16-782
Planning & Decision-making in Robotics

Search Algorithms:
*Heuristic Functions, Multi-Heuristic A**

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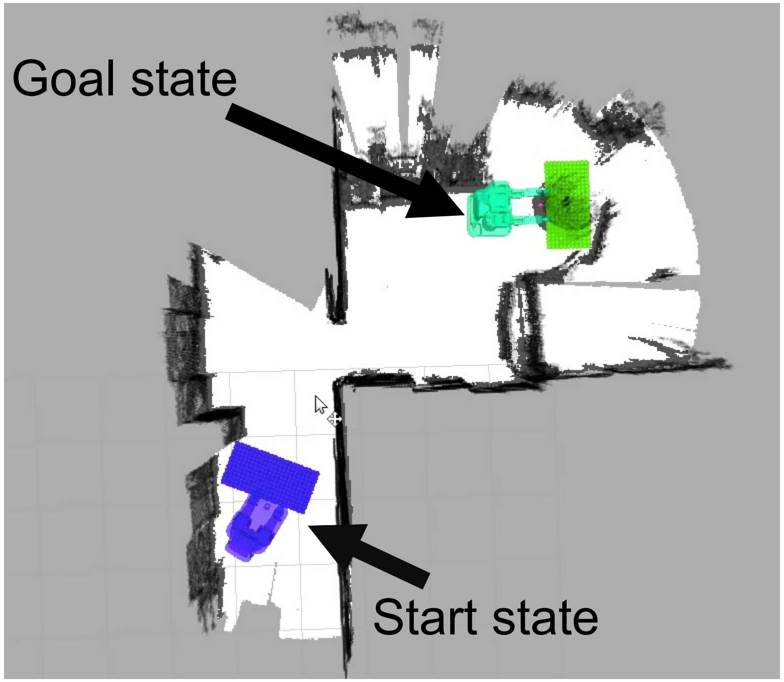
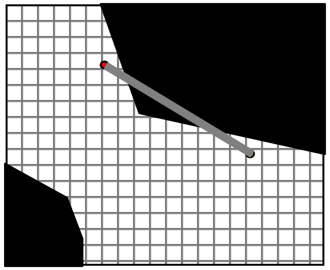
Design of Informative Heuristics

Example problem: *move picture frame on the table*



- Full-body planning
- 12 Dimensions *(3D base pose, 1D torso height, 6DOF object pose, 2 redundant DOFs in arms)*

Design of Informative Heuristics



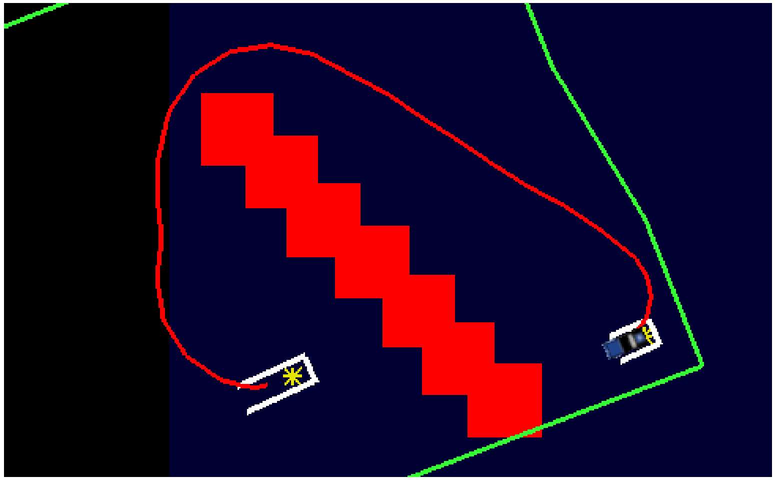
- For grid-based navigation:
  - Euclidean distance
  - Manhattan distance: $h(x,y) = abs(x-x_{goal}) + abs(y-y_{goal})$
  - Diagonal distance: $h(x,y) = max(abs(x-x_{goal}), abs(y-y_{goal}))$
  - More informed distances???

Which heuristics are admissible for
4-connected grid?
8-connected grid?

Design of Informative Heuristics

- For lattice-based 3D (x,y,Θ) navigation:

Any ideas?



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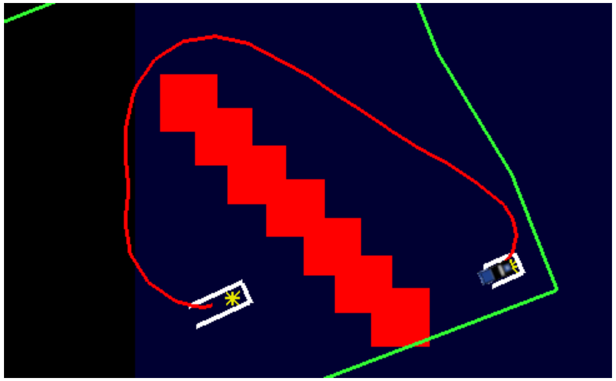
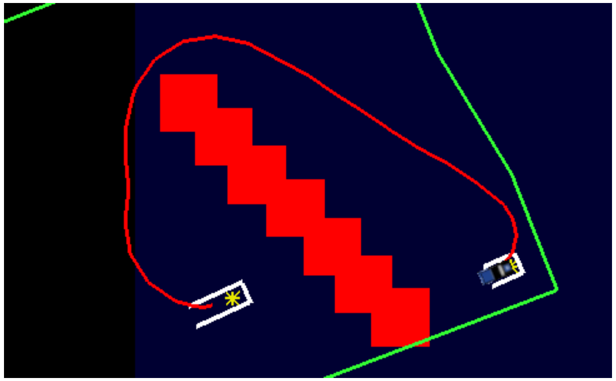


- For lattice-based 3D (x,y,Θ) navigation:
  - 2D (x,y) distance accounting for obstacles (single Dijkstra's on 2D grid cell starting at goalcell will give us these values)

Design of Informative Heuristics



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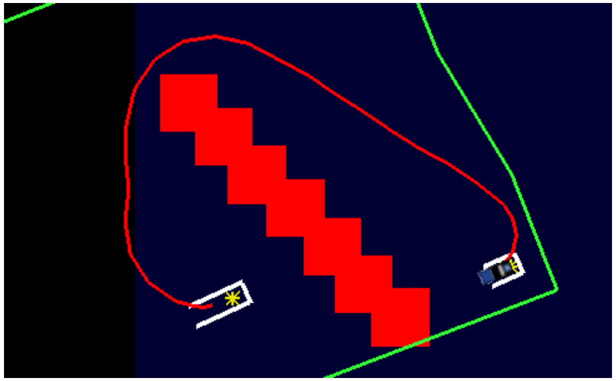

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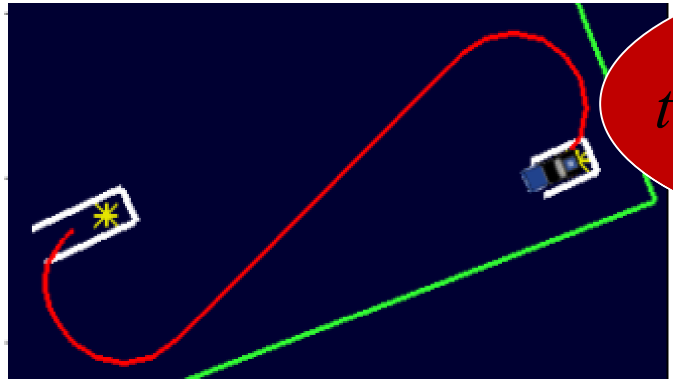
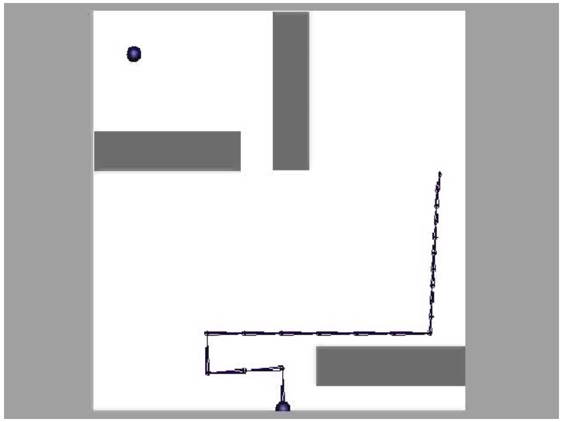


Any heuristic functions that will guide search well in this example?

Design of Informative Heuristics

- 20DoF Planar arm planning *(forget optimal A*, use weighted A*)*:



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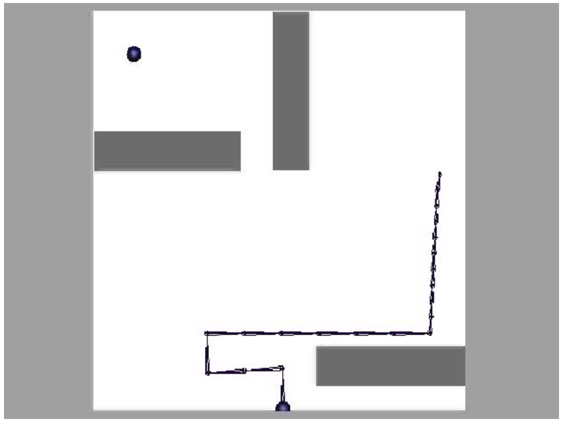

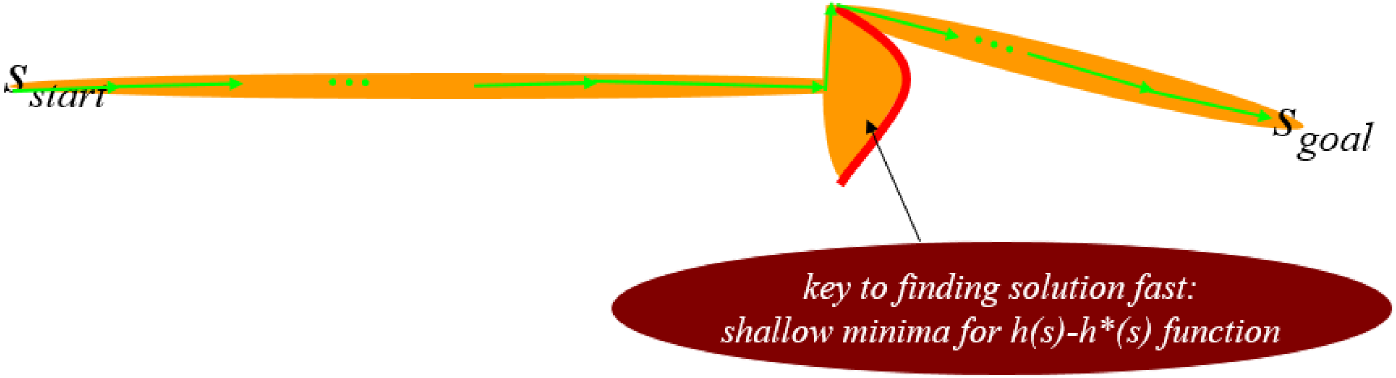

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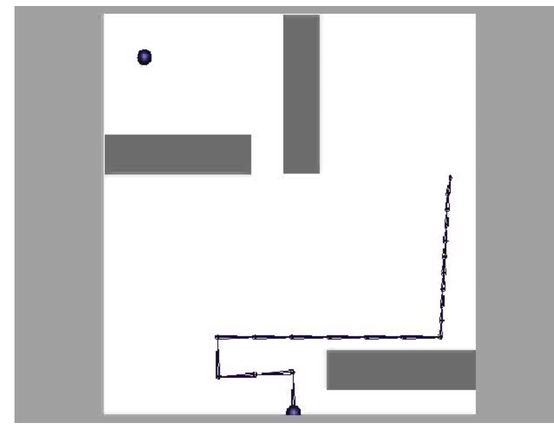

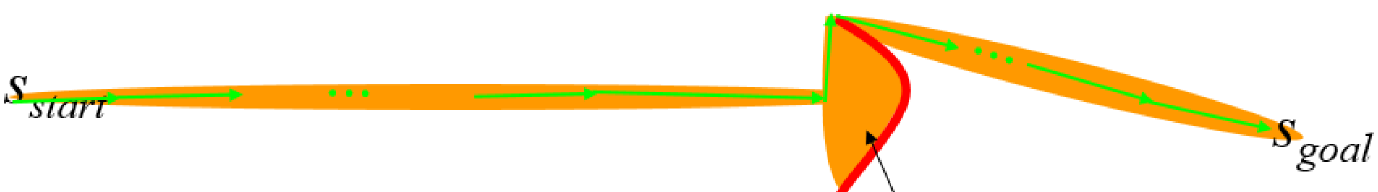

key to finding solution fast:
shallow minima for h(s)-h(s) function*

Design of Informative Heuristics

- 20DoF Planar arm planning *(forget optimal A*, use weighted A*)*:
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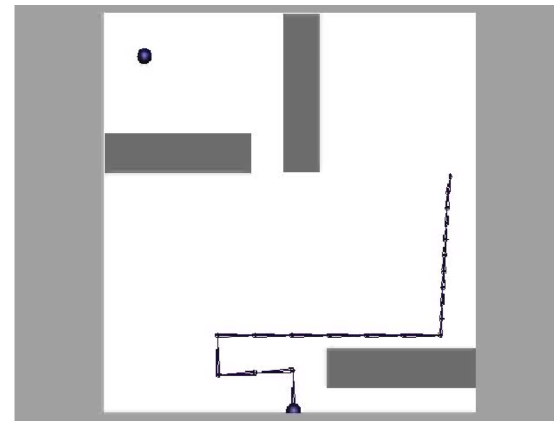

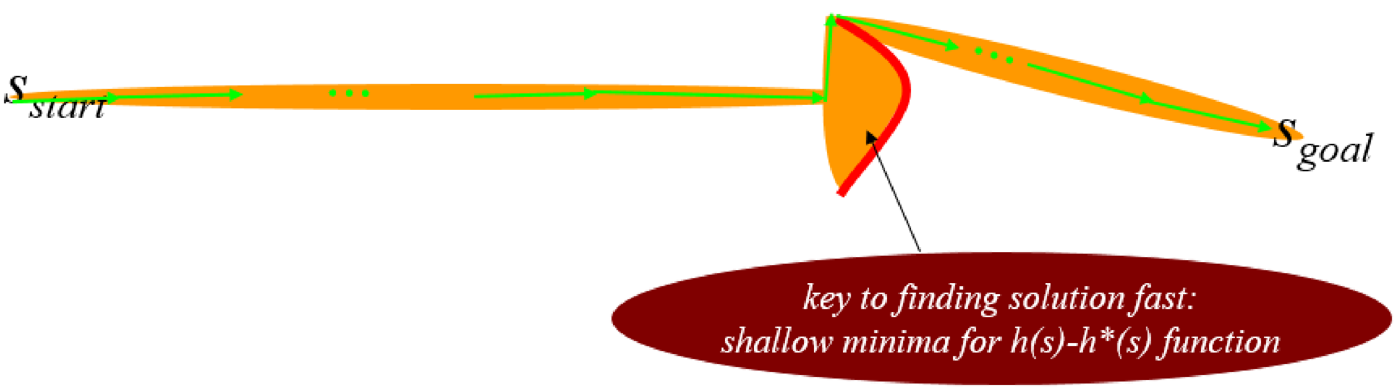

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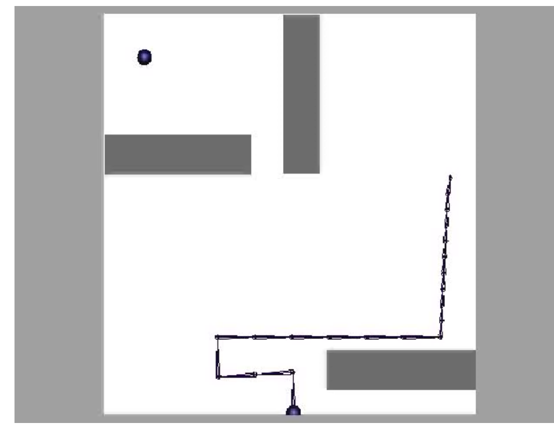
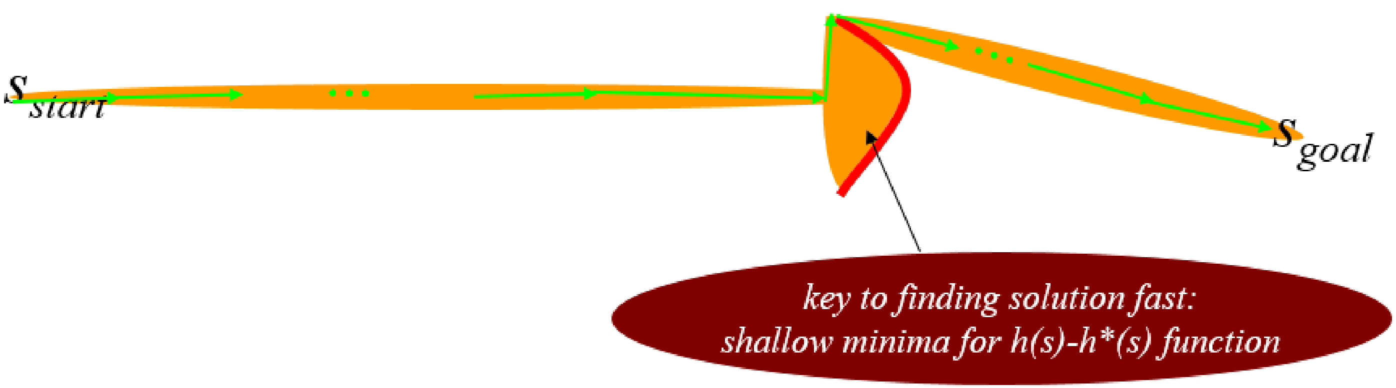
Example where it will miserably fail?

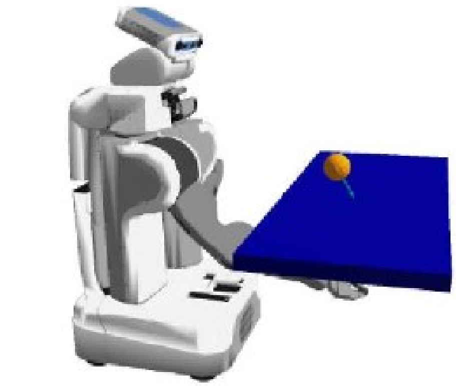


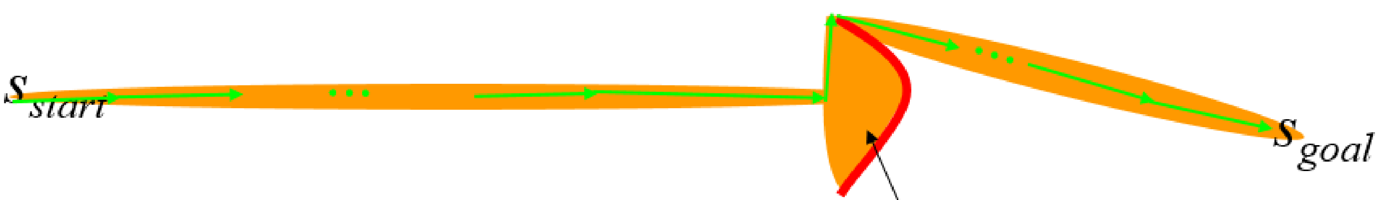


Design of Informative Heuristics

- Arm planning in 3D:

Any ideas?





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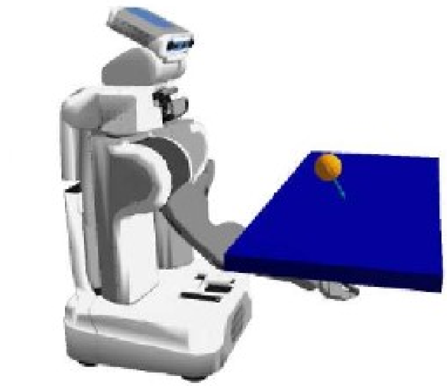
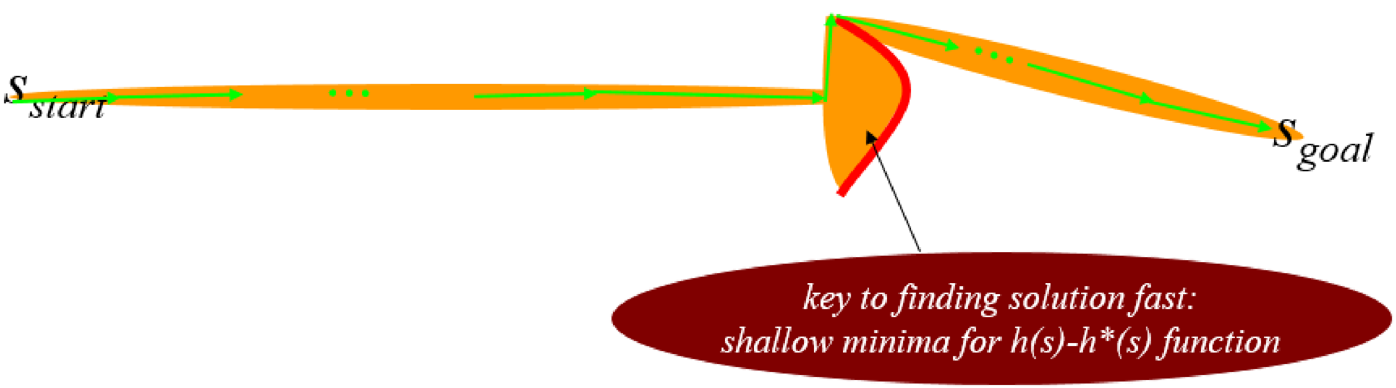


  – 3D (*x*,*y*,*z*) end-effector distance accounting for obstacles



Few Properties of Heuristic Functions

- Useful properties to know:

  - $h_1(s)$, $h_2(s)$ – consistent, then:

    $h(s) = max(h_1(s),h_2(s))$ – consistent

  - if A* uses ε-consistent heuristics:

    $h(s_{goal}) = 0$ *and* $h(s) \leq \varepsilon\ c(s,succ(s)) + h(succ(s)$ *for all* $s \neq s_{goal}$,

    then A* is ε-suboptimal:

    $cost(solution) \leq \varepsilon\ cost(optimal\ solution)$

  - weighted A* is A* with ε-consistent heuristics

  - $h_1(s)$, $h_2(s)$ – consistent, then:

    $h(s) = h_1(s)+h_2(s)$ – ε-consistent

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    Proof?

  - $h_1(s)$, $h_2(s)$ – consistent, then:

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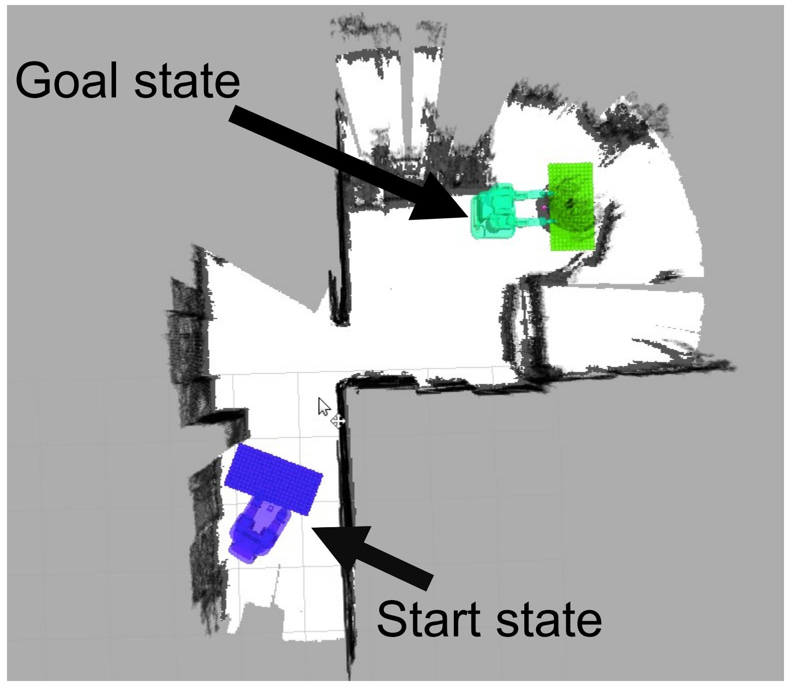
    What is ε? Proof?

Back to the Example

Example problem: *move picture frame on the table*



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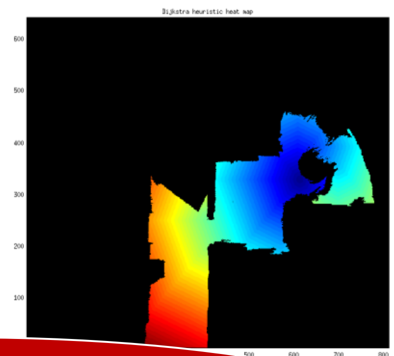
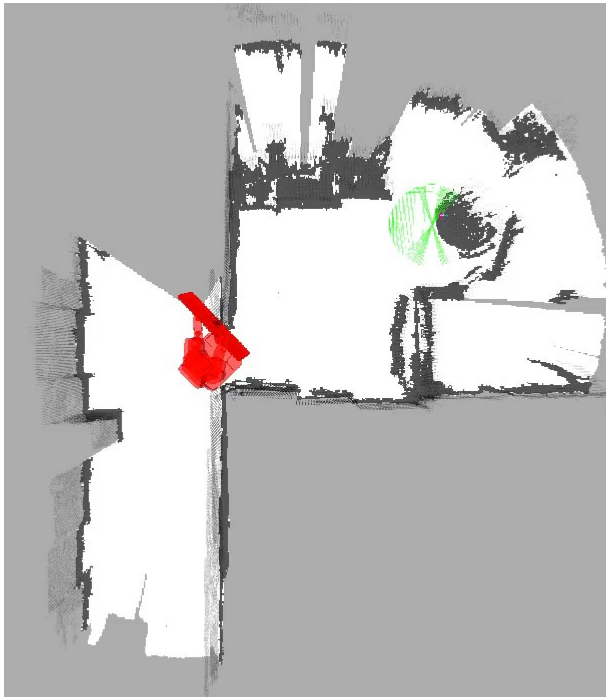
Back to the Example

Admissible and Consistent Heuristic

- h_0: base distance
  - 2D BFS from goal state



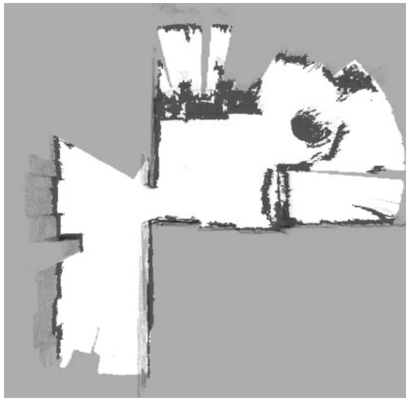


Do you think it will guide search well?

Any other ideas for good heuristics?

Inadmissible Heuristics

- ***h_1***: base distance + object orientation difference with goal



- ***h_2***: base distance + object orientation difference with vertical

Back to the Example

More generally: we can often easily generate N arbitrary heuristic functions that estimate costs-to-goal

Solutions to N lower-dimensional manifolds
Solutions to N problems with different constraints relaxed
….

- ***h₂***: base distance + object orientation difference with vertical

Utilizing Multiple Heuristic Functions

Can we utilize a bunch of inadmissible heuristics simultaneously, *leveraging their individual strengths while preserving guarantees on completeness and bounded sub-optimality?*

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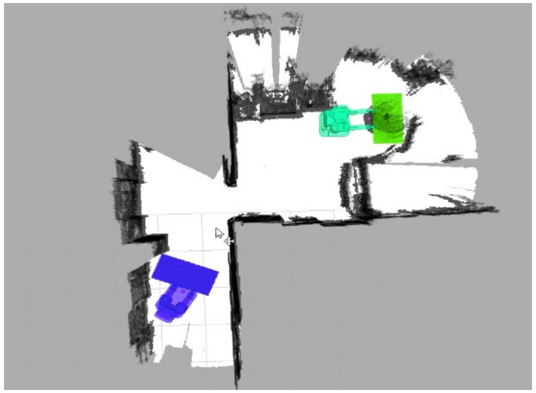
Can we utilize a bunch of inadmissible heuristics simultaneously, *leveraging their individual strengths while preserving guarantees on completeness and bounded sub-optimality?*

Combining multiple heuristics into one (e.g., taking max) is often inadequate



- *information is lost*
- *creates local minima*
- *requires all heuristics to be admissible*

Multi-Heuristics A*: version 1

- Given N inadmissible heuristics
- Run N independent searches
- Hope one of them reaches goal

Within the while loop of the ComputePath function:

for i=1…N

  remove s with the smallest $[f(s) = g(s)+w_1*h(s)]$ *from* $OPEN_i$ *;*

  expand s;

Inad. Search 1	Inad. Search 2	Inad. Search 3
priority queue: $OPEN_1$ key = $g + w_1*h_1$	priority queue: $OPEN_2$ key = $g + w_1*h_2$	priority queue: $OPEN_3$ key = $g + w_1*h_3$

Multi-Heuristics A*: version 1

- Given N inadmissible heuristics
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Problems:

- Each search has its own local minima
- N times more work
- No completeness guarantees or bounds on solution quality

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Multi-Heuristics A*: version 2

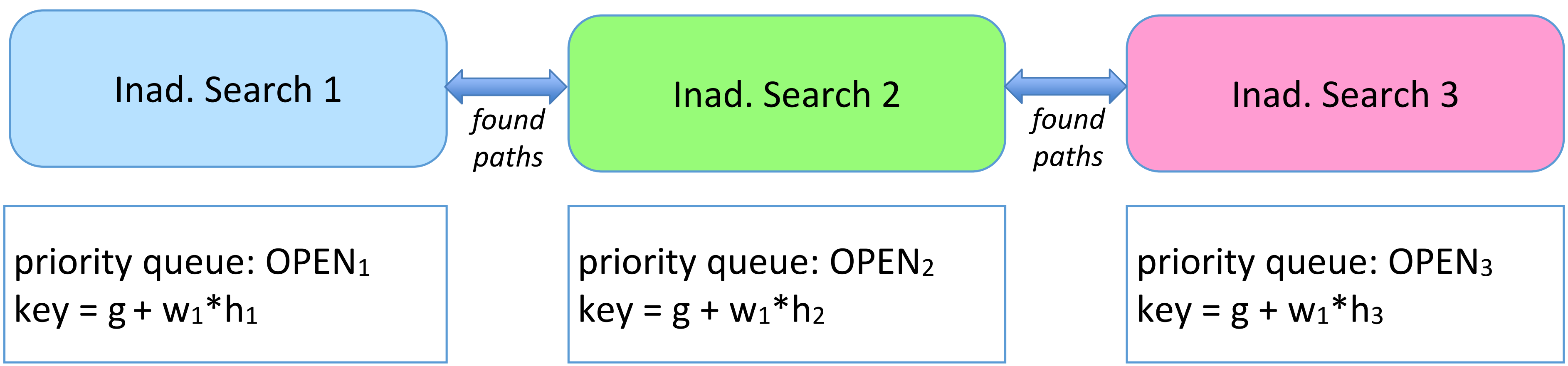
- Given N inadmissible heuristics
- Run N independent searches
- Hope one of them reaches goal
- Key Idea #1: Share information (g-values) between searches!

Within the while loop of the ComputePath function (note: CLOSED is shared):

for i=1…N

*remove s with the smallest [f(s) = g(s)+w_1*h(s)] from OPEN$_i$;*

expand s ***and also insert/update its successors into all other OPEN lists****;*



Multi-Heuristics A*: version 2

- Given N inadmissible heuristics
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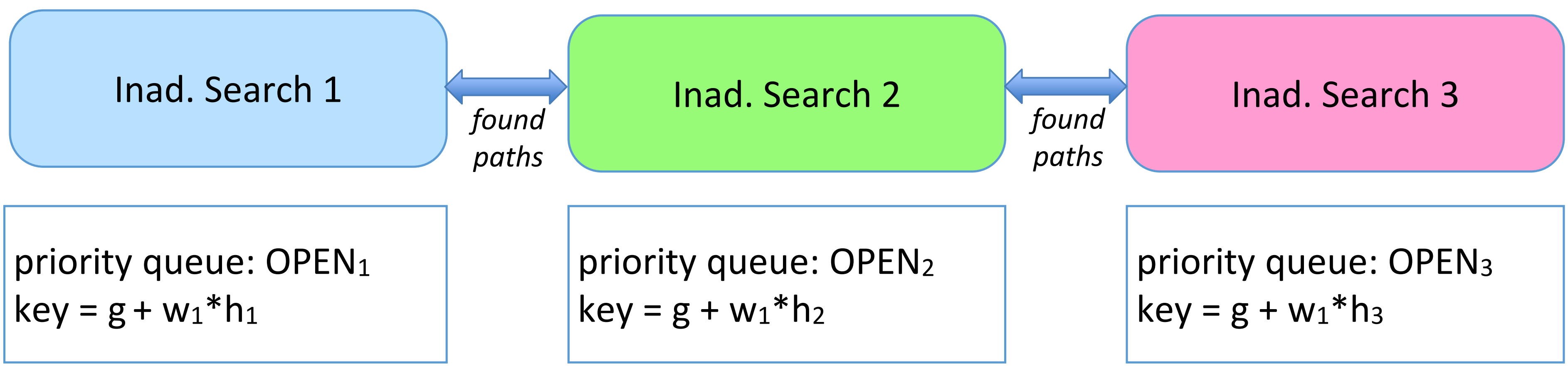
Benefits:

- Searches help each other to circumvent local minima
- States are expanded at most once across ALL searches

Remaining Problem:

- No completeness guarantees or bounds on solution quality



Multi-Heuristics A* [Aine et al.,'14]

- Given N inadmissible heuristics
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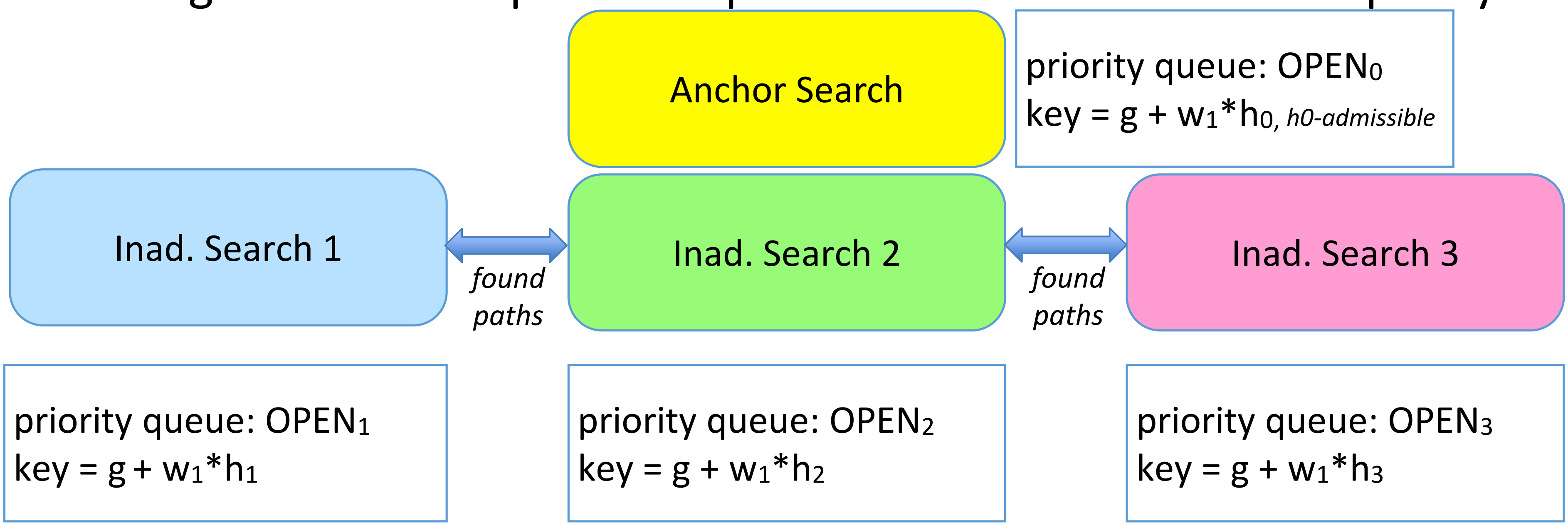
- Key Idea #1: Share information (g-values) between searches!
- Key Idea #2: Search with admissible heuristics controls expansions

Benefits:

- Algorithm is complete and provides bounds on solution quality



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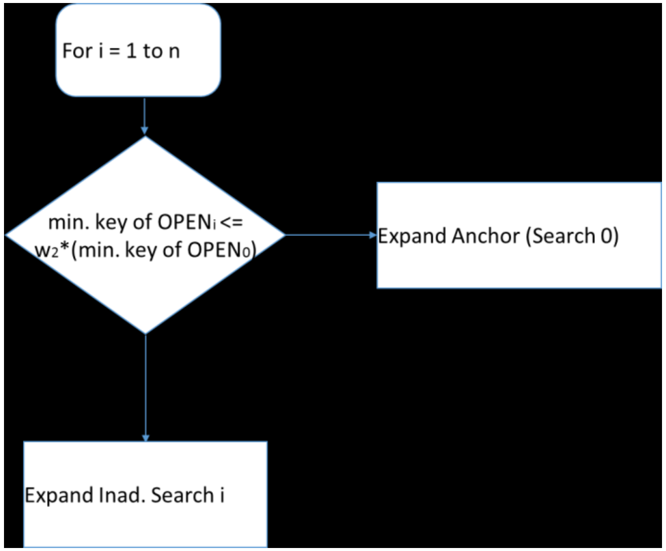

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Within the while loop of the ComputePath function

(note: CLOSED is shared among searches 1…N. Search 0 has its own CLOSED):

for i=1…N

if(min. f-value in $OPEN_i \leq w_2$ ** min. f-value in* $OPEN_0$*)*

remove s with the smallest $[f(s) = g(s)+w_1 * h_i(s)]$ *from* $OPEN_i$ *;*

expand s and also insert/update its successors into all other OPEN lists;

else

remove s with the smallest $[f(s) = g(s)+w_1 * h_0(s)]$ *from* $OPEN_0$ *;*

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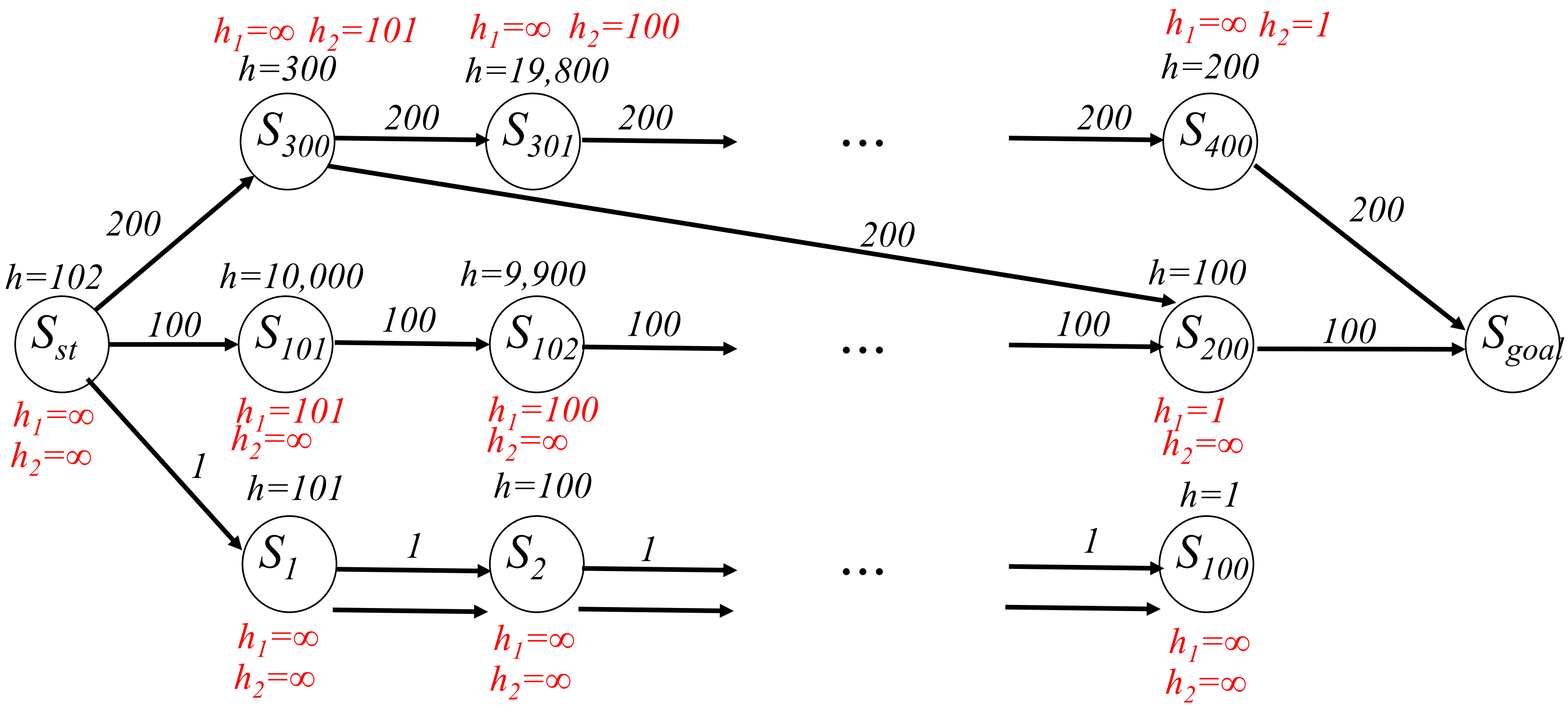

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Benefits:

- Algorithm is c

Theorem 1: *min. key of* $OPEN_0$ <= w_1**optimal solution cost*

Theorem 2: *min. key of* $OPEN_i$ <= w_2*w_1**optimal solution cost*

Theorem 3: *The algorithm is complete and the cost of the found solution is no more than* w_2*w_1**optimal solution cost*

Theorem 4: *Each state is expanded at most twice: at most once by one of the inadmissible searches and at most once by the Anchor search*

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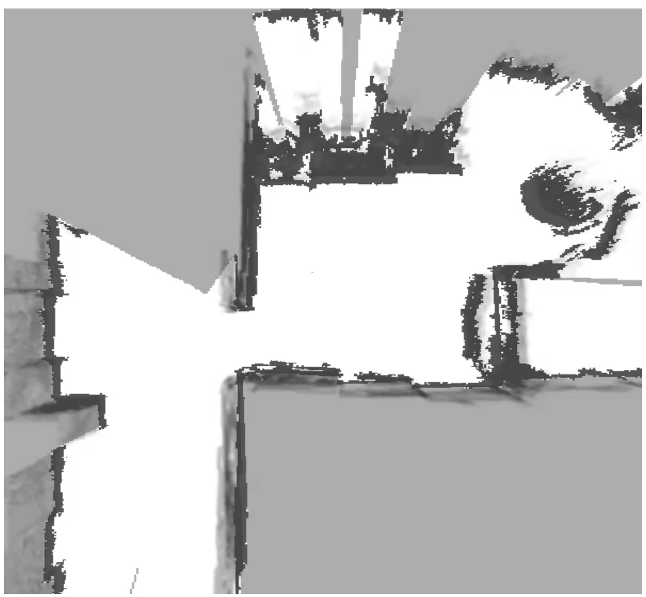

What You Should Know…

- Examples of heuristic functions
  - for X-connected grids
  - For higher dimensional planning problems derived by lower-dimensional search
- Be able to come up with a good heuristic function for a given problem
- Properties of heuristic functions
- How Multi-heuristic A* works