# *16-350*
# *Planning Techniques for Robotics*

# *Planning Representations: Explicit vs. Implicit Graphs Skeletonization-, Grid- and Lattice-based Graphs*

*Maxim Likhachev*

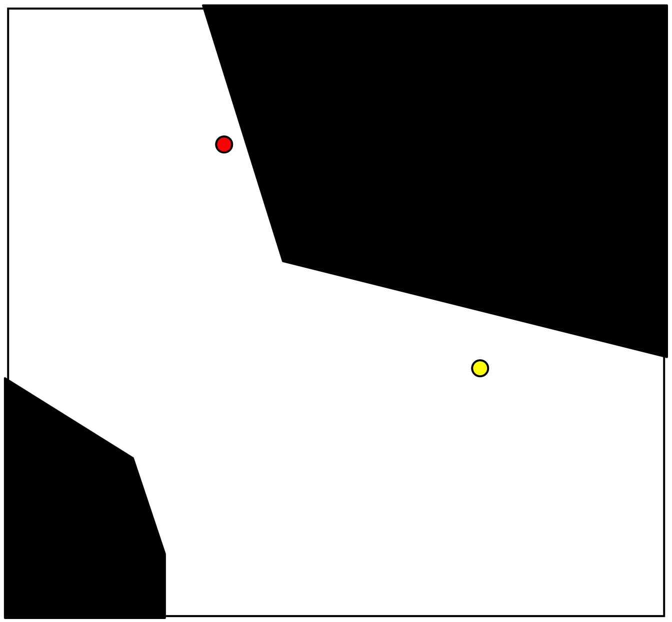
*Robotics Institute*

*Carnegie Mellon University*

# 2D Planning for Omnidirectional Point Robot

Planning for omnidirectional point robot:

*What is* $M^R$ = <x,y>
*What is* $M^W$ = <obstacle/free space>
*What is* $s^R_{current}$ = $<x_{current}, y_{current}>$
*What is* $s^W_{current}$ = constant
*What is* $C$ = Euclidean Distance
*What is* $G$ = $<x_{goal}, y_{goal}>$



# Planning as Graph Search Problem

1. Construct a graph representing the planning problem

2. Search the graph for a (hopefully, close-to-optimal) path

The two steps above are often interleaved

# Planning as Graph Search Problem

1. Construct a graph representing the planning problem

   *This class*

2. Search the graph for a (hopefully, close-to-optimal) path

   *Next lecture*

The two steps above are often interleaved

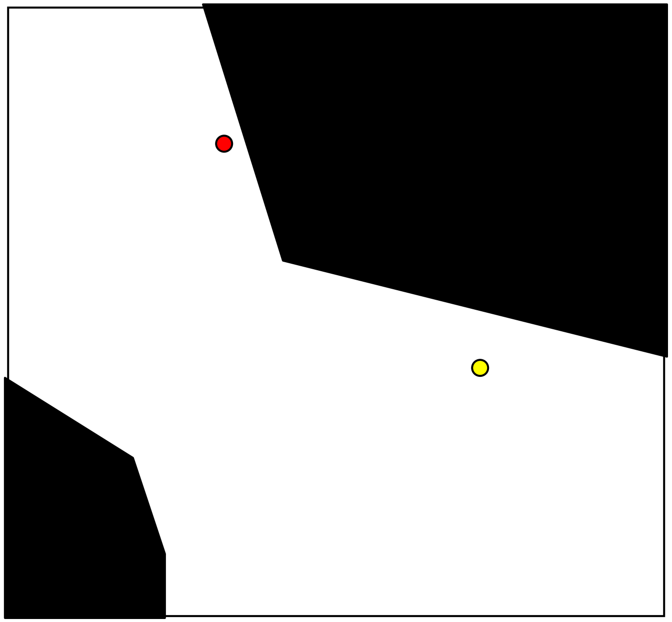
*More on this in this & later classes*

# 2D Planning for Omnidirectional Point Robot

Planning for omnidirectional point robot:

*What is* $M^R = <x,y>$
*What is* $M^W$ = *<obstacle/free space>*
*What is* $s^R_{current} = <x_{current}, y_{current}>$
*What is* $s^W_{current}$ = *constant*
*What is C = Euclidean Distance*
*What is* $G = <x_{goal}, y_{goal}>$

*Any ideas on how to construct a graph for planning?*



# Two Classes of Graph Construction Methods

- Skeletonization

  -Visibility graphs

  -Voronoi diagrams

  - Probabilistic roadmaps

- Cell decomposition

  - X-connected grids

  - lattice-based graphs

# Two Classes of Graph Construction Methods

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*Will be covered in a later class*

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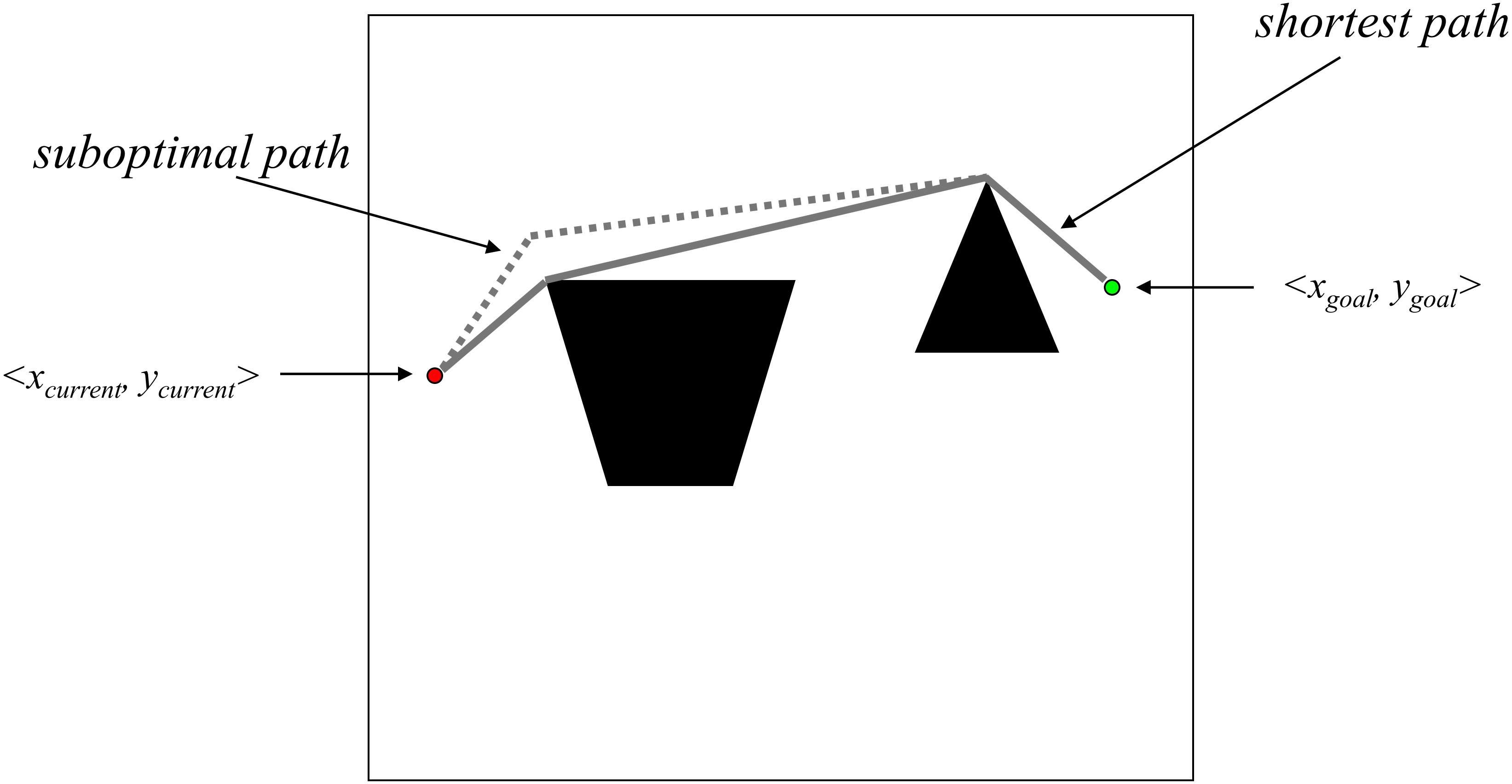
# Skeletonization-based Graphs

- Visibility Graphs [Wesley & Lozano-Perez '79]
  - based on idea that *the shortest path consists of obstacle-free straight line segments connecting all obstacle vertices and start and goal*



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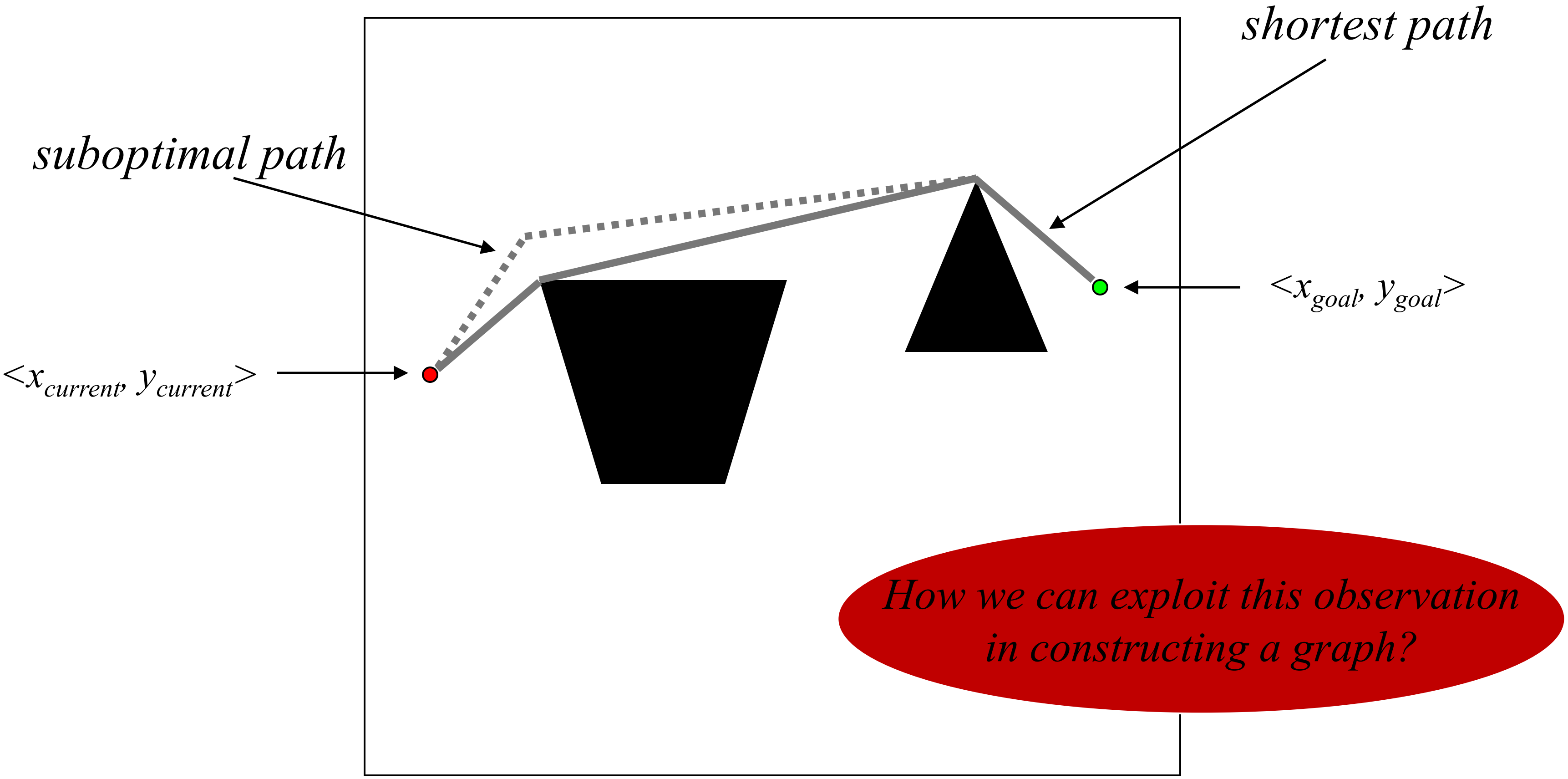
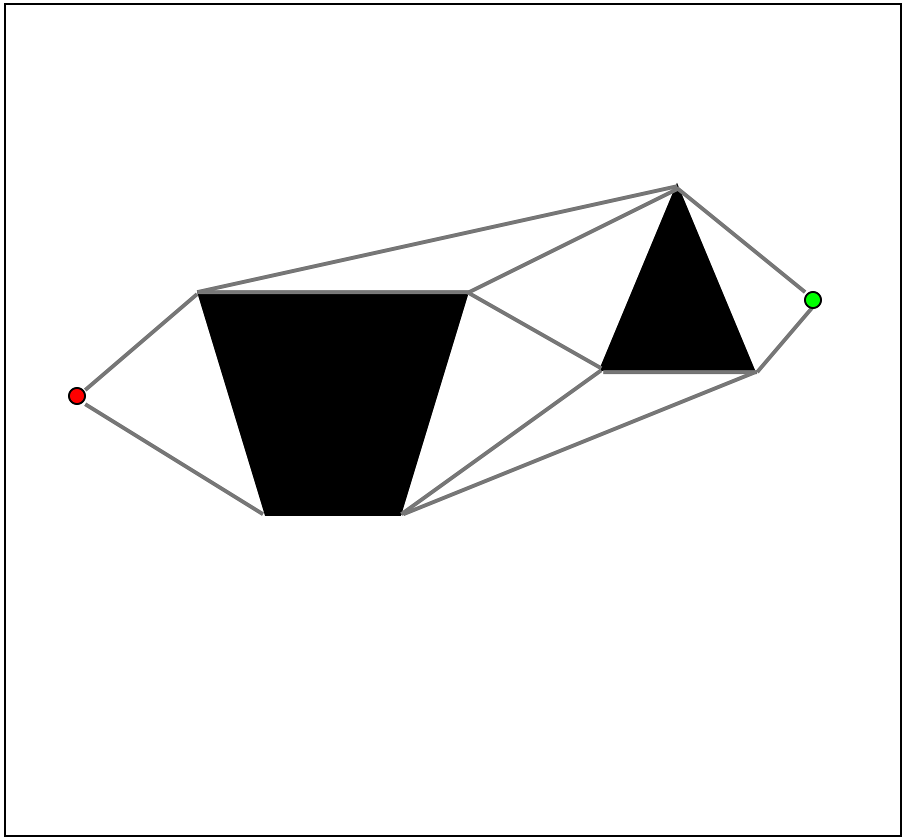


# Skeletonization-based Graphs

- Visibility Graphs [Wesley & Lozano-Perez '79]
  - construct a graph by connecting all vertices, start and goal by obstacle-free straight line segments (graph is $O(n^2)$, where n - # of vert.)



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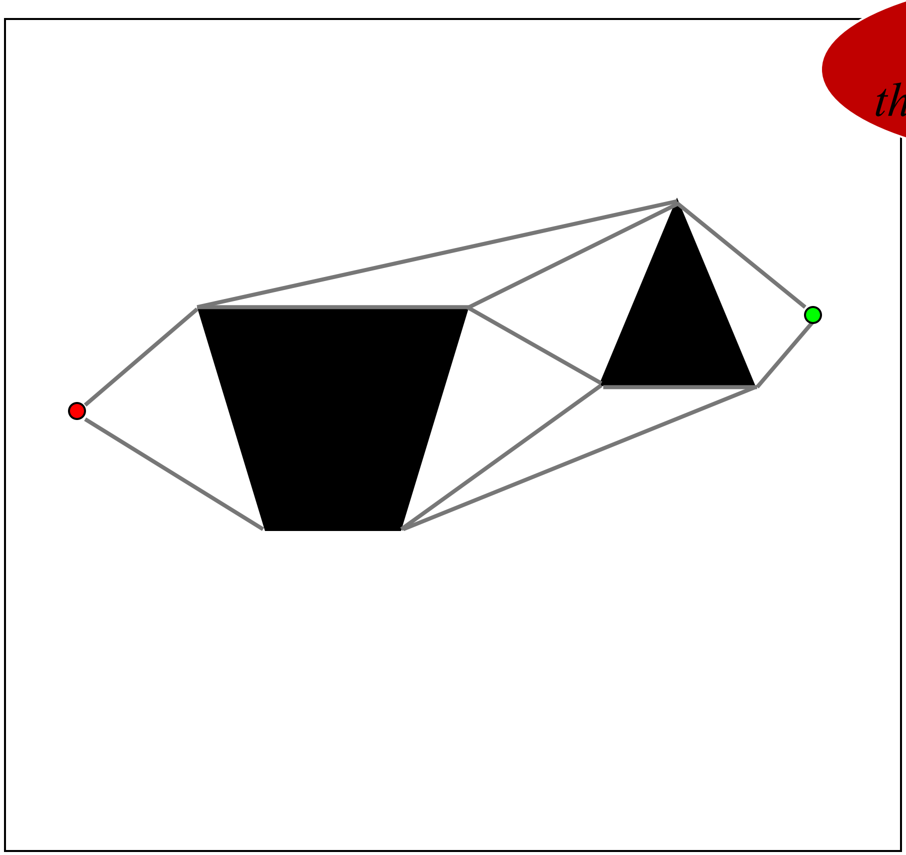
*Disadvantages of the Visibility Graphs?*



# Skeletonization-based Graphs

- Visibility Graphs
  - advantages:
    - independent of the size of the environment
  - disadvantages:
    - path is too close to obstacles
    - hard to deal with the cost function that is not distance
    - hard to deal with non-polygonal obstacles
    - hard to maintain the polygonal representation of obstacles
    - can be expensive in spaces higher than 2D

# Two Classes of Graph Construction Methods

- Skeletonization

  -Visibility graphs

  -Voronoi diagrams

  - Probabilistic roadmaps

- Cell decomposition

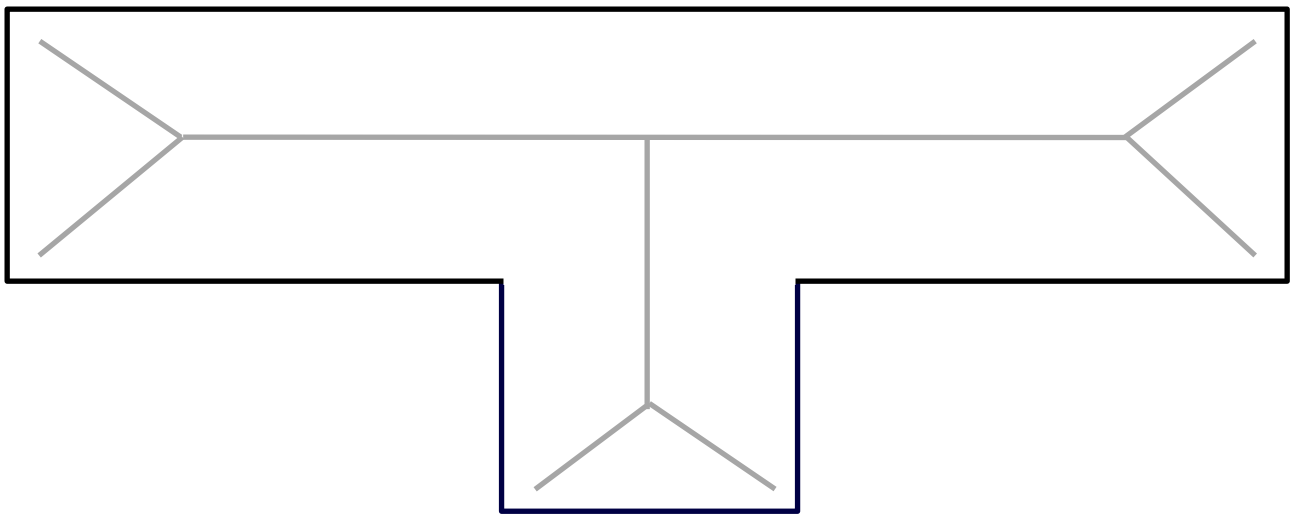
  - X-connected grids

  - lattice-based graphs

# Skeletonization-based Graphs

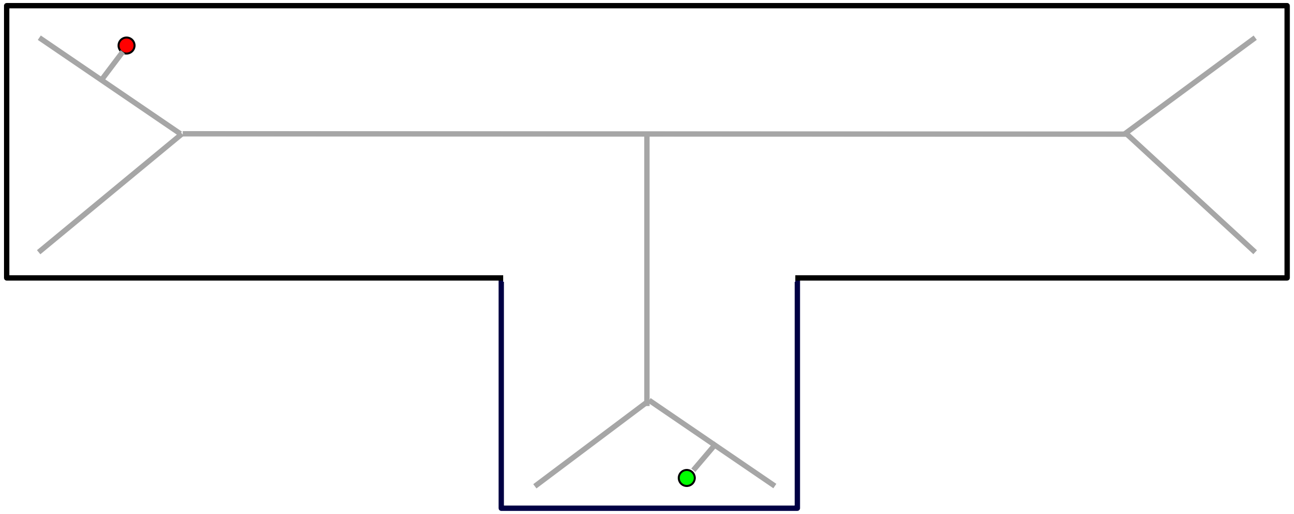
- Voronoi diagram [Rowat '79]
  - set of all points that are equidistant to two nearest obstacles

  (can be computed O (n log n), where n - # of points that represent obstacles)



# Skeletonization-based Graphs

- Voronoi diagram-based graph
  - Edges: Boundaries in Voronoi diagram
  - Vertices: Intersection of boundaries
  - Add start and goal vertices
  - Add edges that correspond to:
    - shortest path segment from start to the nearest segment on the Voronoi diagram
    - shortest path segment from goal to the nearest segment on the Voronoi diagram



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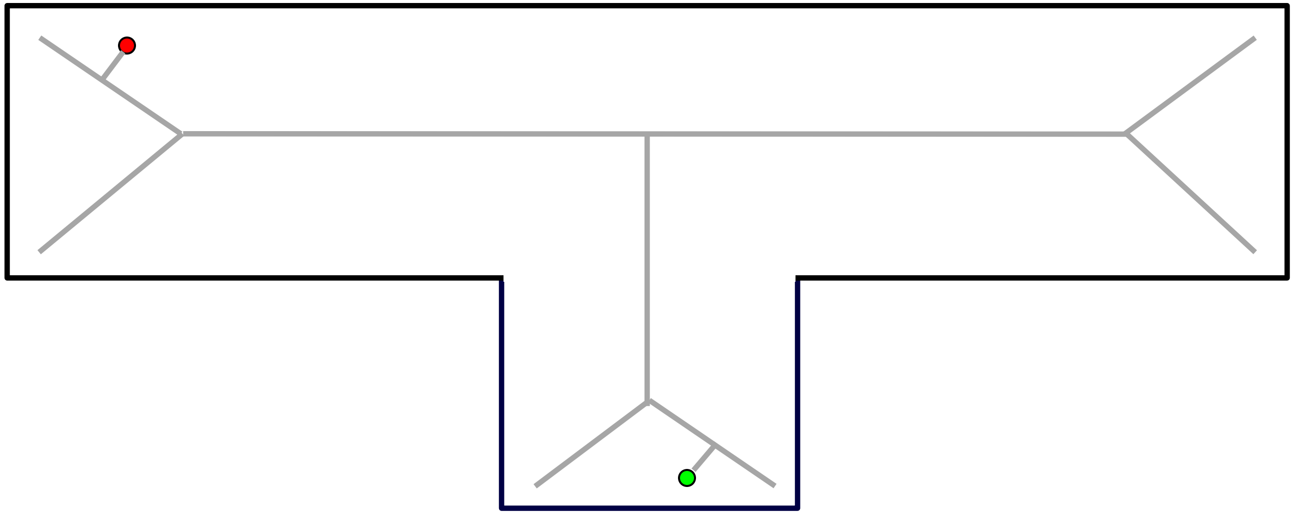
*Disadvantages of the Voronoi diagram-based Graphs?*



# Skeletonization-based Graphs

- Voronoi diagram-based graph
  - advantages:
    - tends to stay away from obstacles
    - independent of the size of the environment
    - can work with any obstacles represented as set of points
  - disadvantages:
    - can result in highly suboptimal paths
    - hard to deal with the cost function that is not distance
    - hard to use/maintain beyond 2D

# Two Classes of Graph Construction Methods

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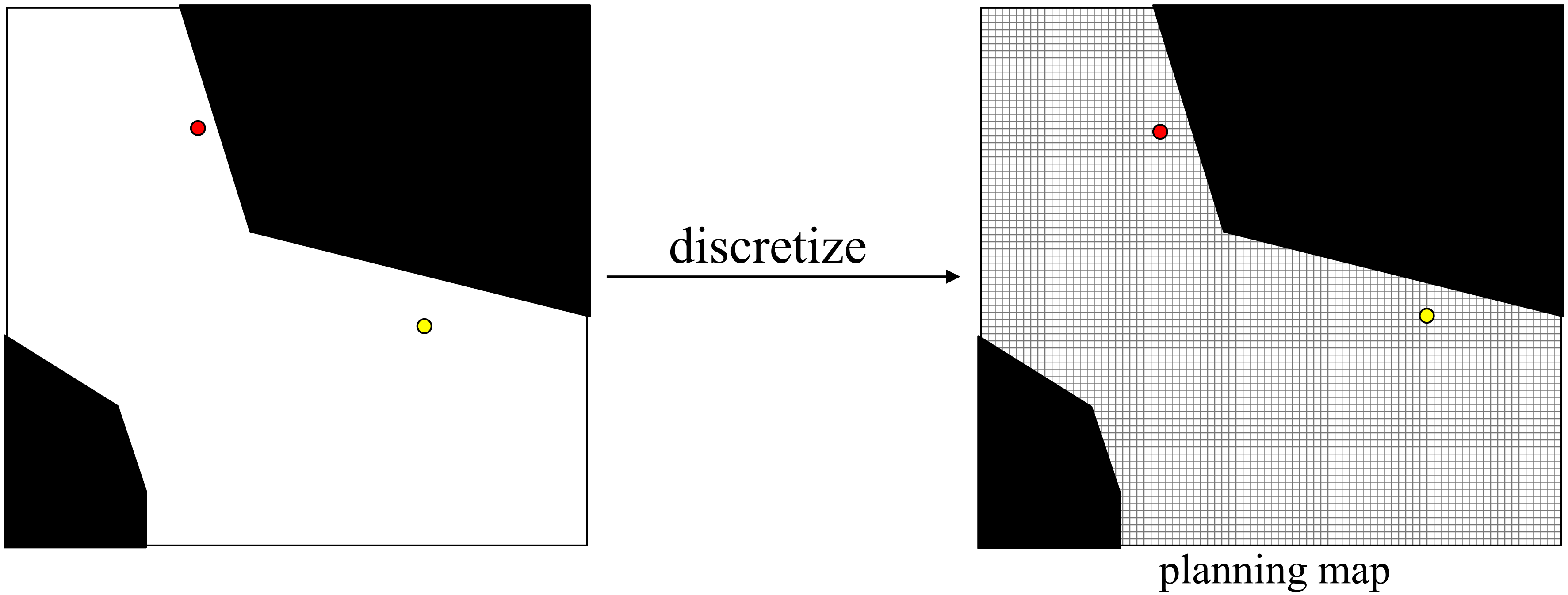
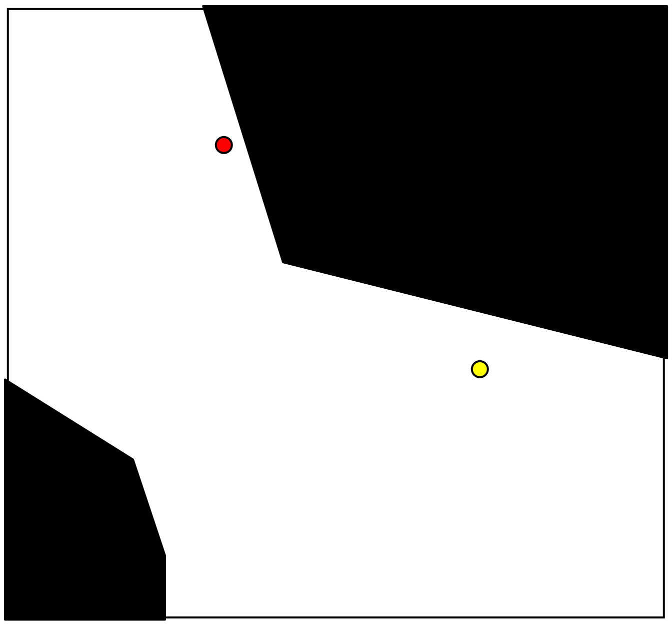
# Grid-based Graphs

- Approximate Cell Decomposition:
  - overlay uniform grid (discretize)

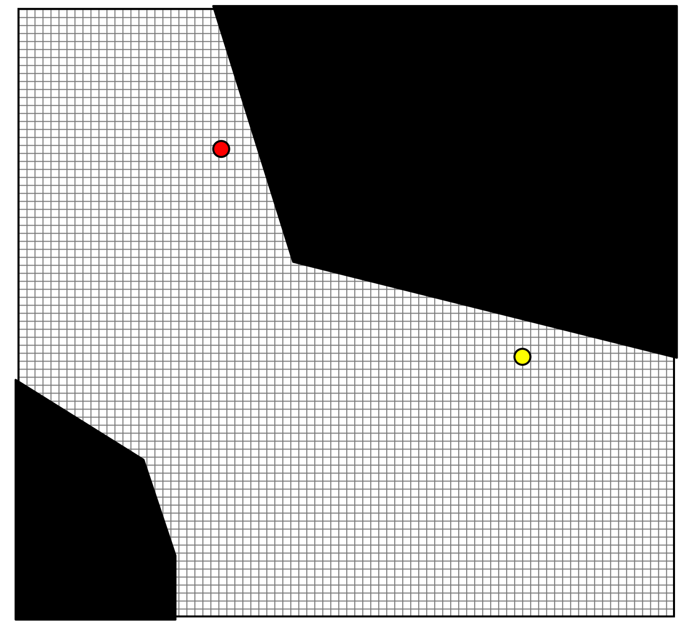


# Grid-based Graphs

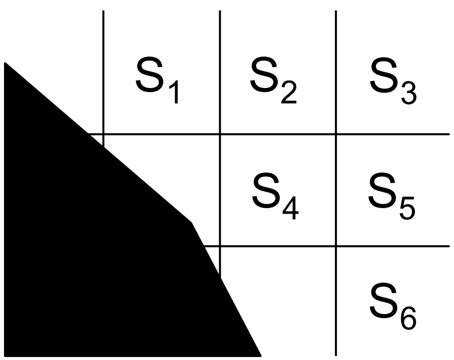
- Approximate Cell Decomposition:
  - construct a graph



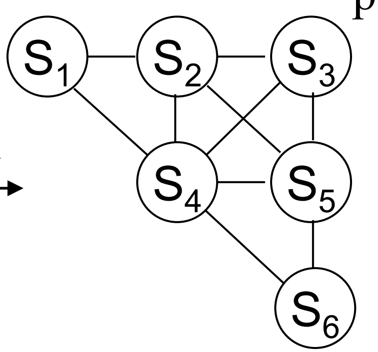
discretize →



planning map




convert into a graph →




search the graph for a least-cost path from $s_{start}$ to $s_{goal}$

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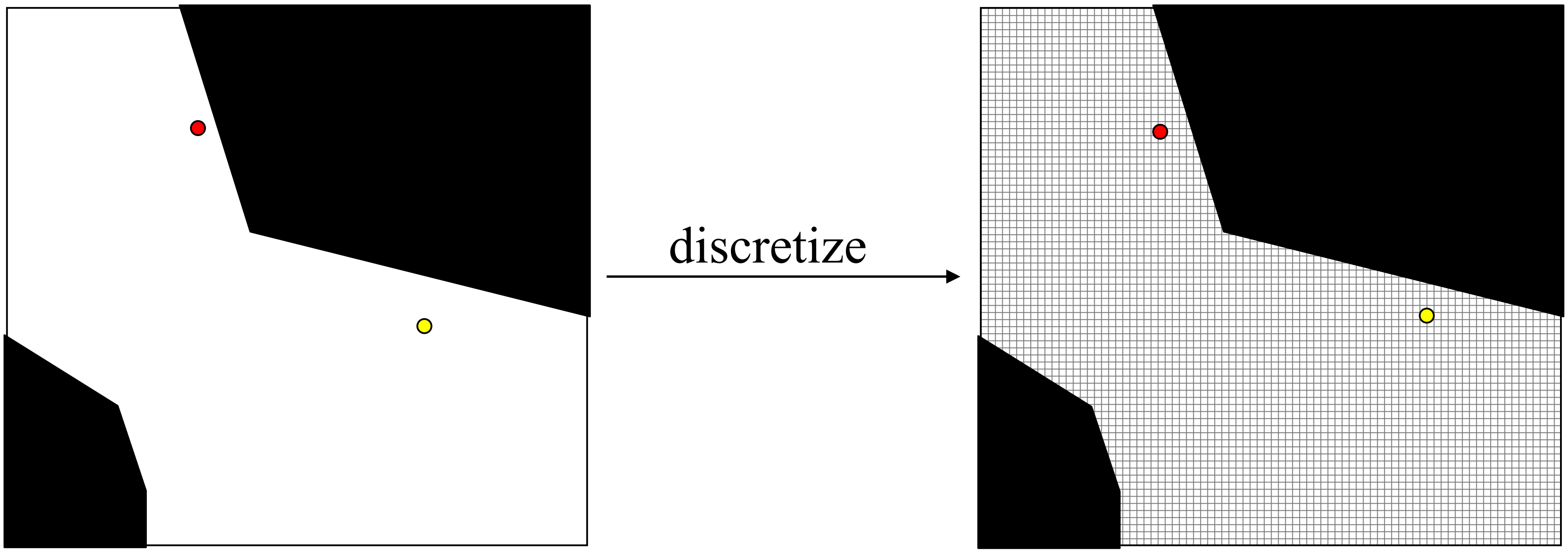


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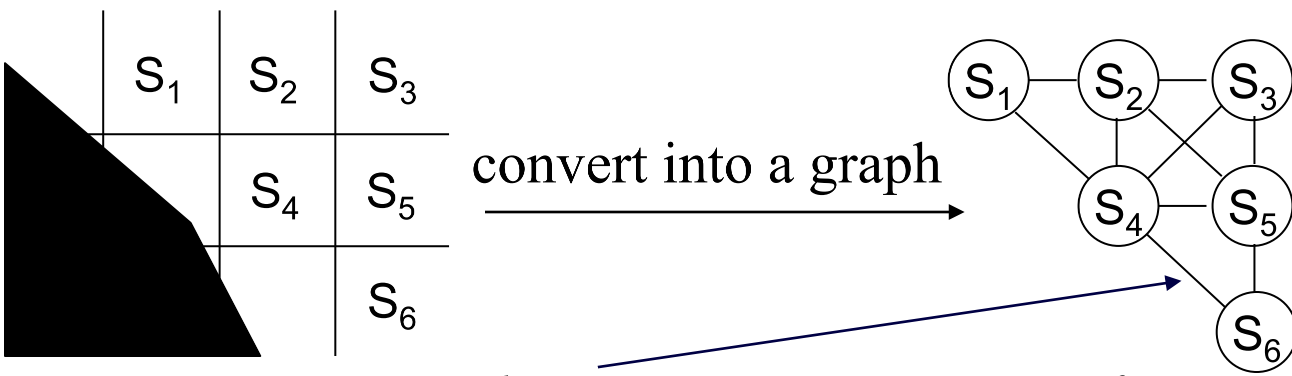


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*edgecosts can represent* ***any*** *cost function*

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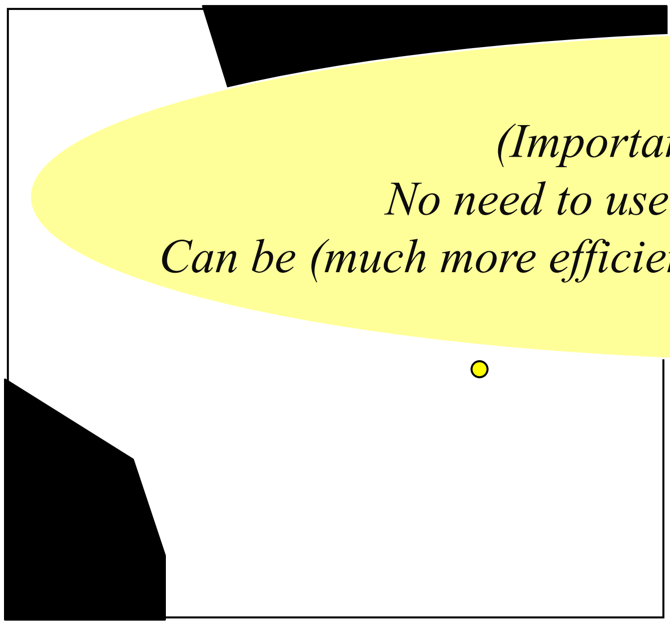
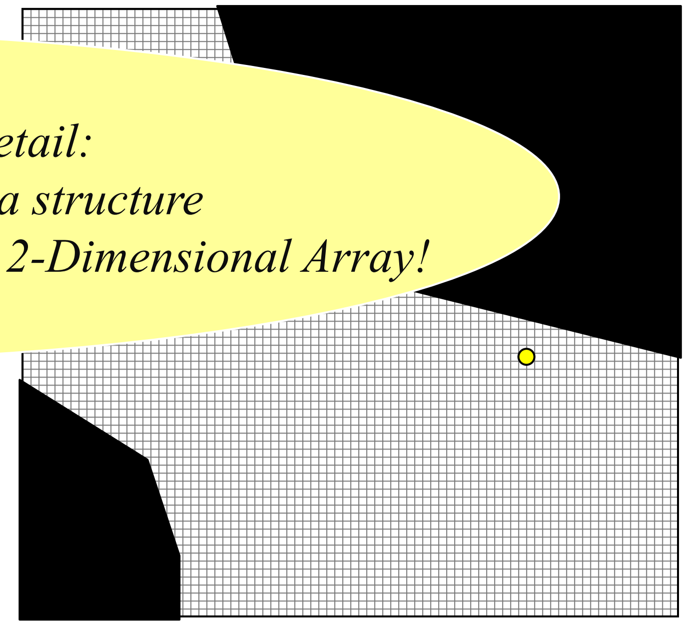


*(Important) Implementation Detail:*
*No need to use an explicit graph data structure*
*Can be (much more efficiently) represented as a 2-Dimensional Array!*



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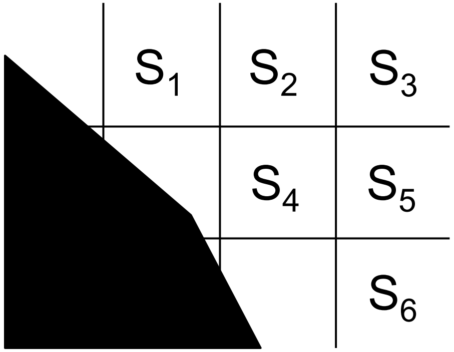


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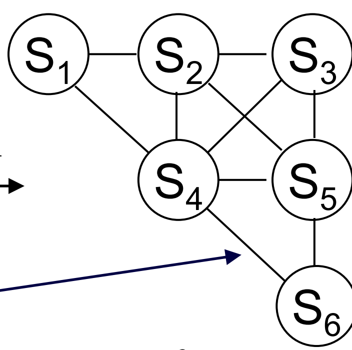


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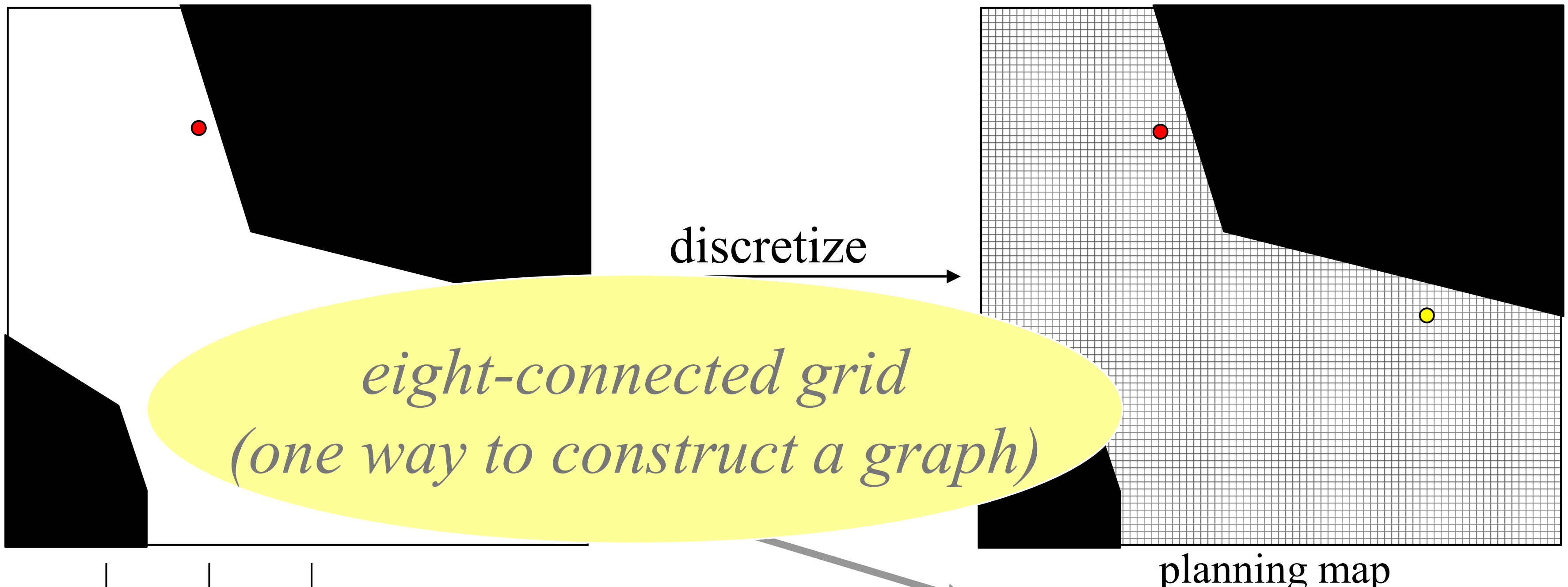


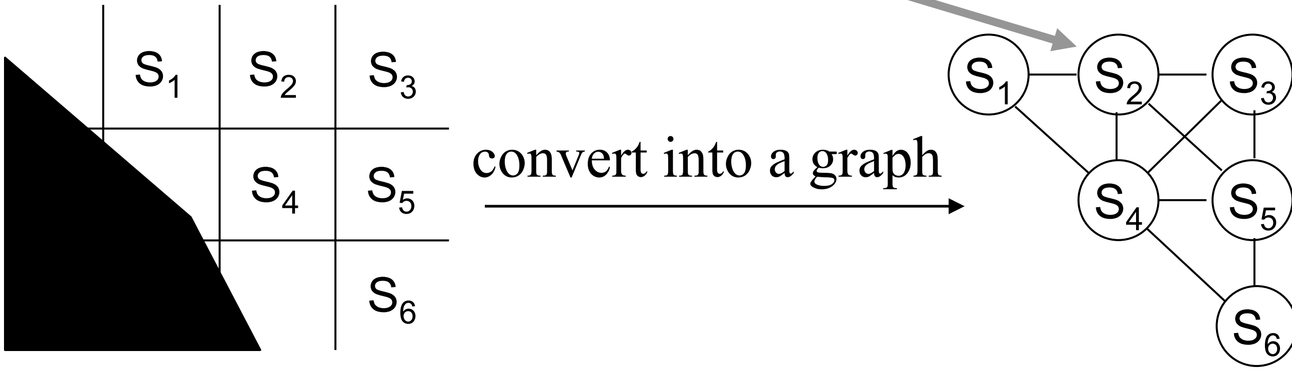


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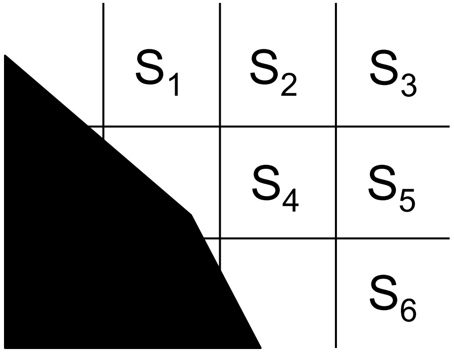
# Grid-based Graphs

- Approximate Cell Decomposition:
  - what to do with partially blocked cells?

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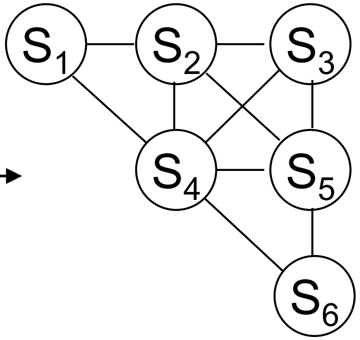
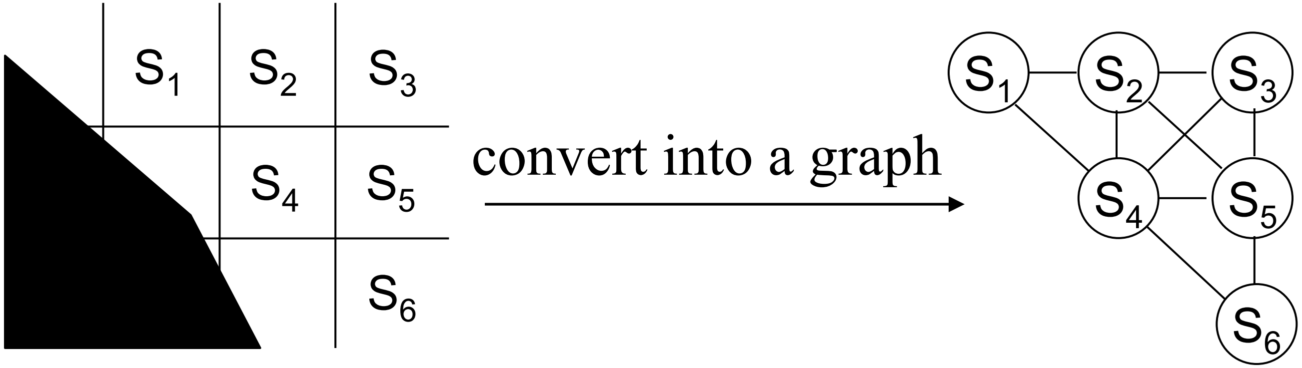



search the graph for a least-cost path from $s_{start}$ to $s_{goal}$

# Grid-based Graphs

- Approximate Cell Decomposition:
  - what to do with partially blocked cells?
  - make it untraversable – incomplete (may not find a path that exists)

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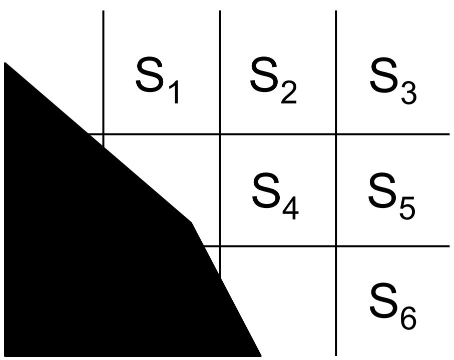
# Grid-based Graphs

- Approximate Cell Decomposition:
  - what to do with partially blocked cells?
  - make it traversable – unsound (may return invalid path)

*so, what's the solution?*

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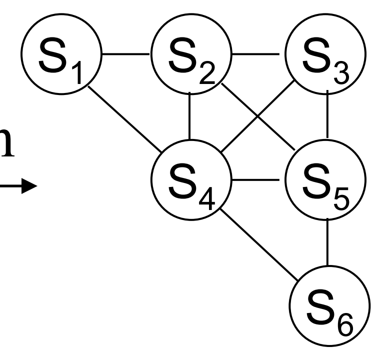


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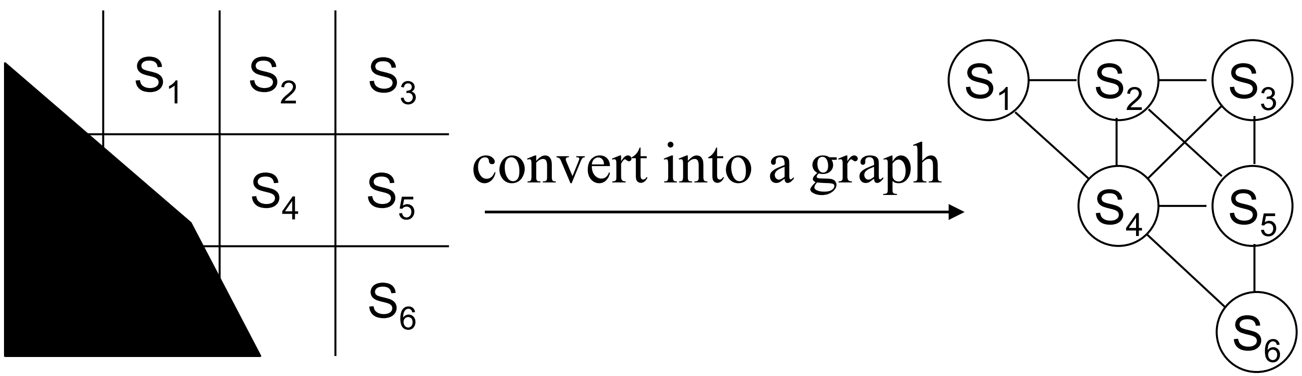
# Grid-based Graphs

- Approximate Cell Decomposition:
  - solution 1:
    - make the discretization very fine
    - expensive, especially in high-D

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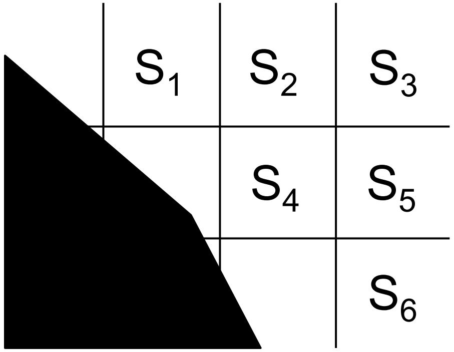
# Grid-based Graphs

- Approximate Cell Decomposition:
  - solution 2:
    - make the discretization adaptive
    - various ways possible

*Any ideas?*

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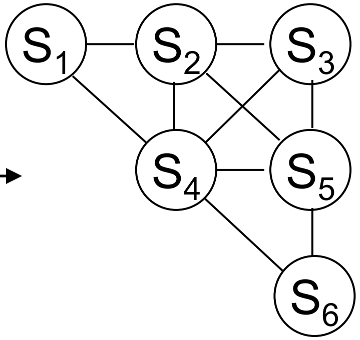


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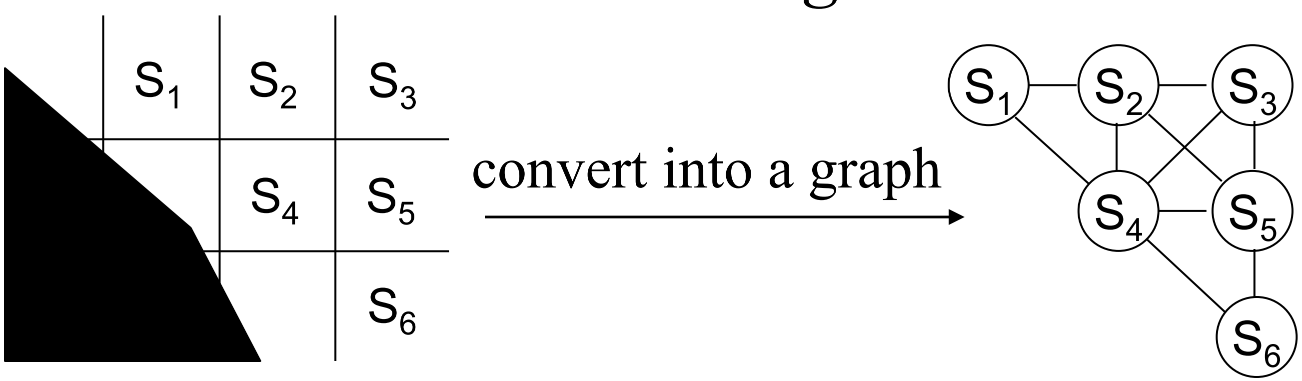
# Grid-based Graphs

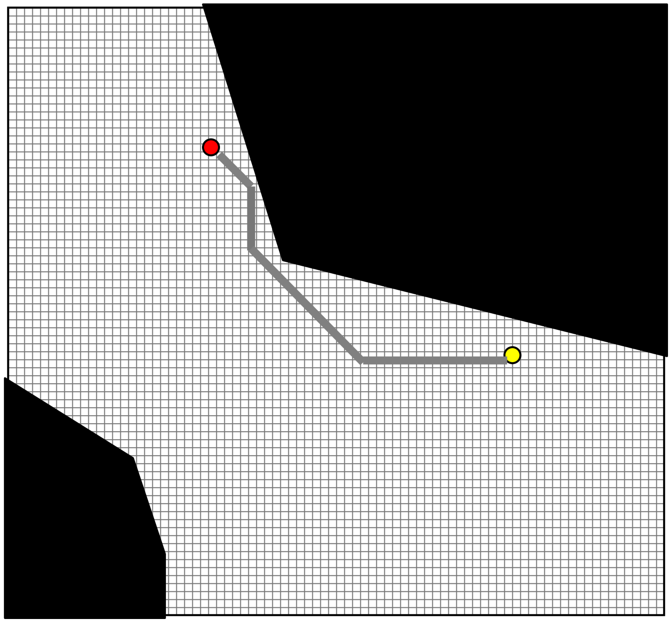
- Graph construction:
  - connect neighbors

***8**-connected grid*



# Grid-based Graphs

- Graph construction:
  - connect neighbors
  - path is restricted to 45$^{o}$ degrees



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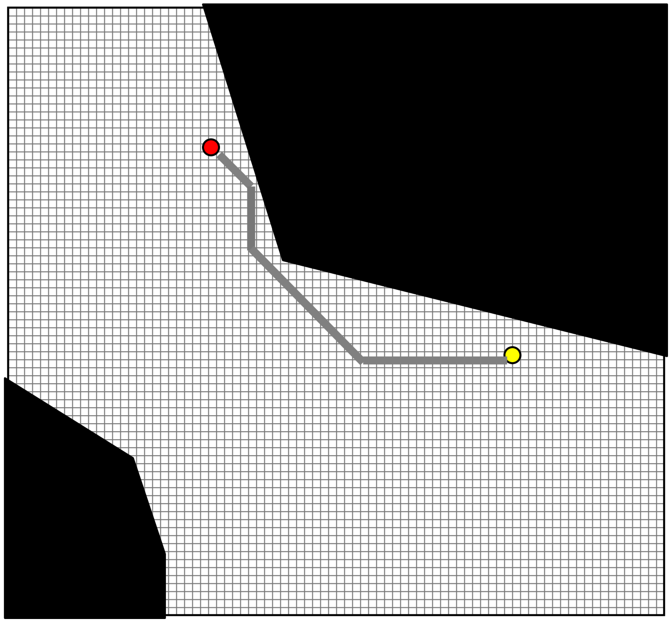
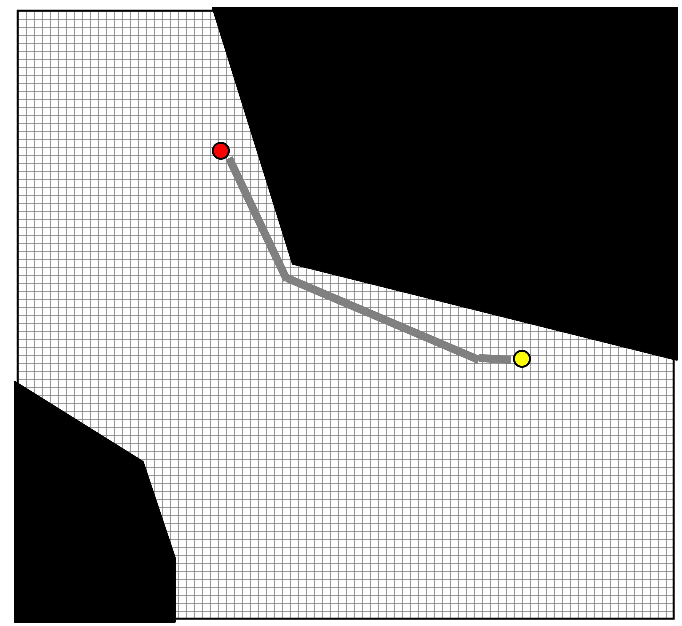
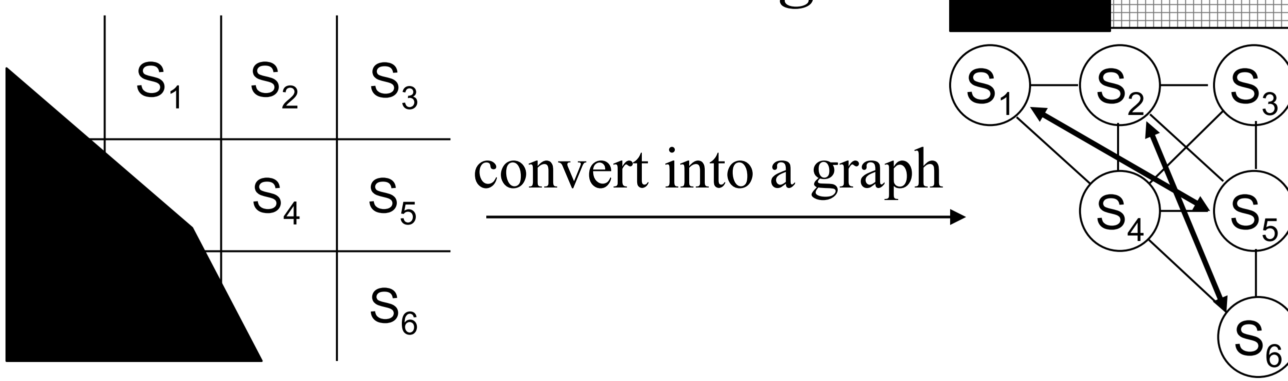


# Grid-based Graphs

- Graph construction:
  - connect cells to neighbor of neighbors
  - path is restricted to ? degrees

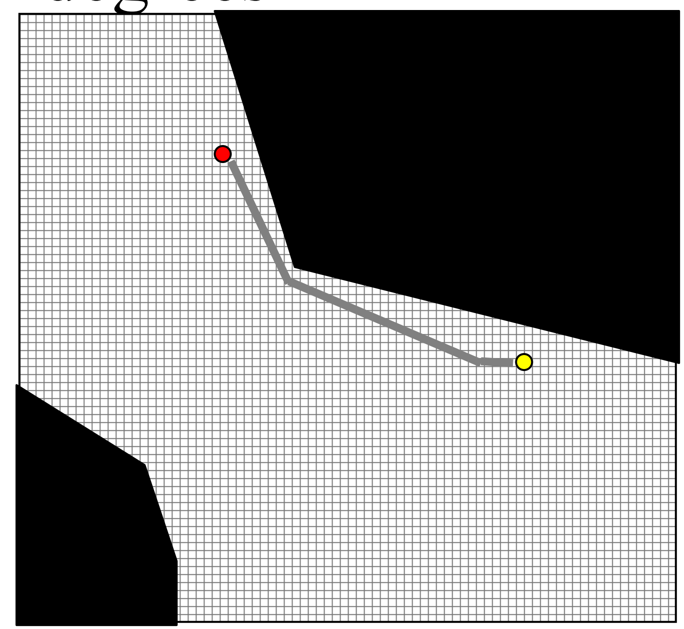


***16*-connected grid**



# Grid-based Graphs

- Graph construction:
  - connect cells to neighbor of neighbors
  - path is restricted to **26.6º/63.4º** degrees



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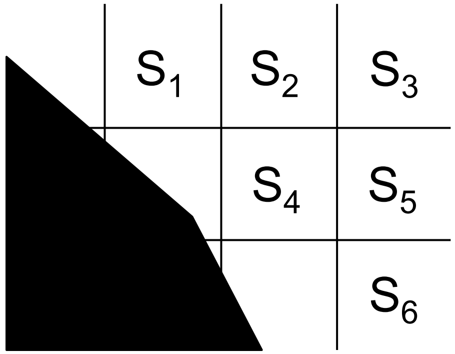
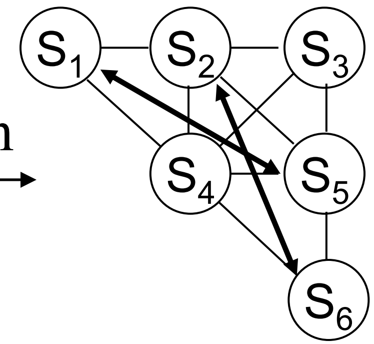



convert into a graph



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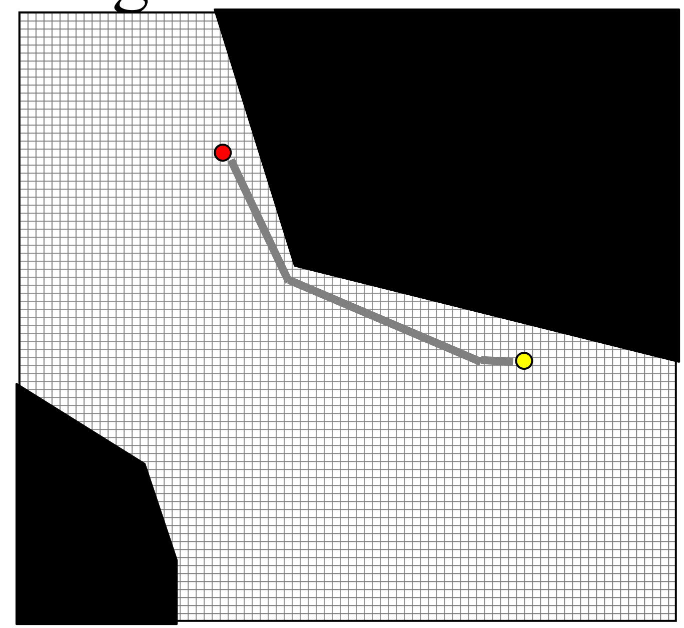
*Disadvantages?*



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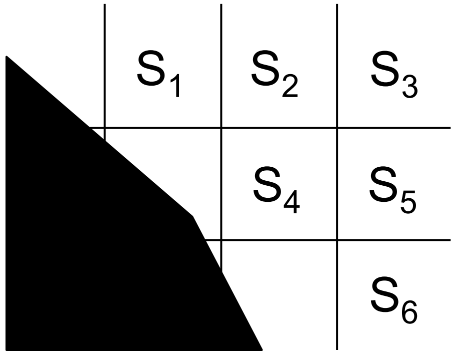


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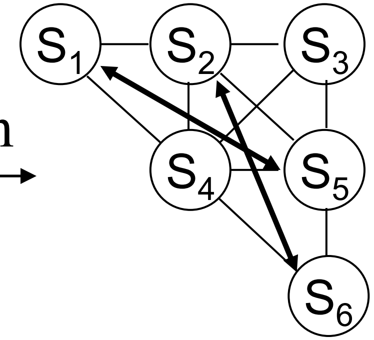

# Cell Decomposition-based Graphs

- Grid-based graph
  - advantages:
    - very simple to implement (super popular)
    - can represent any dimensional space
    - works well with obstacles represented as set of points
    - works with any cost function
  - disadvantages:
    - size does depend on the size of the environment
    - expensive to maintain/compute grids of dimensions > 3

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*More on this later:*
*Implicit vs. Explicit Graph representations*

# 2D Planning for Omnidirectional **Non-Circular Non-point** Robot

Planning for omnidirectional point robot:

*What is* $M^R = \langle x,y \rangle$
*What is* $M^W = \langle \text{obstacle/free space} \rangle$
*What is* $s^R_{current} = \langle x_{current}, y_{current} \rangle$
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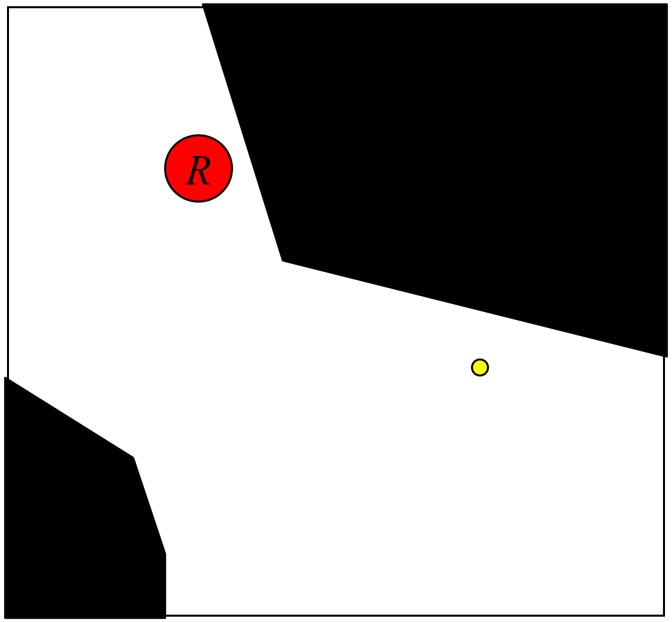
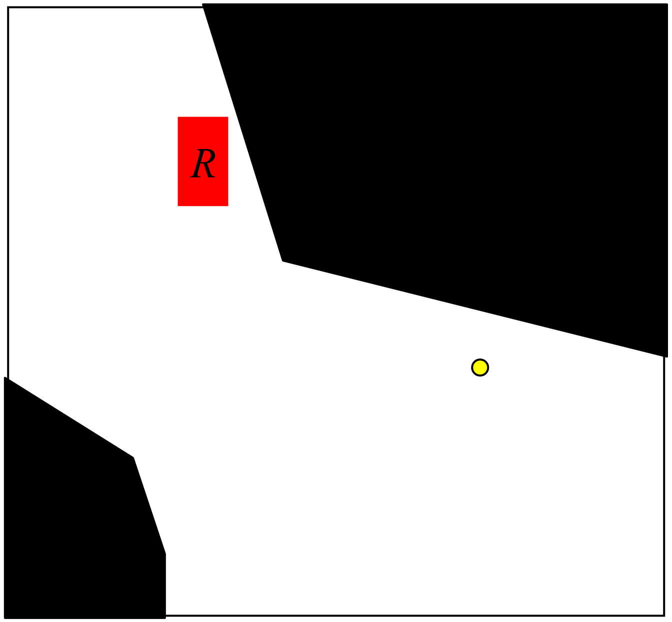
*What is* $G = \langle x_{goal}, y_{goal} \rangle$



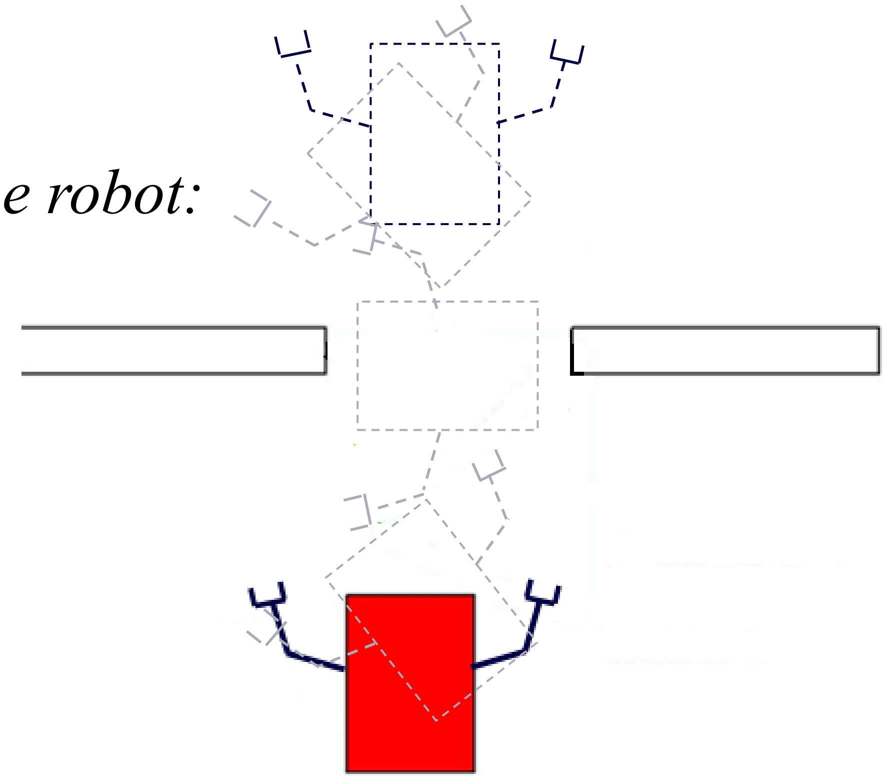



# Configuration Space

- Configuration is legal if it does not intersect any obstacles and is valid

- Configuration Space is the set of legal configurations

*Legal configurations for the base of the robot:*



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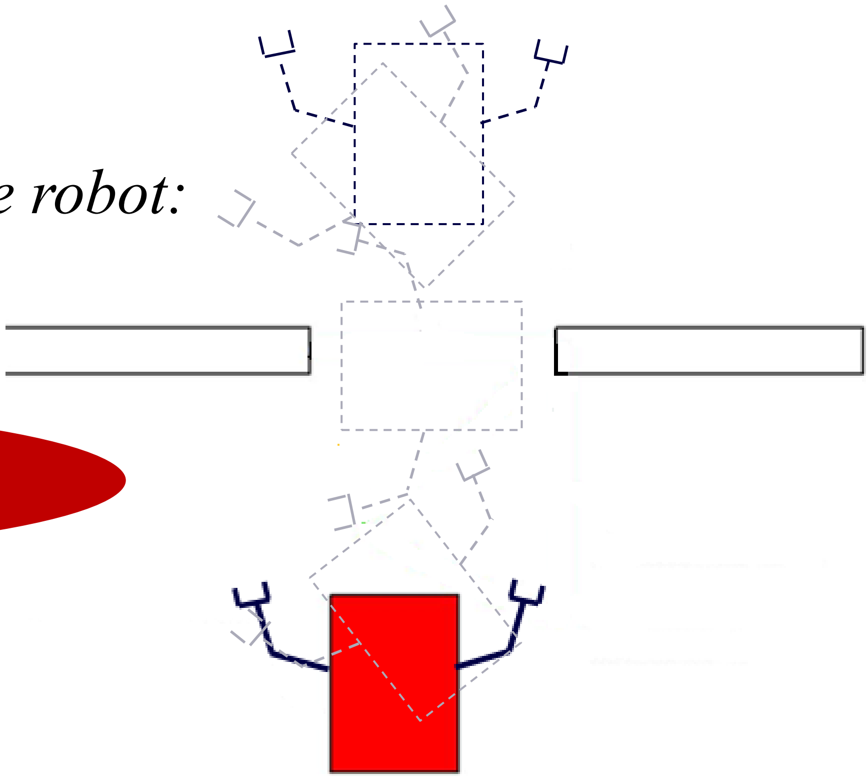
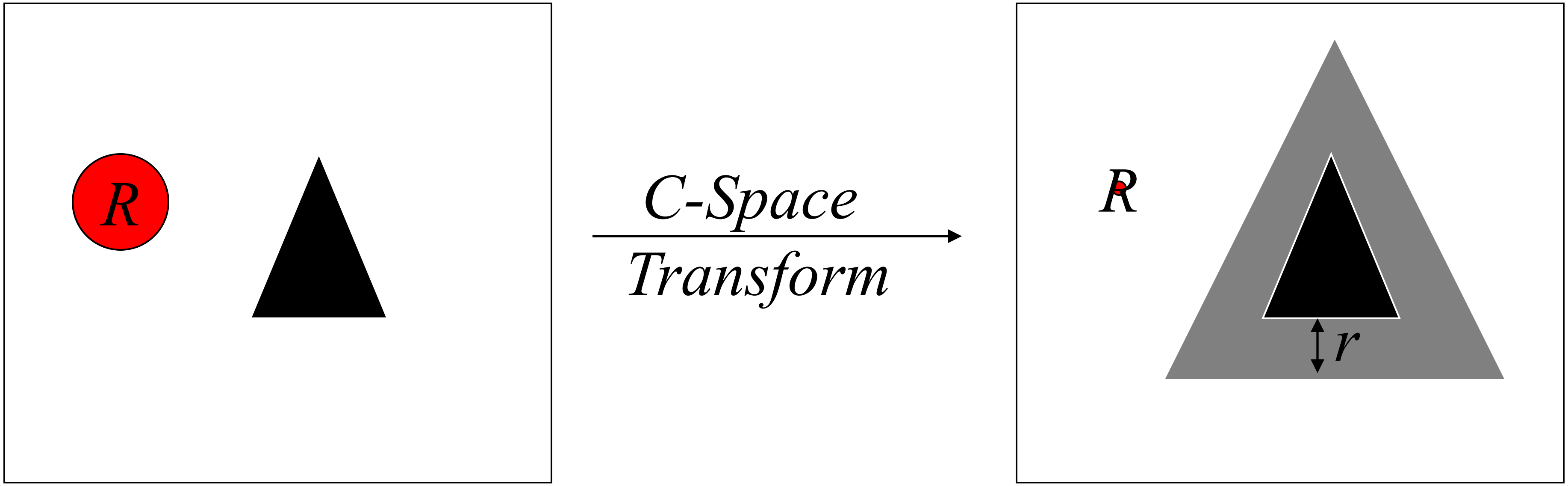


*What is the dimensionality of this configuration space?*

# C-Space Transform

- Configuration space for a robot base in 2D world is:
  - 2D if robot's base is circular




- expand all obstacles by radius r of the robot's base
- graph construction can then be done assuming point robot

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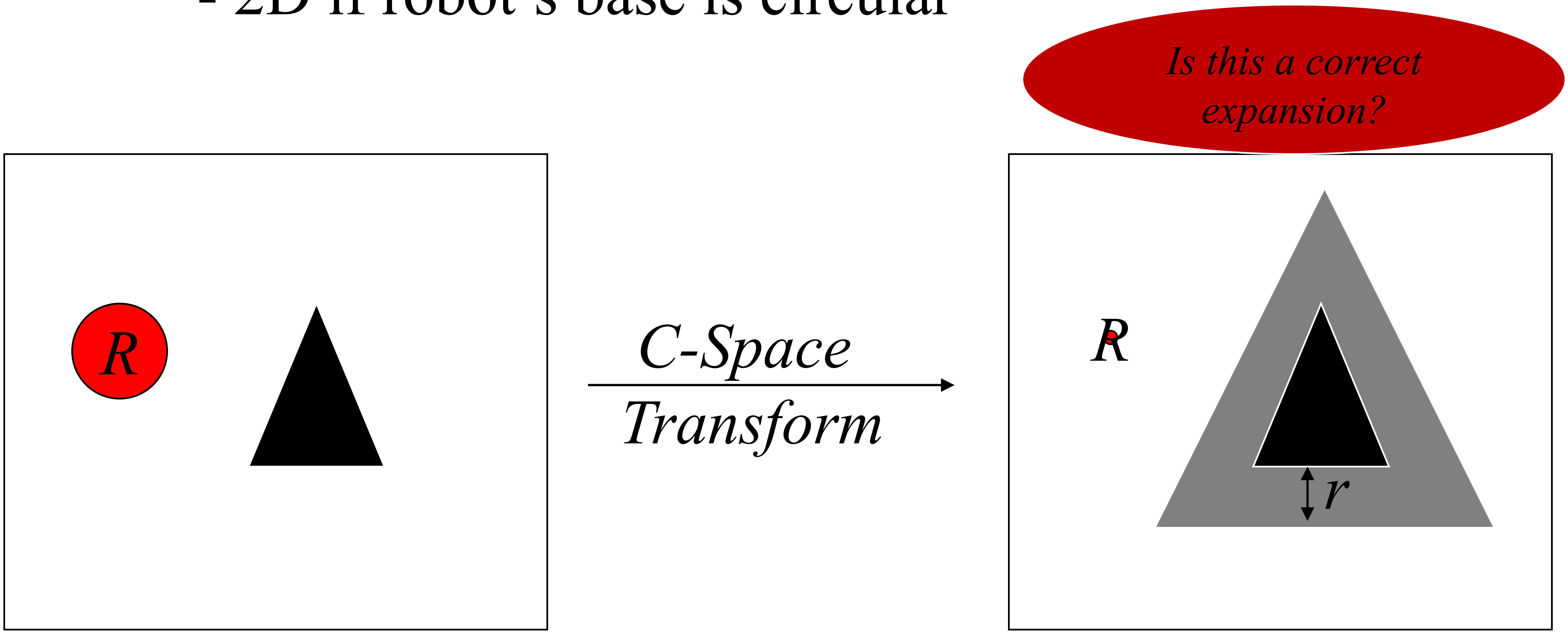


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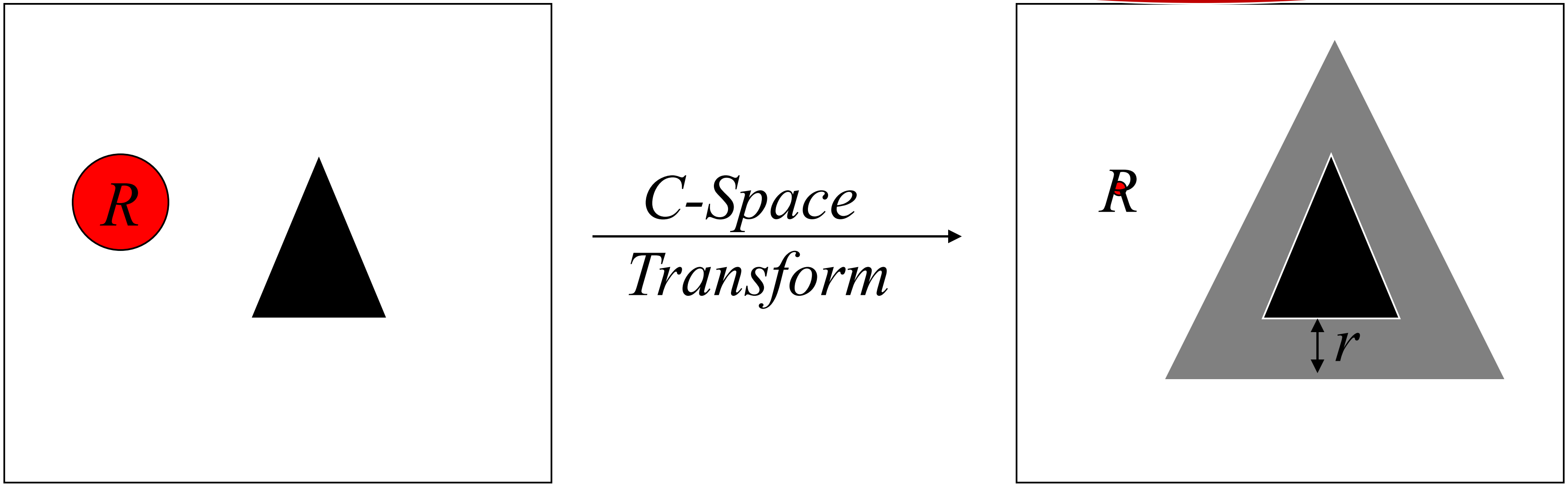
*How to perform expansion of obstacles?*

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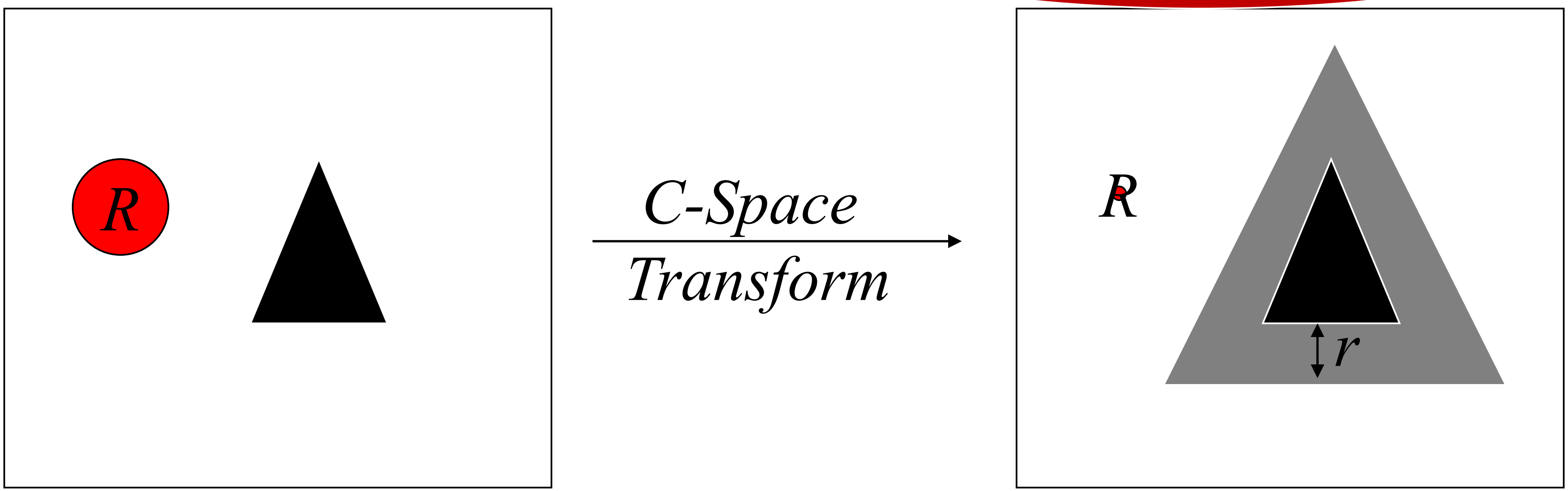
# C-Space Transform

- Configuration space for a robot ba…
  - 2D if robot's base is circula…

*O(n) methods exist to compute distance transforms efficiently*

*How to perform expansion of obstacles?*




- expand all obstacles by radius r of the robot's base
- graph construction can then be done assuming point robot

# 2D Planning for Omnidirectional **Non-Circular Non-point** Robot

Planning for omnidirectional circular robot:

*What is* $M^R = <x,y>$

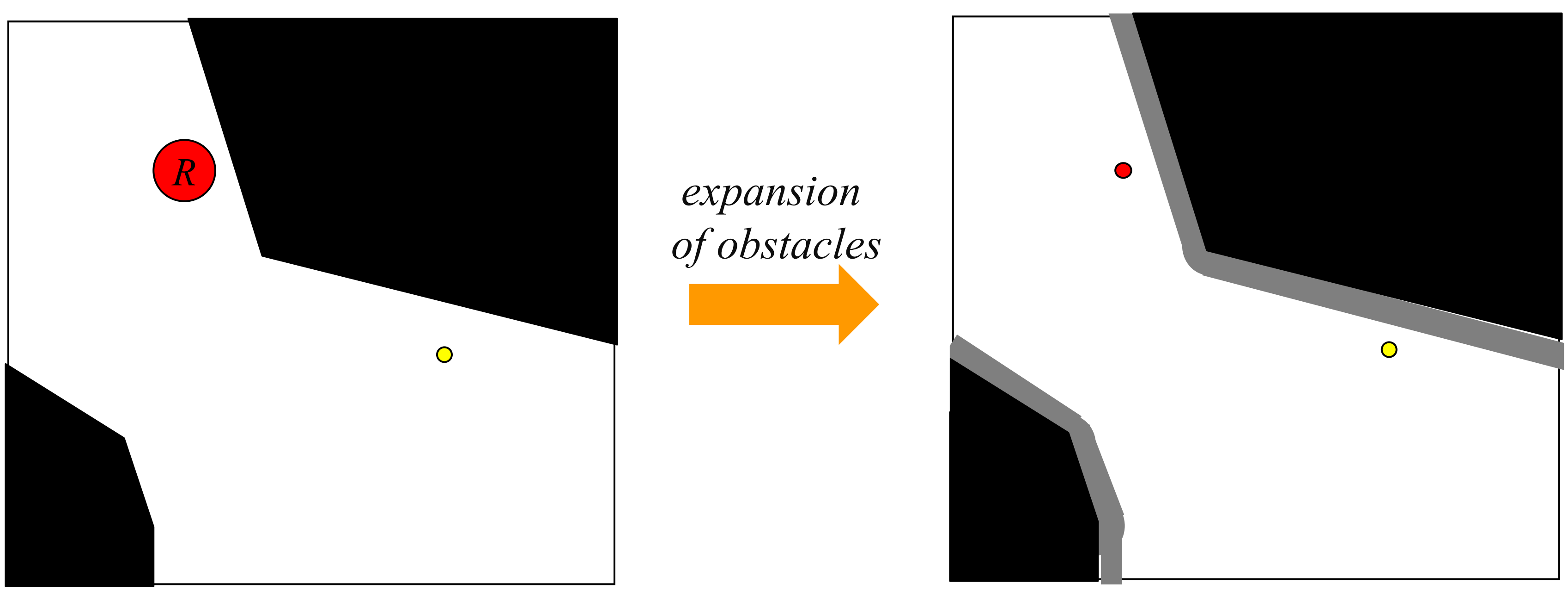
*What is* $M^W = <obstacle/free\ space>$

*What is* $s^R_{current} = <x_{current}, y_{current}>$

*What is* $s^W_{current} = constant$

*What is* $C$ = *Euclidean Distance*

*What is* $G = <x_{goal}, y_{goal}>$



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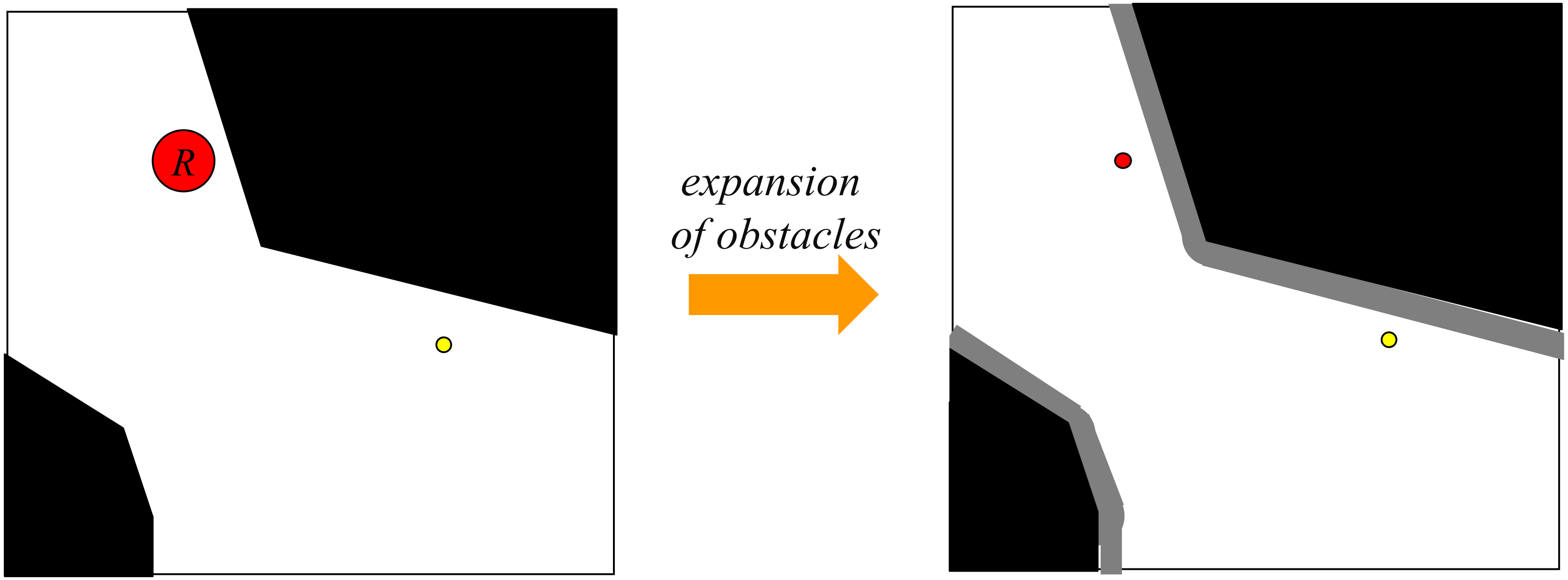
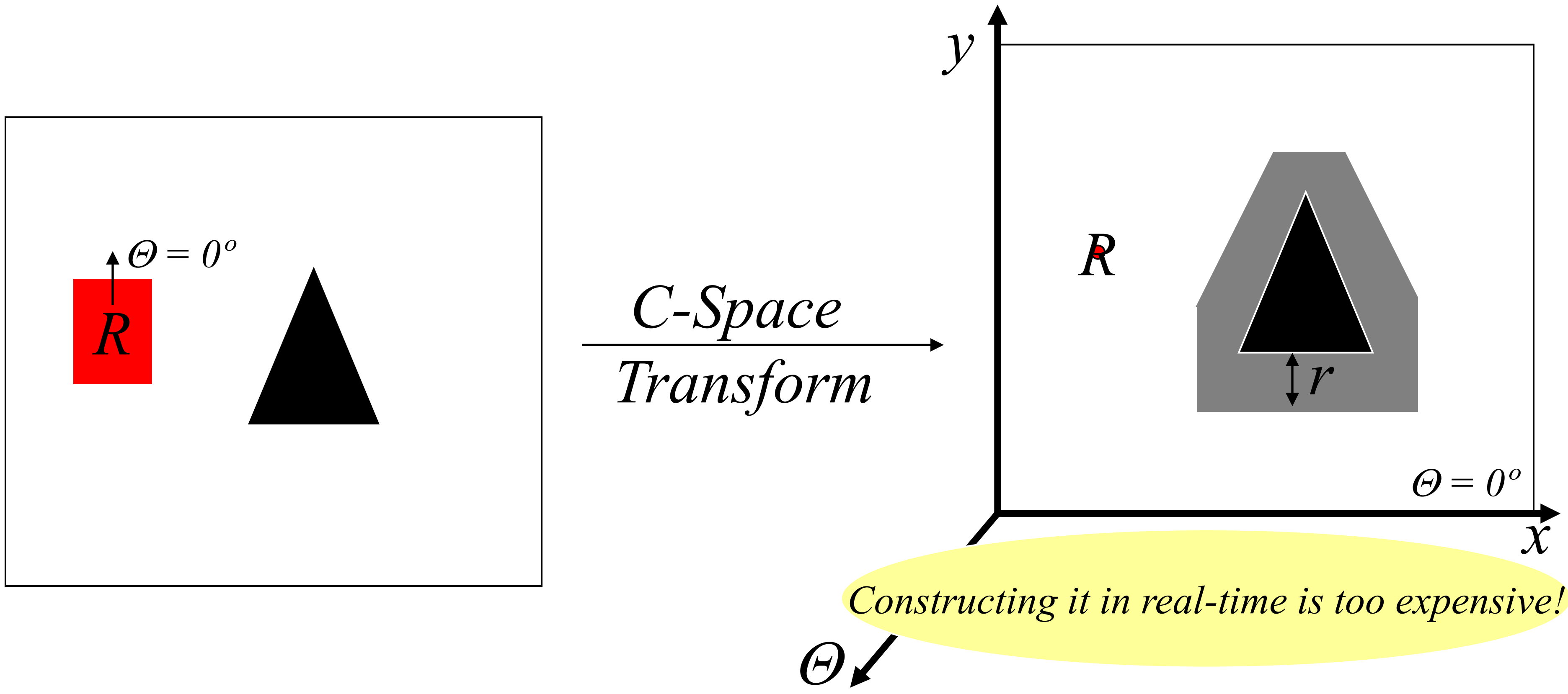
*What is* $G = <x_{goal}, y_{goal}>$

*We can now construct a graph using previously discussed methods (grids, Voronoi graphs, Visibility graphs)*



# C-Space Transform

- Configuration space for a robot base in 2D world is:
  - 3D if robot's base is non-circular



# Planning as Graph Search Problem

1. Construct a graph representing the planning problem

2. Search the graph for a (hopefully, close-to-optimal) path

The two steps above are often interleaved

# Planning as Graph Search Problem

1. Construct a graph representing the planning problem

2. Search the graph for a (hopefully, close-to-optimal) path

The two steps above are often interleaved

# Interleaving Search and Graph Construction

Graph Search using an **Explicit Graph** (allocated prior to the search itself):

1. *Create the graph $G = \{V, E\}$ in-memory*

2. *Search the graph*

*Using Explicit Graphs*
*is typical for low-D (i.e., 2D) problems in Robotics*
*(with the exception of PRMs, covered in a later lecture)*

# Interleaving Search and Graph Construction

Graph Search using an **Implicit Graph** (allocated as needed by the search):

1. *Instantiate Start state*

2. *Start searching with the Start state using functions*

   a) *Succs = GetSuccessors (State s, Action)*
   b) *ComputeEdgeCost (State s, Action a, State s')*

   *and allocating memory for the generated states*

*Using Implicit Graphs*
*is critical for most (>2D) problems*
*in Robotics*

# Interleaving Search and Graph Construction

- **Board example** for deciding whether to use an Explicit graph or Implicit graph

- Planning for *(x,y,Θ)* for
  - 20 by 20 m environment discretized into 25 cm cells with 8 heading *Θ* values

*Is it feasible to use Explicit Graph (memory and pre-computation time reqs)?*

# Interleaving Search and Graph Construction

- **Board example** for deciding whether to use an Explicit graph or Implicit graph

- Planning for *(x,y,Θ)* for
  - 200 by 200 m environment discretized into 25 cm cells with 16 heading *Θ* values for a real vehicle

*Is it feasible to use Explicit Graph (memory and pre-computation time reqs)?*

# 2D Planning for Omnidirectional **Non-Circular Non-point** Robot

Planning for omnidirectional non-circular robot:

*What is* $M^R = <x,y,\Theta>$
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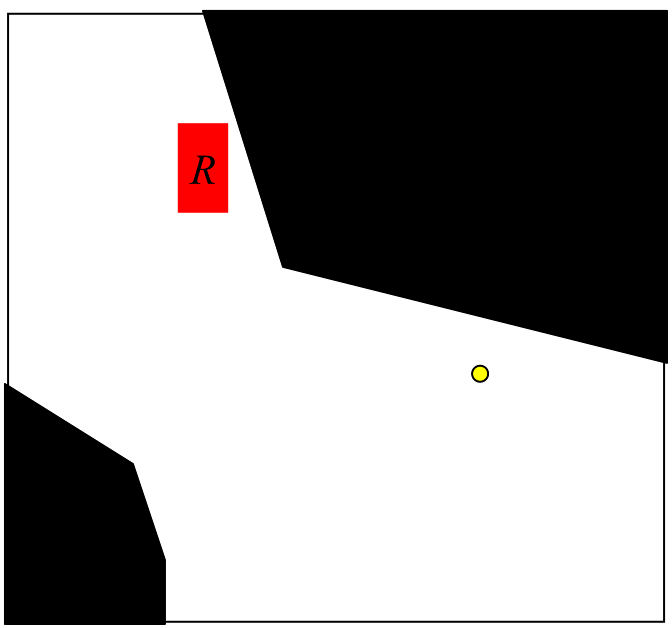
*What is C = Euclidean Distance*
*What is* $G = <x_{goal}, y_{goal}, \Theta_{goal}>$

***Interleave***
***Graph Construction and Graph Search steps!***

*Construct a 3D grid (x,y,Θ) assuming point robot (i.e., a cell (x,y,Θ) is free whenever its (x,y) is free) and compute the **actual** validity of only those cells that get computed by the graph search*



# 2D Planning for Omnidirectional **Non-Circular Non-point** Robot

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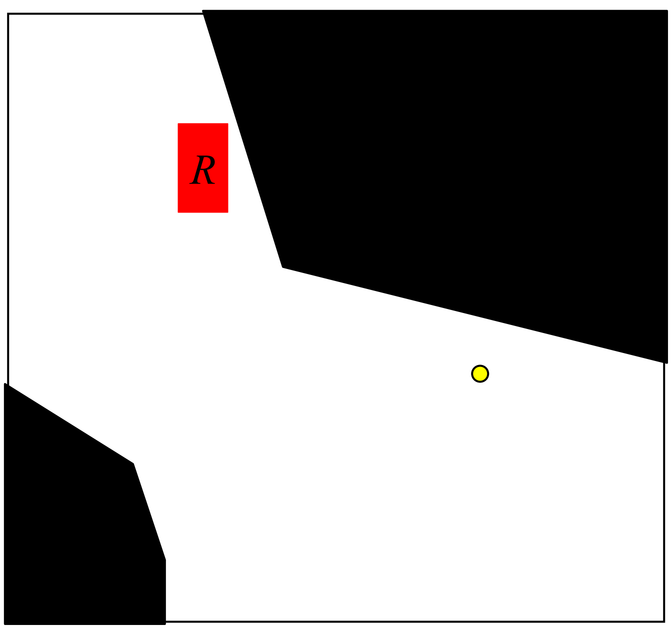


*Construct a 3D grid (x,y,Θ) assuming point robot (i.e., a cell (x,y,Θ) is free whenever its (x,y) is free) and compute the* ***actual*** *validity of only those cells that get computed by the graph search*

*How to compute the actual validity of cell (x,y,Θ)?*

# 2D Planning for Omnidirectional **Non-Circular Non-point** Robot

Planning for omnidirectional non-circular robot:

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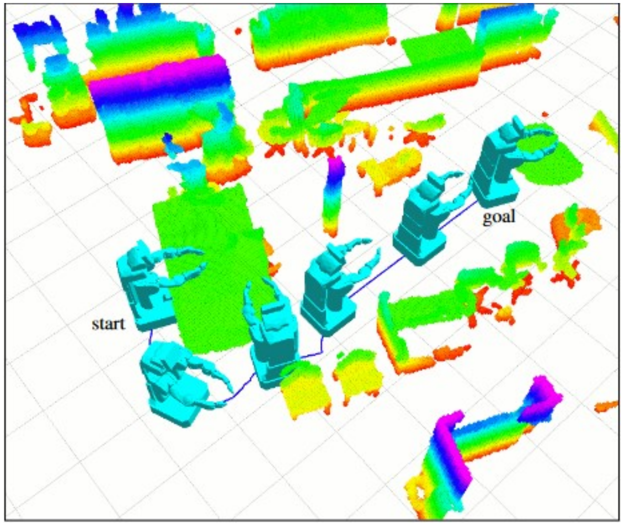
***Interleave Graph Construction and Graph Search steps!***



# Two Classes of Graph Construction Methods

- Skeletonization

  -Visibility graphs

  -Voronoi diagrams

  - Probabilistic roadmaps

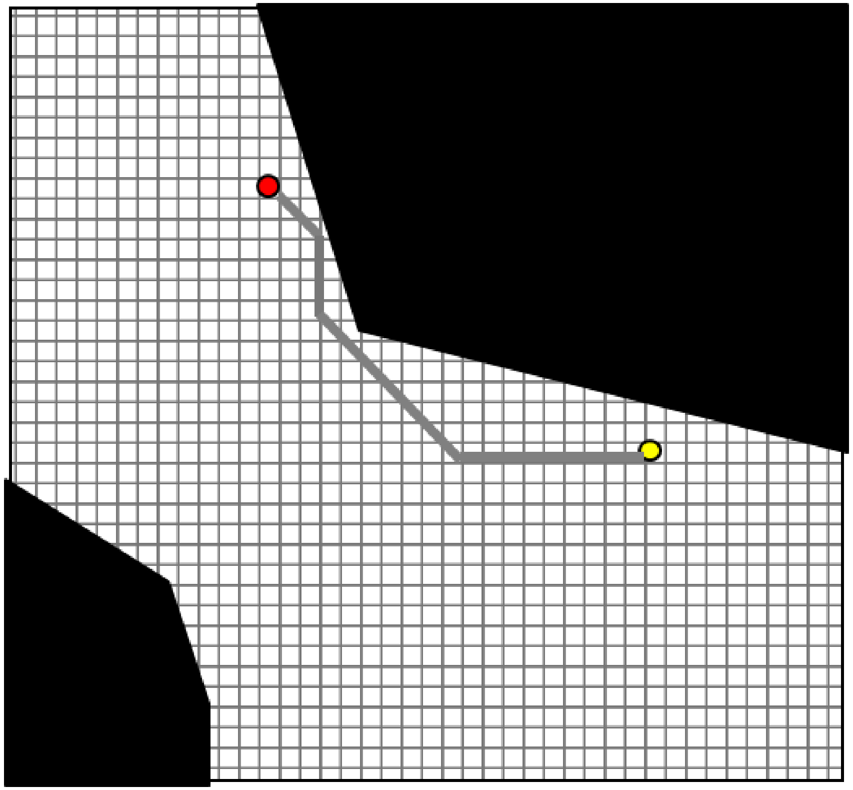
- Cell decomposition

  - X-connected grids

  - lattice-based graphs

# Beyond Planning for Omnidirectional Robots

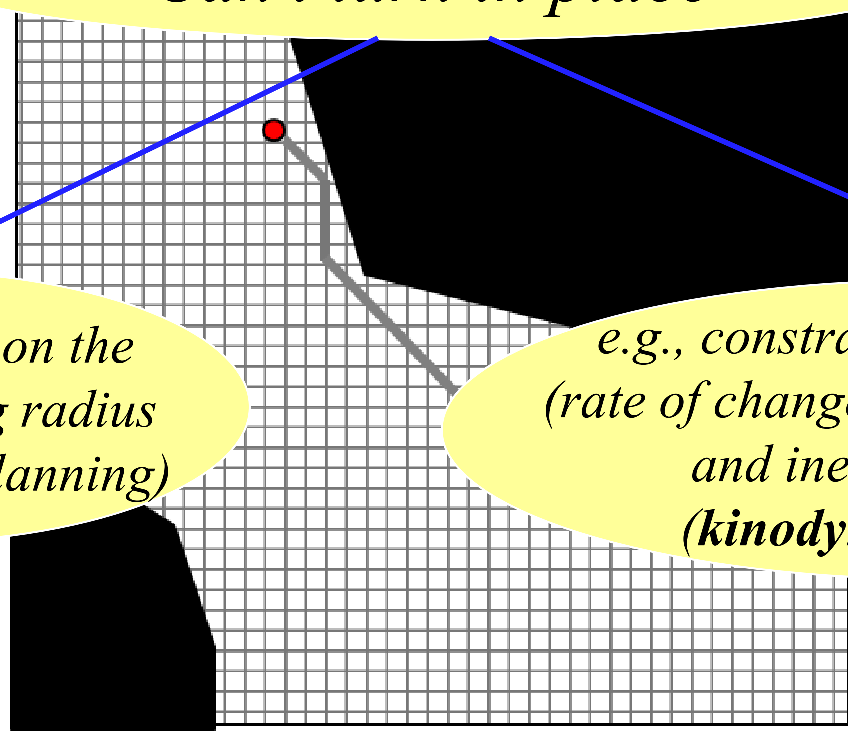
*What's wrong with using*
*Grid-based Graphs when planning*
*for non-omnidirectional robots?*

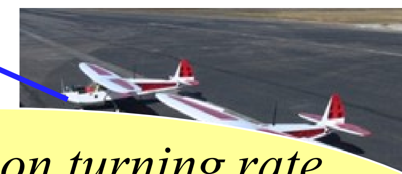



# Beyond Planning for Omnidirectional Robots

*What's wrong with using*
*Grid-based Graphs when planning*
*for non-omnidirectional robots?*


*"Can't turn in place"*





*e.g., constraints on the minimum turning radius (still **kinematic** planning)*

*e.g., constraints on turning rate (rate of change in wheel orientation) and inertial constraints (**kinodynamic** planning)*

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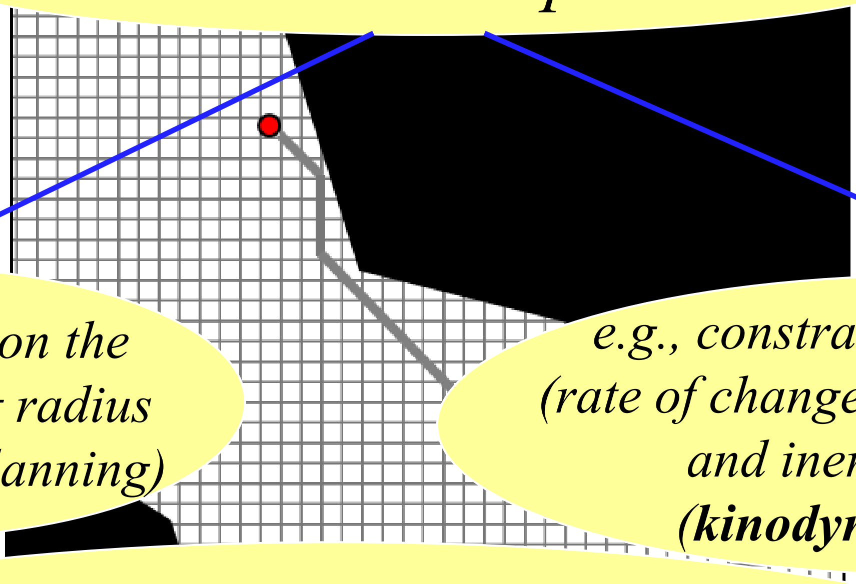


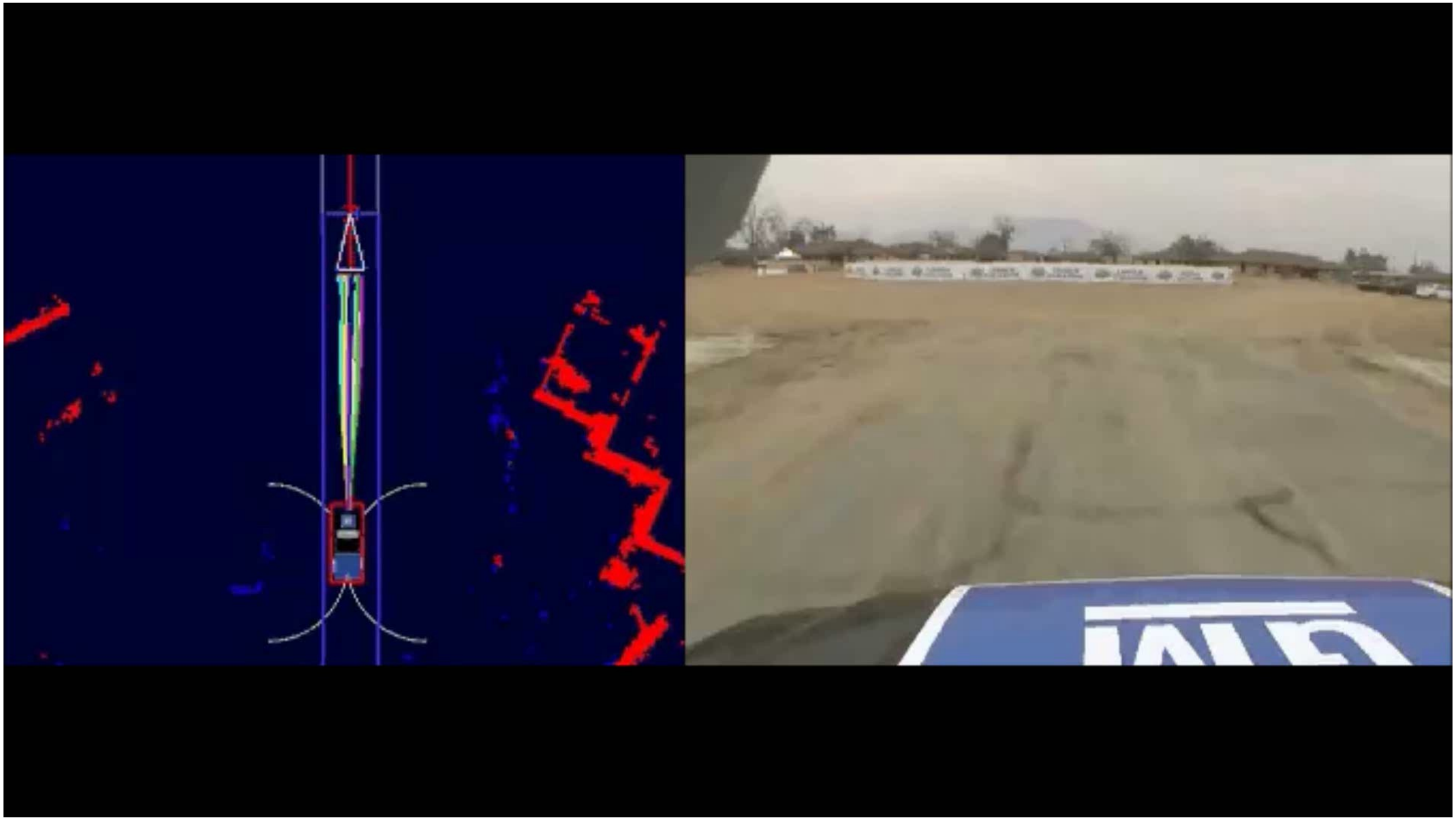
*e.g., constraints on the minimum turning radius (still **kinematic** planning)*

*e.g., constraints on turning rate (rate of change in wheel orientation) and inertial constraints (**kinodynamic** planning)*

***Kinodynamic planning**:*
*Planning representation includes $\{X, \dot{X}\}$,*
*where X-configuration and $\dot{X}$-derivative of X (dynamics of X)*

# Beyond Planning for Omnidirectional Robots

*(x,y,Θ,v) planning*
*with Anytime D* (Anytime Incremental A*) on Lattice Graphs*



# Beyond Planning for Omnidirectional Robots

*(x,y,Θ) planning with ARA*-based algorithm on Lattice Graphs*

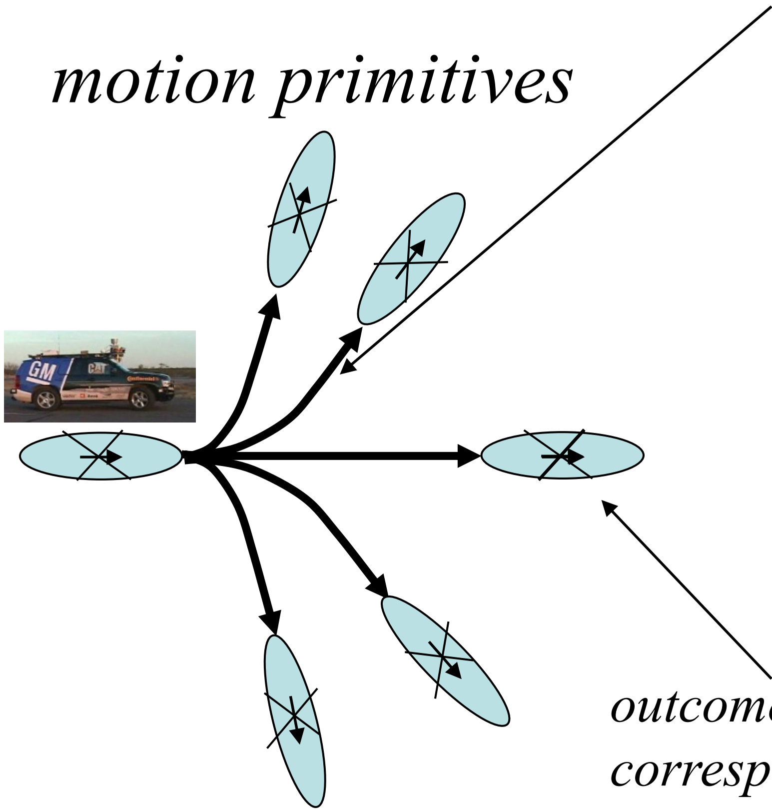
*Joint work with V. Kumar (Upenn), I. Kaminer (NPS) and V. Dobrokhodov (NPS)*
*[thakur et al., '13]*

# Lattice Graphs

- Graph *{V, E}* where
  - *V*: centers of the grid-cells
  - *E*: motion primitives that connect centers of cells via short-term **feasible** motions

*each transition is feasible*
*(typically, constructed beforehand)*

*motion primitives*




*outcome state is the center of the*
*corresponding cell in a grid*

# Lattice Graphs

- Graph *{V, E}* where
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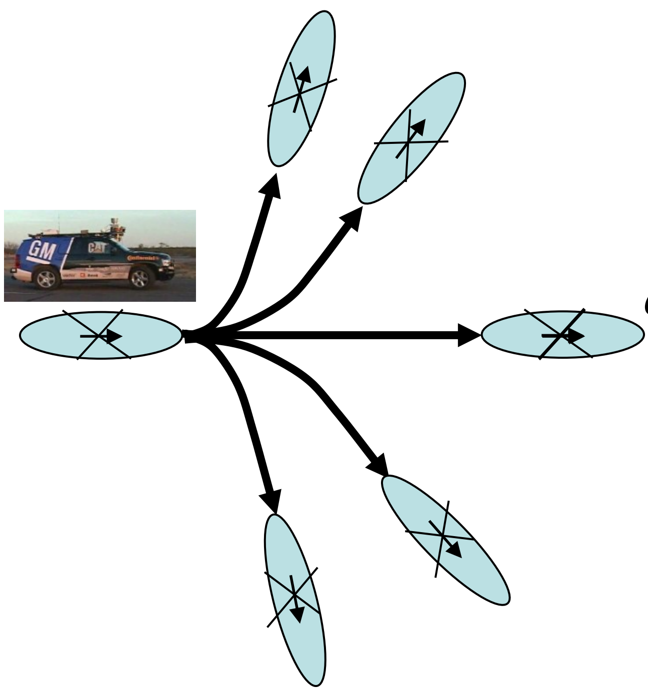
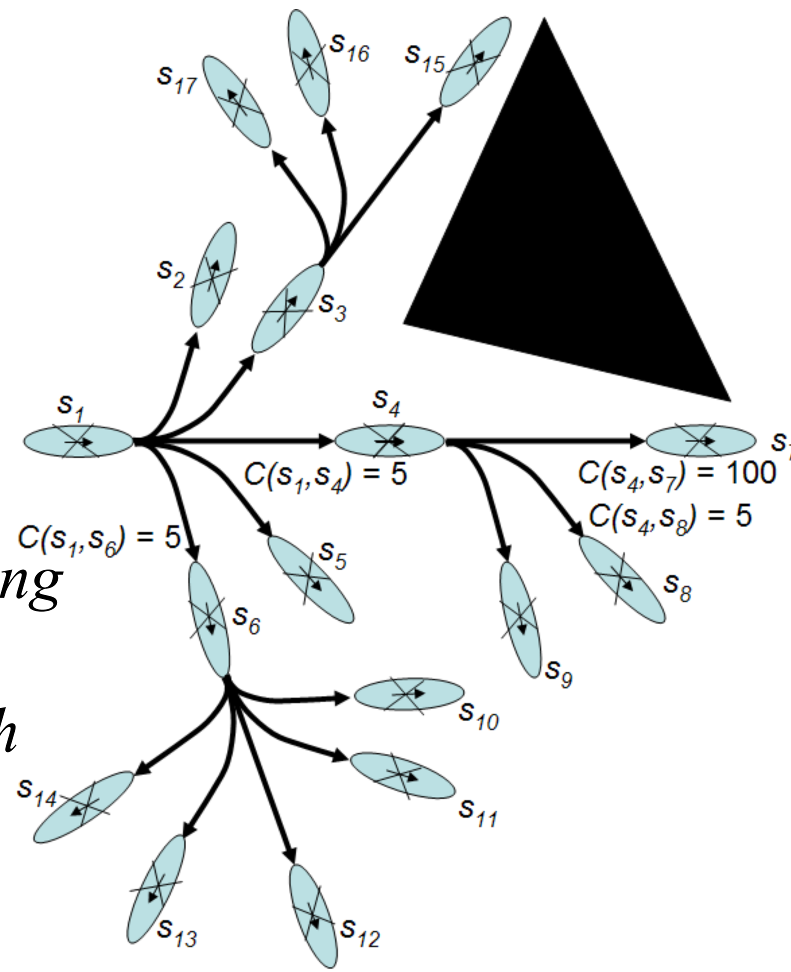
*motion primitives*



*replicate it during planning to generate lattice graph*



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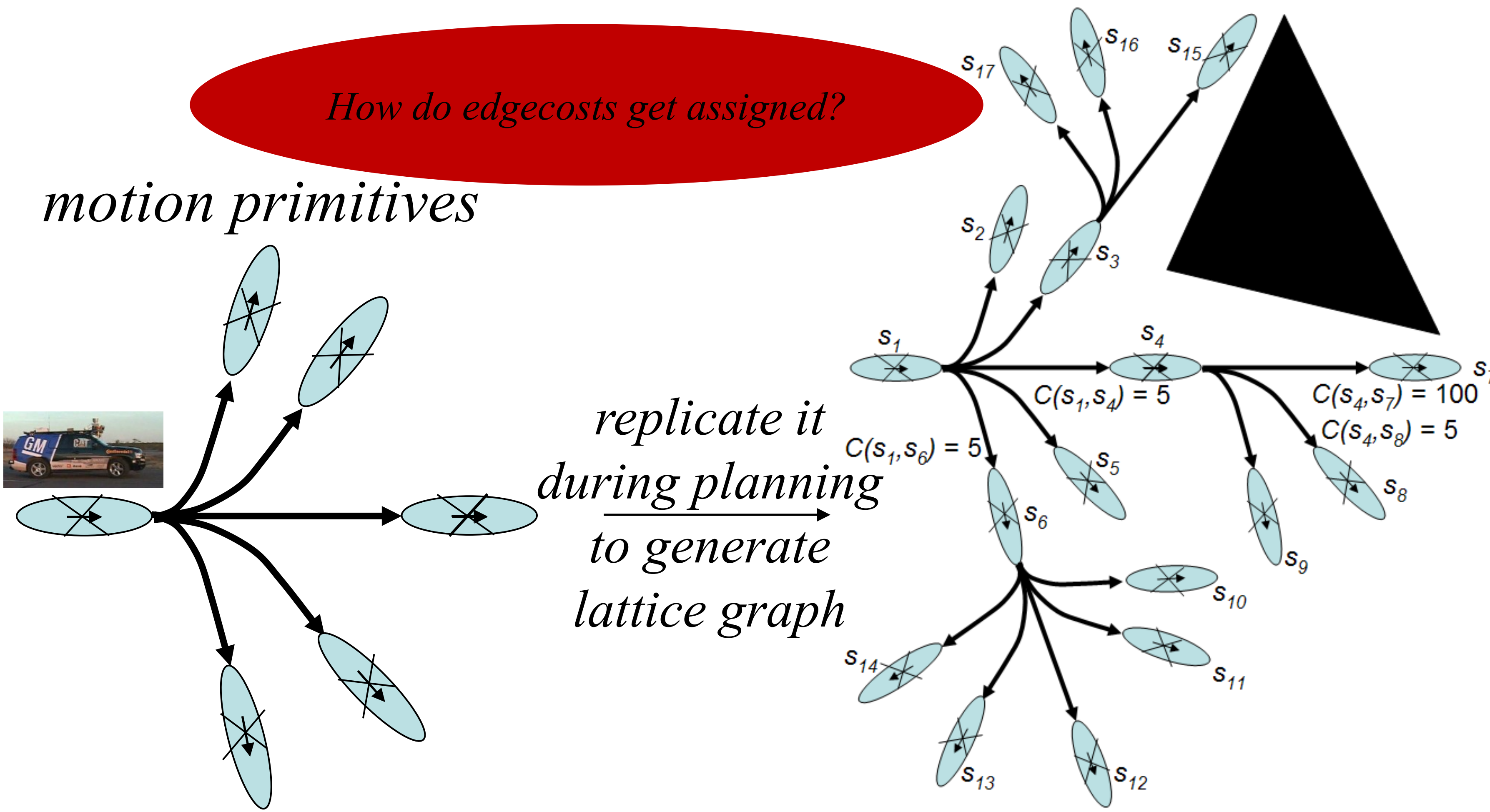


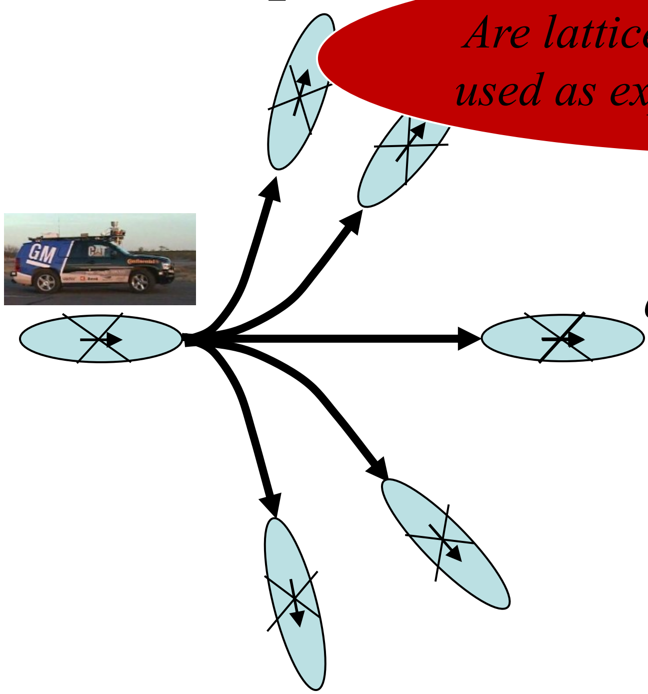
# Lattice Graphs

- Graph *{V, E}* where
  - *V*: centers of the grid-cells
  - *E*: motion primitives that connect centers of cells via short-term **feasible** motions

*How do edgecosts get assigned?*

*motion primitives*

*Are lattice-based graphs typically used as explicit or implicit graphs?*



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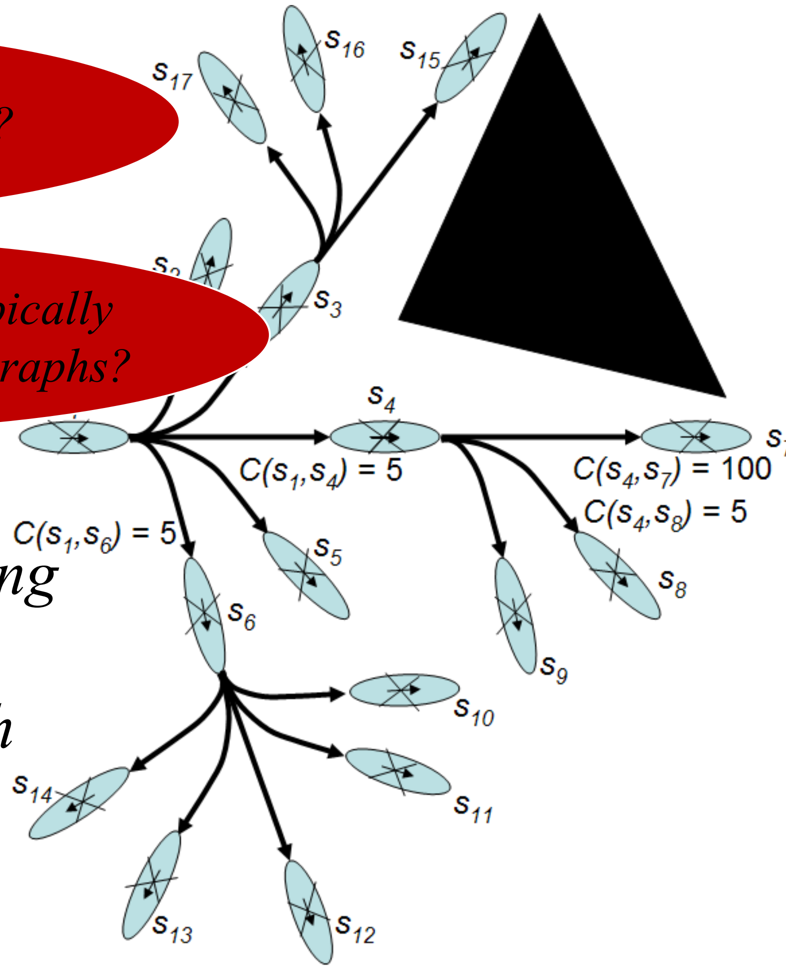

# Lattice Graphs

- **Board example** for *$(x,y,\Theta)$* planning for a unicycle model (minimum turning radius)

# What You Should Know…

- What visibility graphs are
- What Voronoi diagram-based graphs are
- X-connected N-dimensional grids
- Configuration Space, C-Space Transform
- Lattice-based graphs
- Explicit vs. Implicit graphs and pros/cons of each