

*16-350*

*Planning Techniques for Robotics*

*Planning Representations:*

*Lattice-based Graphs, Explicit vs. Implicit Graphs*

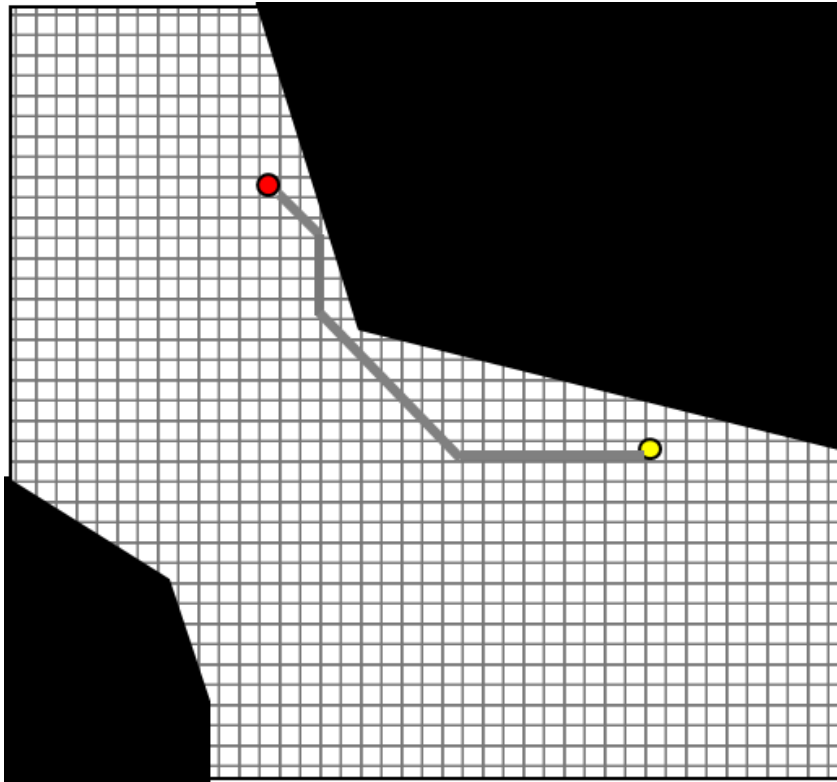
*Maxim Likhachev*

*Robotics Institute*

*Carnegie Mellon University*

# Beyond Planning for Omnidirectional Robots

*What's wrong with using  
Grid-based Graphs when planning  
for non-omnidirectional robots?*

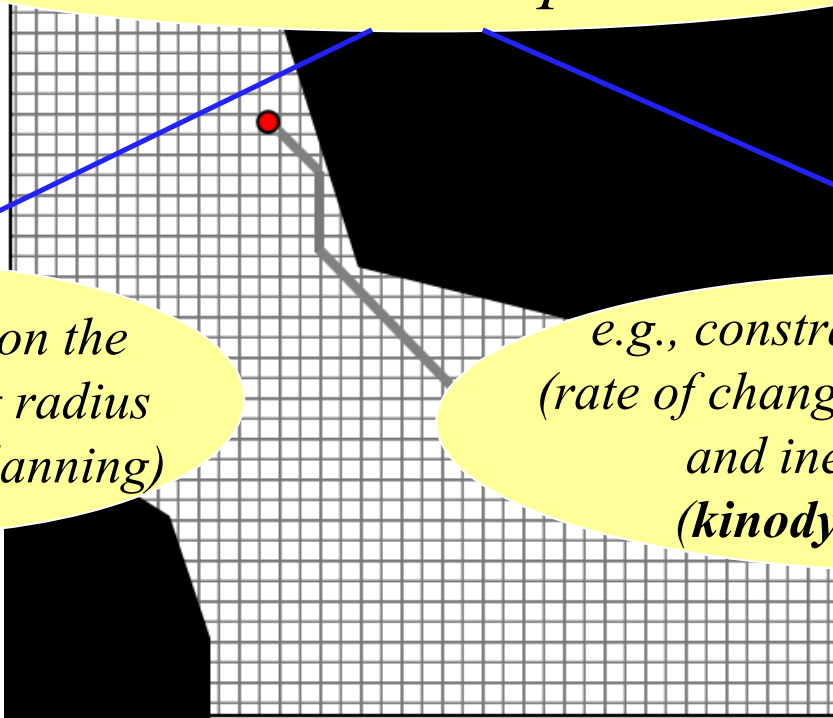


# Beyond Planning for Omnidirectional Robots

*What's wrong with using  
Grid-based Graphs when planning  
for non-omnidirectional robots?*



*“Can't turn in place”*



*e.g., constraints on the  
minimum turning radius  
(still **kinematic** planning)*

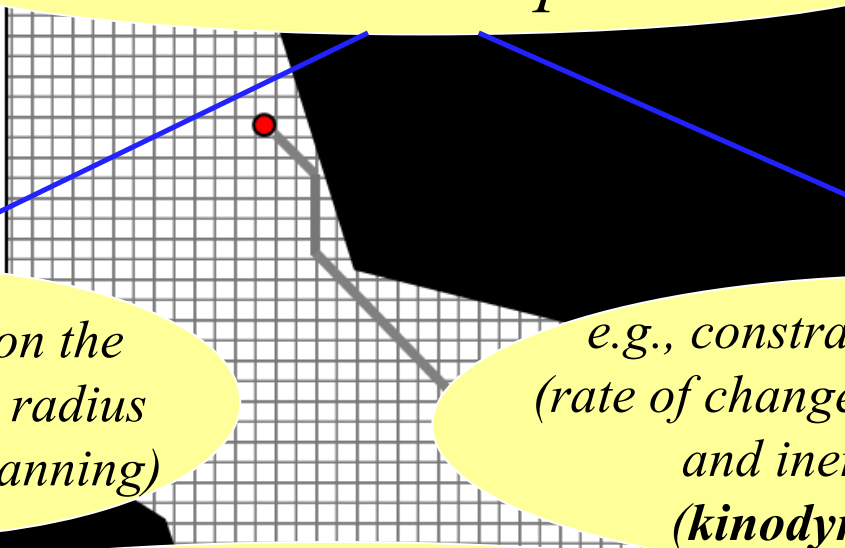
*e.g., constraints on turning rate  
(rate of change in wheel orientation)  
and inertial constraints  
(**kinodynamic** planning)*

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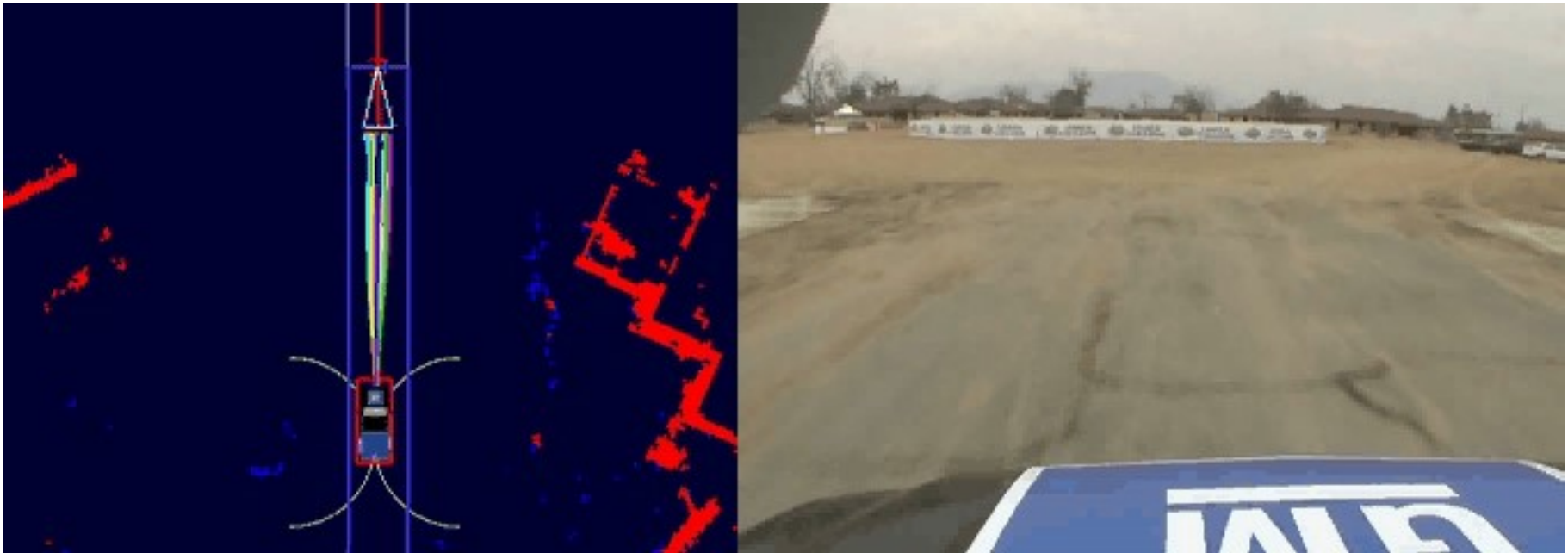
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(still **kinematic** planning)*

*e.g., constraints on turning rate  
(rate of change in wheel orientation)  
and inertial constraints  
(**kinodynamic** planning)*

***Kinodynamic planning:***  
*Planning representation includes  $\{X, \dot{X}\}$ ,  
where  $X$ -configuration and  $\dot{X}$ -derivative of  $X$  (dynamics of  $X$ )*

# Beyond Planning for Omnidirectional Robots

*$(x, y, \theta, v)$  planning  
with Anytime  $D^*$  (Anytime Incremental  $A^*$ ) on Lattice Graphs*



# Beyond Planning for Omnidirectional Robots

*$(x, y, \Theta)$  planning with ARA\*-based algorithm on Lattice Graphs*



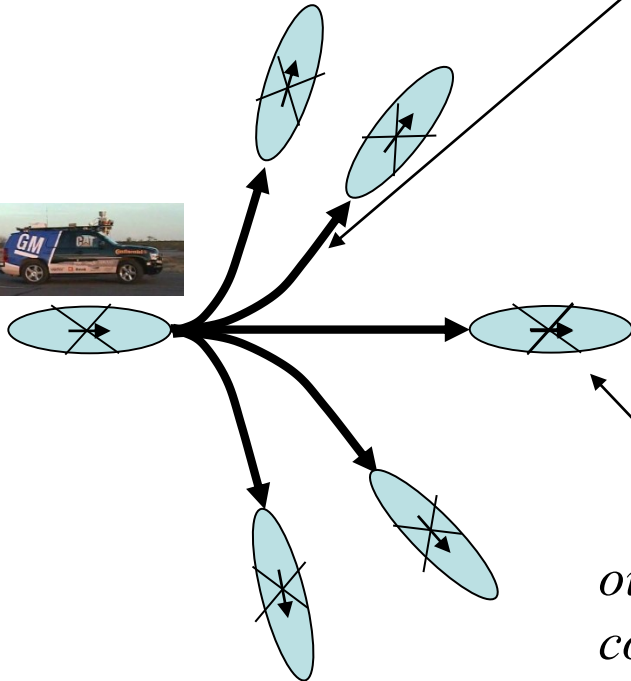
*Joint work with V. Kumar (Upenn), I. Kaminer (NPS) and V. Dobrokhodov (NPS)  
[thakur et al., '13]*

# Lattice Graphs

- Graph  $\{V, E\}$  where
  - $V$ : centers of the grid-cells
  - $E$ : motion primitives that connect centers of cells via short-term **feasible** motions

*each transition is feasible  
(typically, constructed beforehand)*

*motion primitives*



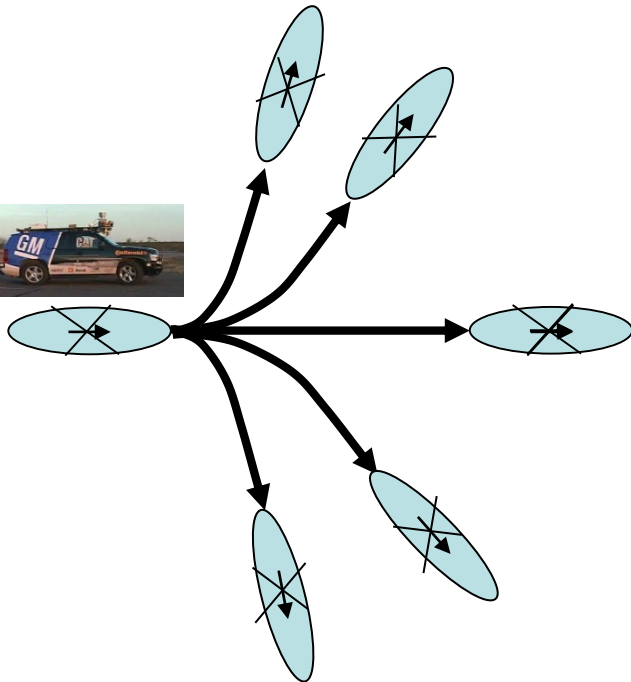
*outcome state is the center of the  
corresponding cell in a grid*

# Lattice Graphs

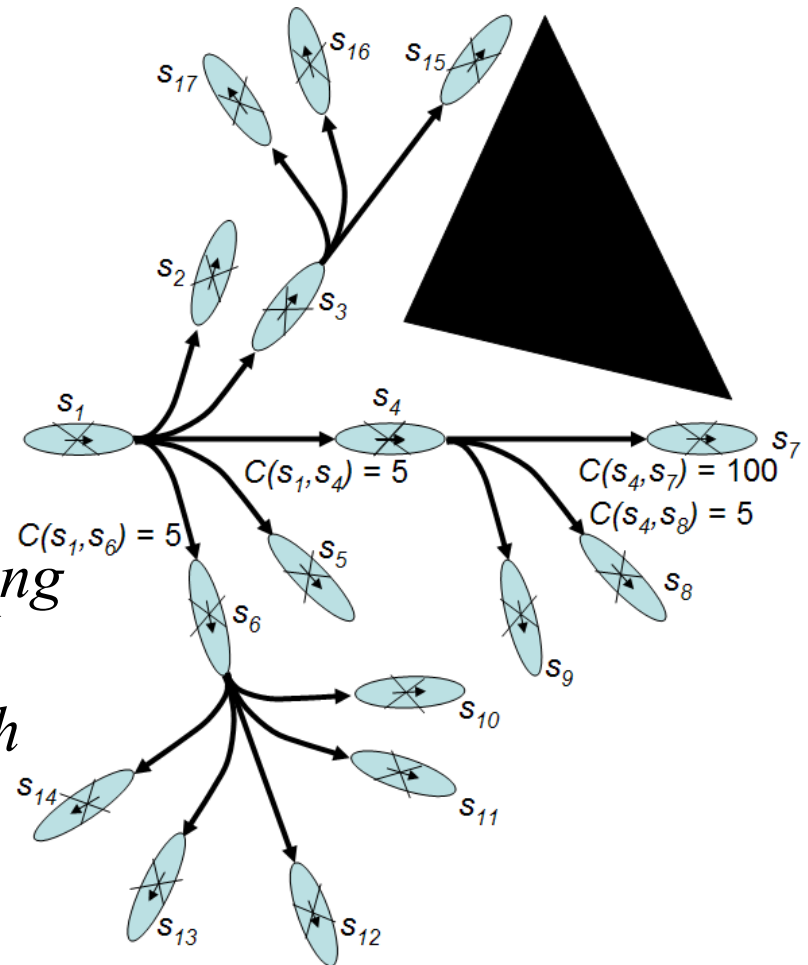
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*motion primitives*



*replicate it  
during planning  
to generate  
lattice graph*





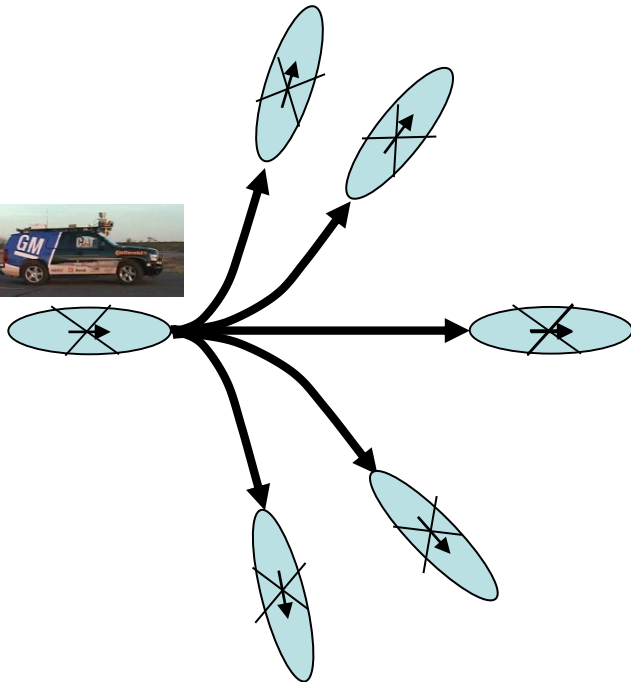
# Lattice Graphs

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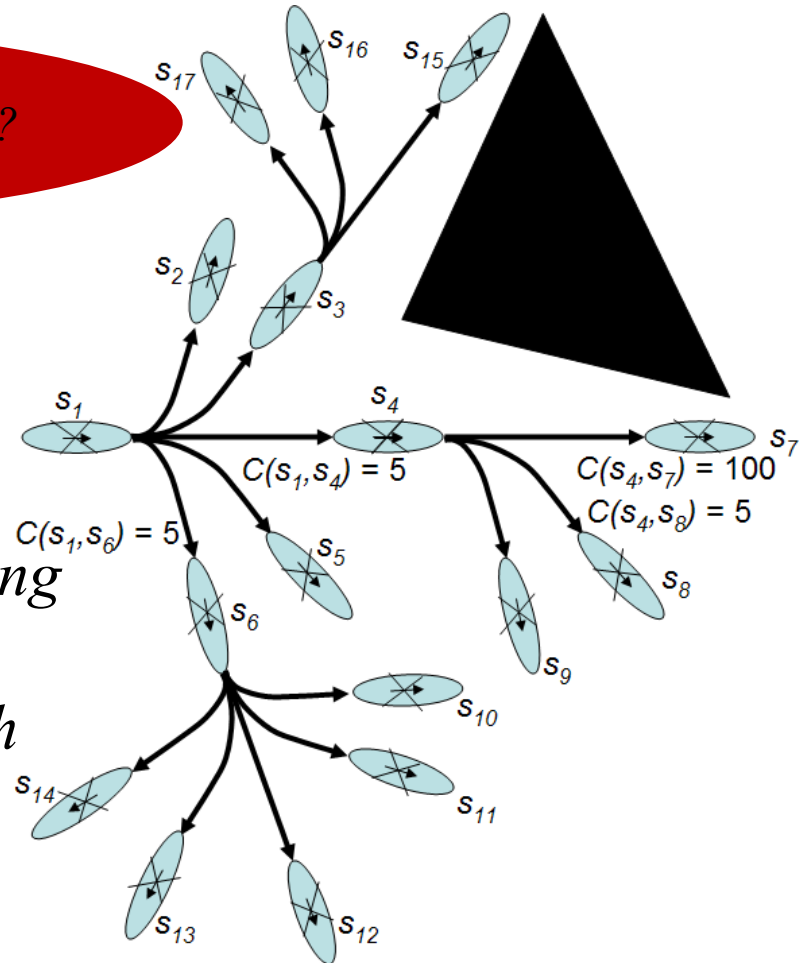
- $V$ : centers of the grid-cells
- $E$ : motion primitives that connect centers of cells via short-term **feasible** motions

*How do edgecosts get assigned?*

*motion primitives*



*replicate it  
during planning  
to generate  
lattice graph*



# Lattice Graphs

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- **Board example** for  $(x, y, \Theta)$  planning for a unicycle model (minimum turning radius)

# Planning as Graph Search Problem

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1. Construct a graph representing the planning problem
2. Search the graph for a (hopefully, close-to-optimal) path

The two steps above are often interleaved

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# Interleaving Search and Graph Construction

Graph Search using an **Explicit Graph** (allocated prior to the search itself):

1. *Create the graph  $G = \{V, E\}$  in-memory*
2. *Search the graph*

*Using Explicit Graphs  
is typical for low-D (i.e., 2D) problems in Robotics  
(with the exception of PRMs, covered in a later lecture)*

# Interleaving Search and Graph Construction

Graph Search using an **Implicit Graph** (allocated as needed by the search):

1. *Instantiate Start state*
2. *Start searching with the Start state using functions*
  - a) *Succs = GetSuccessors (State  $s$ , Action)*
  - b) *ComputeEdgeCost (State  $s$ , Action  $a$ , State  $s'$ )*

*and allocating memory for the generated states*

*Using Implicit Graphs  
is critical for most (>2D) problems  
in Robotics*

# Interleaving Search and Graph Construction

- **Board example** for deciding whether to use an Explicit graph or Implicit graph
- Planning for  $(x, y, \Theta, v)$  for
  - 20 by 20 m environment discretized into 25 cm cells with 8 heading  $\Theta$  values and 2 velocity  $v$  values for a point robot

*Is it feasible to use Explicit Graph  
(memory and pre-computation time reqs)?*

# Interleaving Search and Graph Construction

- **Board example** for deciding whether to use an Explicit graph or Implicit graph
- Planning for  $(x, y, \Theta, v)$  for
  - 200 by 200 m environment discretized into 25 cm cells with 16 heading  $\Theta$  values and 2 velocity  $v$  values for a real vehicle

*Is it feasible to use Explicit Graph  
(memory and pre-computation time reqs)?*



# What You Should Know...

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- What are Lattice graphs and how they get constructed
- Explicit vs. Implicit graphs and pros/cons of each