Planning Techniques for Robotics

Multi-Robot Planning

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Different Categorizations of Multi-Robot Planning

• Centralized vs. Decentralized

  – **Centralized**: one central control of (planning for) all the robots

  – **Decentralized**: each robot decides/plans what to do on its own
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Why do we need decentralized planning approaches?
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*Why do we need decentralized planning approaches?*

*Robust to limits on or loss of communication*
*Robust to loosing some robots in the team*
*Computationally more scalable*
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Challenges with decentralized planning?
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Why do we need decentralized planning approaches?

Robust to limits on or loss of communication
Robust to loosing some robots in the team
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Challenges with decentralized planning?

How to guarantee that the overall team accomplishes its goal?
Different Categorizations of Multi-Robot Planning

• Multi-robot Path Planning vs. Multi-robot Cooperative Task Planning

  – **Multi-robot Path Planning**: how to plan paths for $N$ robots so that they don’t collide with each other during execution

  – **Multi-robot Cooperative Task Planning**: how to compute plans for $N$ robots so that they achieve the overall goal that may require cooperation
Different Categorizations of Multi-Robot Planning

• Small teams vs. large teams (swarms) of robots

  – Planning for small teams: Compute plans for $N$ (potentially heterogeneous) robots, where $N$ is typically 2-10

  – Planning for (control of) swarms of robots: how to control a swarm of $N$ (usually homogeneous) robots, where $N$ is typically 10-1000
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*Control of swarms is typically decentralized*
Different Categorizations of Multi-Robot Planning

- Joint state-space vs. distributed planning (within centralized)

  - **Joint state-space planning**: Planning for $N$ robots in a state-space that represents joint configurations of robots

  - **Distributed planning**: Planning is split into $N$ individual planners that share their results (and potentially re-plan) to obtain a final plan for all $N$ robots

What Planning approach to take?

- **Centralized Planning**
- **Distributed Planning**
- **Joint state-space Planning**
- **Distributed Planning**
Different Categorizations of Multi-Robot Planning

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  ![Planning Approach Diagram]

  **What Planning approach to take?**

  - $N$ is small (e.g., < 10) and inter-robot communications is available
    - **Centralized Planning**
  - $N$ is large (swarms) or communications are limited/unavailable
    - **Decentralized Planning**
  - $N$ is very small (e.g., < 3)
    - **Joint state-space Planning**
  - $N$ is not very small (e.g., ≥ 3)
    - **Distributed Planning**
Multi-Robot Path Planning

• Path planning for $N$ robots to get to their goals w/o collisions

\textit{simple example for two omnidirectional point-size robots}

\begin{center}
\begin{tikzpicture}
\begin{scope}
\draw[step=1cm, thin, gray] (0,0) grid (4,4);
\node at (0.5,4.5) {A}; \node at (1.5,4.5) {B}; \node at (2.5,4.5) {C}; \node at (3.5,4.5) {D}; \node at (0.5,3.5) {E}; \node at (1.5,3.5) {F};
\node[anchor=north,rotate=90] at (0.5,4) {1}; \node[anchor=north,rotate=90] at (1.5,4) {2}; \node[anchor=north,rotate=90] at (2.5,4) {3}; \node[anchor=north,rotate=90] at (3.5,4) {4};
\node[anchor=south,rotate=90] at (0.75,4.75) {$R2$}; \node[anchor=south,rotate=90] at (1.75,4.75) {$G1$}; \node[anchor=south,rotate=90] at (2.75,4.75) {$G2$};
\node[anchor=south,rotate=90] at (0.75,3.75) {$R1$}; \node[anchor=south,rotate=90] at (1.75,3.75) {$G1$}; \node[anchor=south,rotate=90] at (2.75,3.75) {$G2$};
\end{scope}
\end{tikzpicture}
\end{center}

\textit{start positions of the robots}

\textit{goal positions of the robots}
Multi-Robot Path Planning

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*simple example for two omnidirectional point-size robots*

Any examples of this in industry?
Multi-Robot Path Planning

- Path planning for $N$ robots to get to their goals w/o collisions

*Joint state-space planning*
Multi-Robot Path Planning

- Path planning for $N$ robots to get to their goals w/o collisions

**Joint state-space planning**

The simplest approach: construct and search a graph, where each state encodes positions of all the robots and each action encodes all possible movements.
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**Joint state-space planning**

- $R2 = A1$, $R1 = A3$  
  - $R2$ moves east, $R1$ moves east
- $R2 = B1$, $R1 = B3$  
  - $R2$ moves east, $R1$ moves north
- $R2 = B1$, $R1 = A2$  
  - $R2$ moves east, $R1$ moves north
- $R2 = F3$, $R1 = F1$  
  - Goal state

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<th>C</th>
<th>D</th>
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</tr>
</tbody>
</table>
Multi-Robot Path Planning

• Path planning for $N$ robots to get to their goals w/o collisions

Assuming 4-connected grid, what is the maximum branching factor (how many actions/successors)?

Joint state-space planning

R2 moves east, R1 moves east

R2 moves east, R1 moves north

R2=A1
R1=A3

R2=B1
R1=B3

R2=B1
R1=A2

... goal state

\[
\begin{array}{cccccc}
A & B & C & D & E & F \\
1 & & & & G1 & \\
2 & & & & & \\
3 & R1 & & G2 & & \\
4 & & & & & \\
\end{array}
\]
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Joint state-space planning

What is the size of the graph?

What is the size of the graph?
Multi-Robot Path Planning

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Joint state-space planning

What is the size of the graph?

Scalability w.r.t. $N$ is clearly an issue!
Multi-Robot Path Planning

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Distributed planning

One popular approach: Prioritized Planning

For $i = 1:N$

Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1..R_{i-1}$
Multi-Robot Path Planning

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Distributed planning

Each planning needs to include time as a dimension!

One popular approach: Prioritized Planning

For $i = 1:N$

Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1...R_{i-1}$
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**Distributed planning**

What will be the plan returned by Prioritized Planning?

One popular approach: Prioritized Planning

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**Distributed planning**

![Graph showing distributed planning]

Is it complete?

Is it optimal?

What is the complexity of Prioritized Planning?

**One popular approach: Prioritized Planning**

For $i = 1:N$

Compute path for robot $R_i$ that avoids collisions with paths for robots $R_1..R_{i-1}$
Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for $N$ robotic arms to move an object

[Cohen et al., '14]
(performs joint state-space planning)
Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for $N$ robotic arms to move an object

(planning is distributed: plan on Roman platform first, then on PR2)
Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for multi-robot exploration/mapping

\[ N \text{ robots need to explore and build a map of unknown environment} \]

One approach: Distributed Greedy Mapping

For \( i = 1 : N \)

Compute a path using Greedy Mapping approach for robot \( R_i \) taking into account what paths were computed for \( R_1..R_{i-1} \) (and what cells they would see)
Multi-Robot Cooperative Planning/Task Allocation

• Example: planning for multi-robot exploration/mapping

\[ N \text{ robots need to explore and build a map of unknown environment} \]

• Greedy Mapping for a single robot:
  – always move the robot on a shortest path to the closest unobserved (or unvisited) cell
  – it always achieves a gain in information.
  – thus, it is guaranteed to map the environment that is reachable (assuming all moves are reversible)

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• Example: planning for multi-robot exploration/mapping

_N robots need to explore and build a map of unknown environment_

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For _i = 1:N_

Compute a path using Greedy Mapping approach for robot _R_i_ taking into account what paths were computed for _R_1.._R_\(i-1\) (and what cells they would see)

[Butzke et al., ’11]
Multi-Robot Cooperative Planning/Task Allocation

• Example: planning for multi-robot exploration/mapping

\( N \) robots need to explore and build a map of unknown environment

\[ \text{A Planning Framework for Persistent, Multi-UAV Coverage with Global Deconfliction} \]

Submitted to the 12th Conference on Field and Service Robotics

Collaboration between

Search-Based Planning Lab, CMU (headed by M. Likhachev) and
Mitsubishi Heavy Industries (MHI)

[Kusner et al., '19]

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Multi-Robot Cooperative Planning/Task Allocation

• Market-based approach (very popular distributed approach)
  – Consider planning the allocation of tasks to $N$ robots
  – General scheme: *robots auction out their tasks to their teammates with the goal of increasing their own revenue*
Market-based Approach

Given \( N \) robots \( R_1 \ldots R_N \), \( M \) tasks \( T_1 \ldots T_M \), and \( C_{iR_j} \) – cost of executing task \( i \) by robot \( R_j \) (cost may depend on other tasks executed by this robot)

**Planner needs to decide:** Which task gets executed by which robot?

Find a plan (mapping) \( \pi^* : T_i \rightarrow R_j \) such that \( \pi^* = \arg\min \sum C_{i\pi(T_i)} \)
Market-based Approach

Given $N$ robots $R_1...R_N$, $M$ tasks $T_1...T_M$, and $C_{iR_j}$ – cost of executing task $i$ by robot $R_j$ (cost may depend on other tasks executed by this robot)

Iterate over steps 1-4 until convergence or planning time expires

Step 1: start with an arbitrary plan $\pi$

Step 2: all robots offer their tasks $T_i$ at auction at the max. price of $C_{Ti}^{\pi(Ti)} - \epsilon$

Step 3: all robots $R_j$ bid on the offered tasks $T_i$ with the bid $= C_{iR_j} + \epsilon$

Step 4: robots sell to the lowest bidders if they are below max. price and get profit: $C_{Ti}^{\pi(Ti)} - C_{iR_j}$
Market-based Approach

Given $N$ robots $R_1...R_N$, $M$ tasks $T_1...T_M$, and $C_{i}^{R_j}$ — cost of executing task $i$ by robot $R_j$ (cost may depend on other tasks executed by this robot)

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When does it converge in one iteration?
Planning for Leader-based Coordination

- **Fully decentralized approach** (doesn’t rely on the presence of communication between robots)
- Plan for the “leader” robot (sometimes leader can be just a centroid of the team or some other reference point)
- All other robots execute either “follow the leader” or “follow neighbors within field-of-view” behaviors while avoiding collisions
What You Should Know…

• Different styles of multi-robot planning
  – Centralized vs. decentralized
  – Joint state-space planning vs. distributed planning
  – Multi-robot path planning vs. cooperative task planning

• Prioritized Multi-robot Path Planning

• Market-based Approach to multi-robot planning