16-782
Planning & Decision-making in Robotics

Multi-Robot Planning

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Different Categorizations of Multi-Robot Planning

• Centralized vs. Decentralized
  
  – **Centralized**: one central control of (planning for) all the robots
  
  – **Decentralized**: each robot decides/plans what to do on its own
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*Why do we need decentralized planning approaches?*
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Why do we need decentralized planning approaches?

Robust to limits on or loss of communication

Robust to losing some robots in the team

Computationally more scalable
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**Challenges with decentralized planning?**
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*Why do we need decentralized planning approaches?*

*Robust to limits on or loss of communication*

*Robust to loosing some robots in the team*

*Computationally more scalable*

*Challenges with decentralized planning?*

*How to guarantee that the overall team accomplishes its goal?*
Different Categorizations of Multi-Robot Planning

- Multi-robot Path Planning vs. Multi-robot Cooperative Task Planning

  - **Multi-robot Path Planning**: how to plan paths for $N$ robots so that they don’t collide with each other during execution

  - **Multi-robot Cooperative Task Planning**: how to compute plans for $N$ robots so that they achieve the overall goal that may require cooperation
Different Categorizations of Multi-Robot Planning

• Small teams vs. large teams (swarms) of robots

  – **Planning for small teams**: Compute plans for $N$ (potentially heterogeneous) robots, where $N$ is typically 2-10

  – **Planning for (control of) swarms of robots**: how to control a swarm of $N$ (usually homogeneous) robots, where $N$ is typically 10-1000
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*Control of swarms is typically decentralized*
Different Categorizations of Multi-Robot Planning

- Joint state-space vs. distributed planning (within centralized)
  
  - **Joint state-space planning:** Planning for \( N \) robots in a state-space that represents joint configurations of robots
  
  - **Distributed planning:** Planning is split into \( N \) individual planners that share their results (and potentially re-plan) to obtain a final plan for all \( N \) robots

What Planning approach to take?

- **Centralized Planning**
  - Joint state-space Planning
  - Distributed Planning

- **Decentralized Planning**
Different Categorizations of Multi-Robot Planning

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What Planning approach to take?

- **Centralized Planning**
  - $N$ is small (e.g., $< 10$) and inter-robot communications is available
  - $N$ is very small (e.g., $< 3$)

- **Joint state-space Planning**
  - $N$ is not very small (e.g., $\geq 3$)

- **Distributed Planning**
  - $N$ is large (swarms) or communications are limited/unavailable
Multi-Robot Path Planning

- Path planning for \( N \) robots to get to their goals w/o collisions

**simple example for two omnidirectional point-size robots**
Multi-Robot Path Planning

- Path planning for $N$ robots to get to their goals w/o collisions

Simple example for two omnidirectional point-size robots

Any examples of this in industry?
Multi-Robot Path Planning

• Path planning for $N$ robots to get to their goals w/o collisions

Joint state-space planning
Multi-Robot Path Planning

• Path planning for $N$ robots to get to their goals w/o collisions

Joint state-space planning

The simplest approach: construct and search a graph, where each state encodes positions of all the robots and each action encodes all possible movements
Multi-Robot Path Planning

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Joint state-space planning

- $R_2 = A1, R_1 = A3$
  - $R_2$ moves east, $R_1$ moves east
- $R_2 = B1, R_1 = B3$
- $R_2 = B1, R_1 = A2$
  - $R_2$ moves east, $R_1$ moves north

... goal state
Multi-Robot Path Planning

- Path planning for $N$ robots to get to their goals w/o collisions

Assuming 4-connected grid, what is the maximum branching factor (how many actions/successors)?

Joint state-space planning

```
R2=A1
R1=A3

R2 moves east, R1 moves east

R2=B1
R1=B3

R2=B1
R1=A2

```

\[ \cdots \]

\[ \cdots \]

\[ \begin{array}{cccccc}
A & B & C & D & E & F \\
1 & & & & R2 & G1 \\
2 & & & & & \\
3 & R1 & & & G2 & \\
4 & & & & & \\
\end{array} \]

\[ \text{goal state} \]

Maxim Likhachev

Carnegie Mellon University
Multi-Robot Path Planning

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Joint state-space planning

What is the size of the graph?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>R2</td>
<td></td>
<td>G1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>R1</td>
<td></td>
<td></td>
<td>G2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

goal state

$R2 = A1$
$R1 = A3$

$R2 = B1$
$R1 = B3$

$R2 = B1$
$R1 = A2$

$R2$ moves east, $R1$ moves north

$R2$ moves east, $R1$ moves east

$\cdots$

$R2 = F3$
$R1 = F1$
Multi-Robot Path Planning

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Joint state-space planning

What is the size of the graph?

Scalability w.r.t. $N$ is clearly an issue!
Multi-Robot Path Planning

• Path planning for $N$ robots to get to their goals w/o collisions

Distributed planning

One popular approach: Prioritized Planning

For $i = 1:N$

Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1..R_{i-1}$
Multi-Robot Path Planning

- Path planning for \( N \) robots to get to their goals w/o collisions

**Distributed planning**

Each planning needs to include time as a dimension!

One popular approach: Prioritized Planning

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What will be the plan returned by Prioritized Planning?

One popular approach: Prioritized Planning

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Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1..R_{i-1}$
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Distributed planning

Is it complete?

Is it optimal?

What is the complexity of Prioritized Planning?

One popular approach: Prioritized Planning

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Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1..R_{i-1}$
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Distributed planning

**complete & optimal** approach

Conflict-based Search (CBS)

$C(S_{\text{start}}) = \{\};$ //no constraints on paths at the start state

$OPEN = \{S_{\text{start}}\}; \ g(S_{\text{start}}) = 0;$

while ($OPEN \neq \emptyset$)

- remove $S$ with the smallest $g$-value from $OPEN$
- if no collisions between paths associated with $S$, then return them as the overall plan
- for each (or at least one) collision of robots $R_i$ and $R_j$ at vertex $v$ at time $t$
  - $C_1 = C(S) \cup \{R_i(t) \neq v\}$ //$R_i$ can’t be at vertex $v$ at time $t$
  - compute $N$ paths that satisfy $C_1$ constraints (w/o collision-checking between paths) and the overall cost $\text{PLANCOST}_1$
  - $C_2 = C(S) \cup \{R_j(t) \neq v\}$ //Rj can’t be at vertex v at time t
  - compute $N$ paths that satisfy $C_2$ constraints (w/o collision-checking between paths) and the overall cost $\text{PLANCOST}_2$

insert state $S_1$ with $C(S_1) = C_1$ and $g(S_1) = \text{PLANCOST}_1$
insert state $S_2$ with $C(S_2) = C_2$ and $g(S_2) = \text{PLANCOST}_2$
Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for $N$ robotic arms to move an object

[Cohen et al., ’14]
(performs joint state-space planning)
Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for $N$ robotic arms to move an object

(planning is distributed: plan on Roman platform first, then on PR2)
Multi-Robot Cooperative Planning/Task Allocation

• Example: planning for multi-robot exploration/mapping

\[ N \text{ robots need to explore and build a map of unknown environment} \]

One approach: Distributed Greedy Mapping

For \( i = 1:N \)

Compute a path using Greedy Mapping approach for robot \( R_i \) taking into account what paths were computed for \( R_1..R_{i-1} \) (and what cells they would see)
Multi-Robot Cooperative Planning/Task Allocation

• Example: planning for multi-robot exploration/mapping

\[ N \text{ robots need to explore and build a map of unknown environment} \]

• Greedy Mapping for a single robot:
  – always move the robot on a shortest path to the closest unobserved (or unvisited) cell
  – it always achieves a gain in information.
  – thus, it is guaranteed to map the environment that is reachable (assuming all moves are reversible)

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[Butzke et al., '11]
Multi-Robot Cooperative Planning/Task Allocation

• Example: planning for multi-robot exploration/mapping

\[ N \text{ robots need to explore and build a map of unknown environment} \]

\[ A \text{ Planning Framework for Persistent, Multi-UAV Coverage with Global Deconfliction} \]

Submitted to the 12th Conference on Field and Service Robotics

Collaboration between
Search-Based Planning Lab, CMU (headed by M. Likhachev) and Mitsubishi Heavy Industries (MHI)

[Kusner et al., '19]

One approach: Distributed Greedy Mapping

For \( i = 1 : N \)

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Multi-Robot Cooperative Planning/Task Allocation

- Market-based approach (very popular distributed approach)
  - Consider planning the allocation of tasks to $N$ robots
  - General scheme: \textit{robots auction out their tasks to their teammates with the goal of increasing their own revenue}
Market-based Approach

Given $N$ robots $R_1 \ldots R_N$, $M$ tasks $T_1 \ldots T_M$, and $C_i^{R_j}$ – cost of executing task $i$ by robot $R_j$ (cost may depend on other tasks executed by this robot)

Planner needs to decide: Which task gets executed by which robot?

Find a plan (mapping) $\pi^*$: $T_i \rightarrow R_j$ such that $\pi^* = \arg\min \sum C_i^{\pi(T_i)}$
Market-based Approach

Given $N$ robots $R_1...R_N$, $M$ tasks $T_1...T_M$, and $C_{iR_j}$ – cost of executing task $i$ by robot $R_j$ (cost may depend on other tasks executed by this robot)

**Iterate over steps 1-4 until convergence or planning time expires**

Step 1: start with an arbitrary plan $\pi$

Step 2: all robots offer their tasks $T_i$ at auction at the max. price of $C_{Ti}^{\pi(Ti)} - \epsilon$

Step 3: all robots $R_j$ bid on the offered tasks $T_i$ with the bid $= C_{iR_j} + \epsilon$

Step 4: robots sell to the lowest bidders if they are below max. price and get profit: $C_{Ti}^{\pi(Ti)} - C_{iR_j} - \epsilon$
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When does it converge in one iteration?
Planning for Leader-based Coordination

- **Fully decentralized approach** (doesn’t rely on the presence of communication between robots)
- Plan for the “leader” robot (sometimes leader can be just a centroid of the team or some other reference point)
- All other robots execute either “follow the leader” or “follow neighbors within field-of-view” behaviors while avoiding collisions
What You Should Know…

• Different styles of multi-robot planning
  – Centralized vs. decentralized
  – Joint state-space planning vs. distributed planning
  – Multi-robot path planning vs. cooperative task planning

• Prioritized Multi-robot Path Planning

• Market-based Approach to multi-robot planning