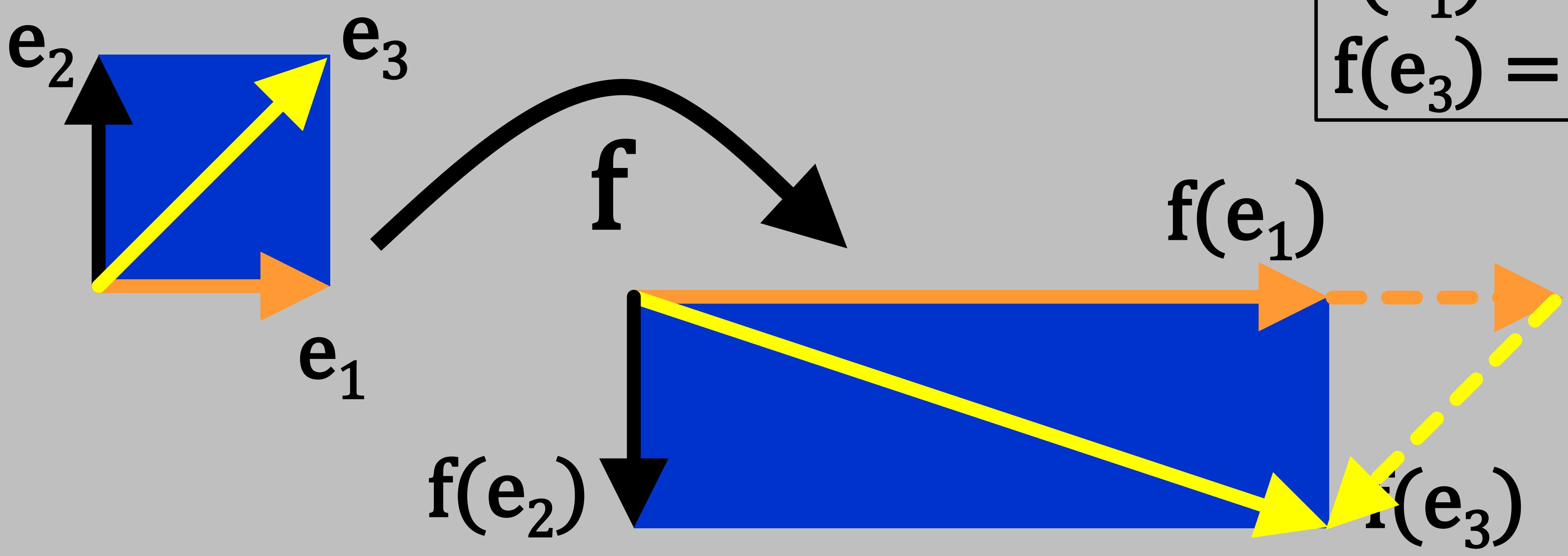

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