SODA Bites

Some interesting ideas from SODA 2003
Announcements

Join theory-students@cs.cmu.edu for discussions
(maintained by Charlie)

Theory Workshop
http://www.cs.cmu.edu/~maverick/TheoryWorkshop/
Disclaimer
“Learn Adobe Illustrator…”

Steve Jobs
My Job

Some Ideas from SODA 2003

-No CMU
-No Approximation
-No Geometry

I am not going over papers, I am going over ideas.
Your Job

Chip in ideas!

(Optionally, go read the papers.)
Certifying Algorithms for Recognizing Interval Graphs and Permutation Graphs

Dieter Kratsch, Ross M. McConnell, Kurt Mehlhorn, Jeremy P. Spinrad
Certifying Algorithms

An algorithm for a decision problem that provides a certificate with each answer it produces.
Bipartite Graph Recognition

Certificate

Show a 2-coloring

Co-certificate

Show an odd cycle
Planar Graph Recognition

Certificate

Show a planar embedding

Co-certificate

Show a $K_5$ or $K_{3,3}$ subdivision
Why Certifying Algorithms?

“In theory there is no difference between theory and practice. In practice there is!”

But when the algorithms get complicated...
Certificate is a form of “bug-free” proof...
Almost Catch-22

How do we verify a certificate?

Use another program as the verifier...
Certificate is a form of “bug-free” proof...

Make the verifier a certifying algorithm...
Hardware Reliability

- Design Mistakes
- Faults

Hard problem!
Software Reliability

“Simple” programs can be checked

Software Reliability via run-time result-checking
Wasserman, Blum [JACM 1997]

“Simple” programs can be proved correct

This simplicity is a subjective measure
An Interesting Perspective

Let's call a certificate “strong” if the verifier takes less time than known algorithms for the original problem.

Otherwise, call it “weak”.

An Example

Bipartite Graph Recognition

Just try to do a 2-coloring

$O(n + m)$ time
Bipartite Graph Co-certificate

What needs to be verified?

- It’s a cycle.
- It’s an odd cycle.
- It’s an odd cycle of the input graph.

\(\langle \text{A sequence of pointers to edges in the input} \rangle\)

Bottom line: \(O(n)\) time
Bipartite Graph Recognition

... has a strong co-certificate.

Q: How about the certificate?

 Mey Still need to read and verify the whole 2-coloring...

A: Weak
A Phenomenon

Authors assert that if one certificate is strong, the other is typically weak (p159).

- Bipartite Graph
- Interval Graph
- Permutation Graph

There are other examples too...
I wonder...

What can we say about this phenomenon?

Lower bound on $O(\max(t, t'))$, or $O(t \times t')$?

How about harder problems like 3SAT?

Anyone?
Improved Bounds on the Average Length of Longest Common Subsequence

George S. Lueker
LCS (not the one in MIT)

Let $L_n$ be the length of the Longest Common Subsequence of two random binary strings of length $n$.

Guess $E[L_n]$. 

Good To Know

Sure, you all guessed it’s $\Theta(n)$.

What’s the constant?

$0.7880 - 0.8263$
Good To Know
Selection with Monotone Comparison Costs

Sampath Kannan, Sanjeeve Khanna
Selection

Given a list of $n$ unsorted elements from a total order, find the $r$-th element in the sorted order.

$r$ is called the rank of the element.
**Classic O(n) Algorithm**

*Time Bounds for Selection*

Blum, Floyd, Pratt, Rivest, Tarjan  

[JCSS 1973]
In Practice

Comparisons are not constant time for complex objects.

- Strings
- Databases
Each object has an associated “size”
Comparison cost is a function of the two sizes

Sum, Product, Minimum, Nuts&Bolts…
Monotone Cost

Increasing the size of one of the objects do not decrease the cost of the comparison

✓ Sum, Product, Minimum

✗ Nuts&Bolts
Competitive Analysis

Cost incurred by our algorithm to compute the function value

vs.

Cost incurred by an optimal proof that the function has such value
Optimal Selection Proof

For each element $x$ whose rank is not $r$, compare $x$ to the smallest $y$ that lies between $x$ and the rank-$r$ element.
The Challenge

Cost of optimal proof is highly sensitive to the rank of the element being selected.

♫ Many algorithms find pivots whose ranks approach \( r \) over iterations.
♫ Let’s say we are given the element of rank \( r+1 \).
Can we do a pivot using it?
Why MoF won’t work

Consider the sum cost function
Let’s Try This

Find the maximum element under the sum cost function.
Almost Old School

10 sort $x_i$’s by weight
20 maxIndex = 1
30 for $i = 2$ to $n$
40 if $x_i > x_{\text{maxIndex}}$ then maxIndex = $i$
50 next $i$
60 return maxIndex
2-competitive Analysis

The $i$-th comparison costs at most $w_i + w_{i+1}$.

The total cost is at most

$$w_1 + 2 \sum_{i=2}^{n-1} w_i + w_n$$

Optimal proof must read each element: $\sum w_i$
Sorting

Selection’s big brother

The optimal proof is even more straightforward

Anyone?
Pursuit-Evasion with Imprecise Target Location

Günther Rote
A Chasing Game

Sheriff and Thief

Travel at the same speed
Thief “Position”

Thief can choose a nearby point to reveal as its location
Realistic Enough

Sheriff makes decision based on the point reveal by Thief

 espos ; Can Thief get away?
 (increase its distance from Sheriff indefinitely)
Alas

We don’t live inside a Hollywood movie...

Bad guys can win.

In Euclidean space, a smart Thief can increase the distance at a rate of

$$\Omega(\frac{3}{\sqrt{t}})$$
My Ideal Solution
I wonder

If the Sheriff can travel \((1 + \varepsilon)\) faster, do we still need a vigilante?
Directed Graphs Requiring Large Numbers of Shortcuts

William Hesse
Transitive Closure

Some algorithms depend on the diameter of the graph.

Shortcuts
Thorup’s Conjecture

The diameter of a directed graph can be made polylog by adding a linear number of shortcuts.

Thorup showed this is true for planar graphs.

This paper says: NO
Efficient Sequences of Trials

Edith Cohen, Amos Fiat, Haim Kaplan
An incompetent attorney can delay a trial for months or years. A competent attorney can delay one even longer.

--- Evelle J. Younger (1918 - 1989)  
LA Times, March 3rd, 1971