Sense Making Alone Doesn't Do It: Fluency Matters Too! ITS Support for Robust Learning with Multiple Representations

Martina A. Rau[1], Vincent Aleven[1], Nikol Rummel[1,2], and Stacie Rohrbach[3]

[1] Human-Computer Interaction Institute, Carnegie Mellon University
[2] Institute of Educational Research, Ruhr-Universität Bochum, Germany
[3] School of Design, Carnegie Mellon University
{marau, aleven}@cs.cmu.edu, nikol.rummel@rub.de, stacie@cmu.edu

Abstract. Previous research demonstrates that multiple representations of learning content can enhance students' learning, but also that students learn deeply from multiple representations only if the learning environment supports them in making connections between the representations. We hypothesized that connection-making support is most effective if it helps students *make* both in *making sense* of the content across representations and in *becoming fluent* in making connections. We tested this hypothesis in a classroom experiment with 599 4th- and 5th-grade students using an ITS for fractions. The experiment further contrasted two forms of support for sense making: *auto-linked* representations and the use of *worked examples* involving one representation to guide work with another. Results confirm our main hypothesis: A combination of worked examples and fluency support lead to more robust learning than versions of the ITS without connection-making support. Therefore, combining different types of connection-making support is crucial in promoting students' deep learning from multiple representations.

Keywords: Multiple representations, fractions, intelligent tutoring system, connection making, classroom evaluation

1 Introduction

Multiple representations, such as charts and diagrams in mathematics, are universally used in instructional materials because they can emphasize important aspects of the learning content. Representations as learning tools may be especially beneficial when incorporated in intelligent tutoring systems (ITSs): rather than working with static representations, students can interact with virtual manipulatives [1], and they can be tutored on their interactions with them. There is extensive evidence in the educational psychology literature that learning with multiple representations can enhance students' deep understanding of the domain [2,3]. However, research has also shown that, in order to benefit from multiple representations, students need to make connections between them [2,4,5]. Yet, students find it difficult to make these connections [2] and tend not to make them spontaneously [2,6]. Therefore, they need to be supported in doing so [7].

In the domain of fractions, multiple representations such as circles, rectangles, and number lines are commonly used [8]. Each representation provides a different conceptual view on fractions [9]. In order to gain a deep understanding of fractions, students need to understand the conceptual views presented by each representation, and they need to relate the representations to one another [8,10]. Being able to relate these different representations is key to developing a deep understanding of fractions (e.g., as numbers that have magnitudes), which is an important educational goal [10].

A crucial question when designing learning environments that use multiple representations is therefore what kind of connection-making support will promote deep learning. Following the KLI theoretical framework for robust learning [11], we distinguish between two types of learning processes: *sense-making* processes and *fluency-building* processes. *Making sense of connections* means (in the case of fractions) that students conceptually understand how different representations relate to each other (e.g., *why* two representations show the same fraction). *Fluently making connections* means to fast and effortlessly relate different representations (e.g., representations that show the same value). Prior research on how best to support students in making connections between multiple representations has focused only on supporting sense-making processes, for instance, by supporting students in relating corresponding elements of representations at a structural level [12]. However, both types of learning processes may be necessary in order to develop competence in a complex domain [11]. Applying this notion to learning with multiple representations, we hypothesize that students learn most robustly when, in addition to being supported in making sense of connections between multiple representations, they are supported in fluently making connections between multiple representations.

A crucial question regarding sense-making support is further: how much automated support should students receive from the system [2]? On the one hand, providing students with auto-linked representations (AL), in which the system, rather than the student, connects and updates representations, has been shown to enhance learning in complex domains [5]. On the other hand, research has demonstrated that students should actively create connections between representations, rather than passively observing correspondences [13]. Thus, we compare two ways of sense-making support, one in which the tutor demonstrates connections (i.e., auto-linked representations, AL), one in which more of that burden falls on the student. A well-researched way of supporting active sense-making processes is to provide students with worked examples (WEs), that is, solved problems with solution steps shown [14]. WEs have been shown to be effective in many domains [14], and have been used in ITSs (e.g., [15]). Berthold and Renkl [16] compared students' learning from multi-representational WEs to single-representation WEs and found that multiple representations can enhance students' learning from WEs. However, to our knowledge, WEs have not yet been used as a means to support students in making connections between multiple representations. In our study, students use a WE that uses a more familiar representation as a guide to solve an isomorphic problem that involves a less familiar representation. As they integrate the example problem and the new problem, they can make connections between the two representations. We hypothesize that WE support (compared to AL support) will be the more effective type of sense-making support in promoting students' learning of fractions, since students have to engage more actively in making connections.

We address these hypotheses in the context of a proven ITS technology, namely, Cognitive Tutors [17]. The Fractions Tutor has been tested and iteratively improved based on five experimental studies with almost 3,000 students. Although Cognitive Tutors have been widely researched with middle- and high-school students [18] (e.g., Rittle-Johnson and Koedinger [19] report on a study in which 6th-graders used a Cognitive Tutor for fractions), the effectiveness of Cognitive Tutors and other ITSs for elementary-school students remains under-researched.

We conducted a classroom experiment to investigate the effects of sense-making support for connection making and of fluency support for connection making on students' understanding of fractions. 599 4th- and 5th-grade students worked with the Fractions Tutor during their regular mathematics class. Students either received sense-making support for connection making (AL or WE) or not. This factor was crossed with a second experimental factor, namely, whether or not students received fluency support for connection making. Since many education researchers and practitioners emphasize the importance of helping students understand number lines [8,10], we included a version of the Fractions Tutor that provides only a number line as a control condition.

2 Methods

2.1 Fractions Tutor

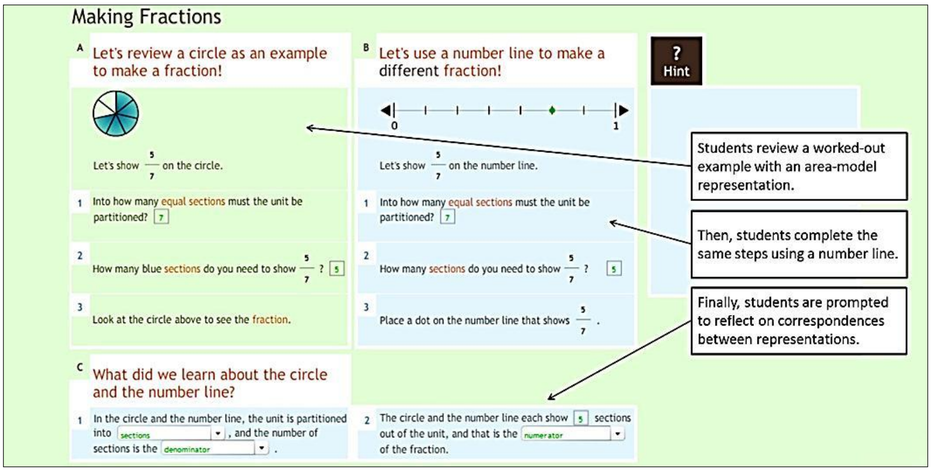

Fig. 1. Example of sense-making support: worked-example problem.

The ITS used in the present study used three different interactive representations of fractions: circles, rectangles, and number lines. Each representation emphasizes certain aspects of different conceptual interpretations of fractions [9]. The circle as a part-whole representation depicts fractions as parts of an area that is partitioned into equally-sized pieces. The rectangle is a more elaborate part-whole representation as it can be partitioned vertically and horizontally. At the same time, it does not have a standard shape for the unit, like the circle does. Finally, the number line is considered a measurement representation and thus emphasizes that fractions can be compared in terms of their magnitude, and that they fall between whole numbers.

The Fractions Tutor covers a comprehensive set of ten topics including interpreting representations, reconstructing the unit of fraction representations, improper fractions from representations, equivalent fractions, fraction comparison, fraction addition and subtraction. In our classroom study, students in all conditions first worked on six introductory problems that introduced the representations. They then worked on eight problems per fractions topic, yielding a total of 80 tutor problems. The sequence of tutor problems included both single-representation problems and (in the connection-making support conditions) multiple-representation problems.

To support students in making connections between the different representations, we created three new types of tutor problems. WE problems and AL problems were designed to provide sense-making support. Each was designed to emphasize conceptual correspondences between the two representations. In the WE problems (see Fig. 1), an example of a solved problem with a familiar representation (i.e., circle or rectangle) was displayed on the left. This worked example contained filled-in answers for all except for the last step. After the student filled in the last step of the worked example, an isomorphic problem with a less familiar representation (number line) showed up on the right. The worked example served to guide students' work on this problem. To solve the problem, students manipulated the interactive number line. The AL problems followed the same side-by-side format with problem steps lined up, but there was no WE. Rather, as students completed the steps in the number line problem, the area model representation updated automatically to mimic the steps the student performed on the number line. In this sense, the more familiar representation provided feedback on the work with the less familiar representation. (To make this work at a technical level, we extended the CTAT tools [20] so that the number line component could serve as a controller for the area model component.) The WE and the AL problems included self-explanation prompts at the end of each problem (see bottom of Fig. 1) which asked students to identify correspondences of the two given representations.

The third type of connection-making problems, mixed representation problems (Mix; see Fig. 2), were designed to help students become fluent in connecting representations. Given a set of representations of fractions, students grouped them (through drag-and-drop) according to the fraction they represent. Students had to drag each individual graphical representation into the correct drop area labeled with a symbolic fraction. Students could drag-and-drop the fraction representations in any order. The drop area was able to detect which graphical representation the student drag-and-dropped into it, and could thereby give error feedback accordingly, when necessary. In each problem, multiple representations matched the same symbolic fraction.

Students received error feedback and hints on all steps. Hint messages and error feedback messages were designed to give conceptually oriented help, often in relation to the representations. The single-representation problems included prompts to help students relate the representations to the symbolic fractions. We had found these prompts to be effective in an earlier experimental study [3].

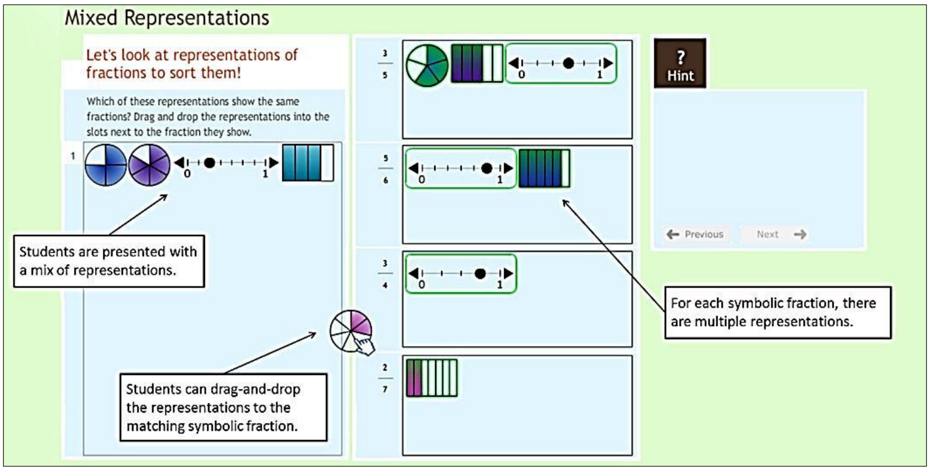

Fig. 2. Example of fluency support: mixed representations problem

2.2 Test Instruments

We assessed students' knowledge of fractions at three test times. We created three equivalent test forms. Based on data from a pilot study with 61 4th-grade students, we made sure that the difficulty level of the test was appropriate for the target age group, and that the different test forms did not differ in difficulty. In our classroom study, we randomized the order in which the different test forms were administered.

The tests targeted two knowledge types: procedural and conceptual knowledge. The conceptual knowledge scale assessed students' principled understanding of fractions. The test items included reconstructing the unit, identifying fractions from graphical representations, proportional reasoning questions, and verbal reasoning questions about comparison tasks. The procedural knowledge scale assessed students' ability to solve questions by applying algorithms. The test items included finding a fraction between two given fractions using representations, finding equivalent fractions, addition, and subtraction. The theoretical structure of the test (i.e., the two knowledge types just mentioned) was based on a factor analysis with the pretest data from the current experiment. We validated the resulting factor structure using the data from the immediate and the delayed posttests.

2.3 Experimental Design and Procedure

In the present paper, we report the data from 599 4th- and 5th-grade students from one school district with 5 different elementary schools (25 classes) in the United States. Students participated in the study as part of their regular mathematics instruction. All students worked with versions of the Fractions Tutor designed and created specifically for this study. Students were randomly assigned to one of the conditions shown in Table 1. We used a 2 (fluency support) x 3 (sense-making support) + 1 (NL control condition) experimental design to investigate the effects of connection making support on students' learning of fractions. The fluency support factor had two levels: students either received Mix problems as fluency support, or no fluency support. The sense-making support factor had three levels: students either received WE problems or AL problems as sense-making support, or no sense-making support.

We assessed students' knowledge of fractions three times. On the first day, students completed a 30-minute pretest. They then worked on the Fractions Tutor for about ten hours, spread across consecutive school days. The day following the tutor sessions, students completed a 30-minute posttest. About one week after the posttest, we gave students an equivalent delayed posttest.

Table 1. Experimental conditions[1] included in the experimental study.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Sense-making support</td><td rowspan="4">Control</td></tr>
<tr><td>None</td><td>Auto-linked representations</td><td>Worked example</td></tr>
<tr><td rowspan="2">Fluency support</td><td>None</td><td>MGR</td><td>AL</td><td>WE</td></tr>
<tr><td>Mixed representations</td><td>Mix</td><td>AL-Mix</td><td>WE-Mix</td></tr>
<tr><td colspan="5">Control</td><td>NL</td></tr>
</table>

3 Results

Table 2. Proportion correct: means (and standard deviation) for conceptual and procedural knowledge at pretest, immediate posttest, delayed posttest. Min. score is 0, max. score is 1.

		pretest	immediate posttest	delayed posttest
conceptual knowledge	MGR	.33 (.20)	.45 (.23)	.48 (.26)
	AL	.38 (.20)	.49 (.23)	.51 (.26)
	WE	.36 (.22)	.43 (.20)	.49 (.26)
	Mix	.31 (.21)	.37 (.22)	.44 (.24)
	AL-Mix	.36 (.20)	.43 (.24)	.49 (.25)
	WE-Mix	.39 (.21)	.52 (.24)	.58 (.26)
	NL	.37 (.20)	.43 (.25)	.48 (.20)
procedural knowledge	MGR	.25 (.25)	.30 (.28)	.30 (.26)
	AL	.21 (.18)	.26 (.24)	.26 (.24)
	WE	.26 (.21)	.29 (.24)	.31 (.27)
	Mix	.19 (.17)	.23 (.20)	.25 (.22)
	AL-Mix	.20 (.18)	.25 (.21)	.26 (.21)
	WE-Mix	.26 (.20)	.32 (.26)	.33 (.26)
	NL	.21 (.20)	.25 (.22)	.27 (.23)

[1] MGR = multiple graphical representations, AL = auto-linked representations, WE = worked examples, Mix = mixed representations, NL = number line

Students who completed all tests, and who completed their work on the tutoring system were included in the analysis, yielding a total of $N = 428$. The number of students who were excluded from the analysis did not differ between conditions, χ^2 (6, $N = 169$) = 4.34, $p > .10$. Table 2 shows the means and standard deviations for the conceptual and procedural knowledge scales by test time and condition.

A hierarchical linear model (HLM; [21]) with four nested levels was used to analyze the data. HLMs are regression models that take into account nested sources of variability [21]. HLMs allow for significance testing in the same way as regular regression analyses do. We modeled performance for each of the three tests for each student (level 1), differences between students (level 2), differences between classes (level 3), and between schools (level 4). More specifically, we fit the following HLM:

$$score_{ij} = test_j + sense_k + fluency_l + sense_k*fluency_l + pre_i*sense_k + pre_i*fluency_l + student(class)_i + class(school)_i + school_i, \quad \textbf{1)}$$

with the dependent variable $score_{ij}$ being $student_i$'s score on the dependent measures at $test_j$ (i.e., immediate or delayed posttest). $Sense_k$ indicates whether or not $student_i$ received sense-making support, and $fluency_l$ indicates whether $student_i$ received fluency support. In order to analyze whether students with different levels of prior knowledge benefit differently from connection-making support, we included students' pretest scores as a covariate (pre_i), and modeled the interaction of pretest score with sense-making support ($pre_i*sense_k$), and with fluency support ($pre_i*fluency_l$). $Student(class)_i$, $class(school)_i$, and $school_i$ indicate the nested sources of variability due to the fact that $student_i$ was in a particular class of a particular school. The reported *p*-values were adjusted for multiple comparisons using the Bonferroni correction. We report partial η^2 for effect sizes on main effects and interactions between factors, and Cohen's *d* for effect sizes of pairwise comparisons. An effect size partial η^2 of .01 corresponds to a small effect, .06 to a medium effect, and .14 to a large effect. An effect size *d* of .20 corresponds to a small effect, .50 to a medium effect, and .80 to a large effect.

3.1 Effects of Connection-Making Support

We had expected that a combination of fluency support and sense-making support for connection making would lead to better results than either sense-making or fluency support alone. The results confirm our hypothesis for conceptual knowledge: we found a significant interaction effect between sense-making and fluency support on conceptual knowledge, $F(2, 351) = 3.97$, $p < .05$, p. η^2 =.03, such that students who received both types of support performed best on the conceptual knowledge posttests. The main effects of sense-making and fluency support were not significant (*F*s < 1). There was no significant interaction effect on procedural knowledge ($F < 1$).

We had further predicted that WE problems would be the more effective type of sense-making support compared to AL problems. The results confirm this hypothesis for the conditions that received fluency support. Effect slices for the effect of sense-making support (i.e., a test of the effect of sense-making support for each level of the fluency support factor) showed that there was a significant effect of sense-making support within the conditions with fluency support on conceptual knowledge, *F*(2,

343) = 4.34, $p < .05$, p. η^2 =.07, but not within the conditions without fluency support ($F < 1$). *Post-hoc* comparisons between the Mix, AL-Mix, and the WE-Mix conditions confirmed that the WE-Mix condition significantly outperformed the Mix condition, $t(341) = 2.82$, $p < .01$, $d = .32$, and the AL-Mix condition $t(342) = 2.20$, $p < .05$, $d = .26$, on conceptual knowledge. In summary, WE problems are more effective in supporting sense-making of connections than AL problems, provided that students also receive fluency support.

Finally, to verify the advantage of receiving connection-making support over the NL control condition, we compared the most successful condition (WE-Mix) to the NL condition using *post-hoc* comparisons. The advantage of the WE-Mix condition over the NL was significant on conceptual knowledge, $t(115) = 2.41$, $p < .05$, $d = .27$.

3.2 Learning Effects

To investigate whether students learned from the pretest to the immediate posttest and to the delayed posttest across conditions, we modified the HLM and treated pretest scores as dependent variables, not as covariates (i.e., pre_i, pre_i*$sense_k$, and pre_i*$fluency_l$ were excluded from the model in equation 1). The main effect for test was significant on procedural knowledge, $F(2, 842) = 43.04$, $p < .01$, p. η^2 =.01, and conceptual knowledge, $F(2, 842) = 98.56$, $p < .01$, p. η^2 =.11. Students in all conditions performed significantly better at the immediate posttest than at the pretest on conceptual knowledge, $t(842) = 9.15$, $p < .01$, $d = .40$ and on procedural knowledge, $t(842) = 7.15$, $p < .01$, $d = .20$. Similarly, students performed significantly better at the delayed posttest than at the pretest on conceptual knowledge, $t(842) = 13.80$, $p < .01$, $d = .60$ and on procedural knowledge, $t(842) = 8.70$, $p < .01$, $d = .24$.

4 Discussion and Conclusion

We had hypothesized that students would learn most robustly about fractions when being supported both in making sense of connections and in fluently making connections between multiple representations. Our results confirm this hypothesis for students' conceptual understanding of fractions: robust conceptual learning with multiple representations is enhanced by a combination of fluency support and sense-making support for connection making. We did not find effects of connection-making support on procedural knowledge. This finding is not surprising: it is conceivable that making connections between multiple representations benefits students' principled understanding of fractions but not their algorithmic knowledge of operations.

The fact that we did not find main effects of sense-making support and fluency support for connection making, on the other hand, is surprising: it shows that each type of connection-making support alone is not effective, but that the combination of both is needed to enhance students' conceptual understanding of fractions. This finding is particularly interesting because prior research on connection making has mostly focused on sense-making processes by supporting connection making of structurally equivalent elements. Our results suggest that standard sense-making support for connection making should be extended by also supporting fluency in making connections. It is possible that fluency activities allow students to deepen the conceptual knowledge about connections they acquired through sense-making activities.

With respect to *how* best to support sense making, our finding that WE support leads to better learning than AL support demonstrates, in line with earlier research on connection making [13], that students need to actively create connections between representations. We show that a novel application of WEs is effective in supporting active connection making. This finding extends the existing literature on WEs by showing that they can help students benefit from multiple representations when used as a means to support sense-making of connections.

As predicted, the advantage for combining fluency and sense-making support for connection making was also significant compared to the control condition who worked only with number lines. Number lines are often considered the most important graphical representation of fractions [10], which may lead teachers to use only number lines in fractions instruction. However, our findings show that with effective connection-making support, multiple representations of fractions can facilitate the acquisition of conceptual knowledge more so than practicing only the number line.

Finally, our results demonstrate significant learning gains for students who worked with the Fractions Tutor during their regular mathematics class. The gains persist at least until one week after the study when we administered the delayed posttest. This finding extends the ITS literature by demonstrating the effectiveness of a Cognitive Tutor for elementary-school students. Evaluation studies with ITSs have focused far more on high schools and middle schools than elementary schools [18,19]. Furthermore, the substantial and robust learning gains are encouraging, given that fractions are a difficult topic for elementary and middle-school students – a fact that provides a major obstacle for later mathematics learning, such as in algebra [8]. Our ITS for fractions is effective in helping students overcome some of these difficulties.

In conclusion, the present experiment extends the ITS and educational psychology literature on learning with multiple representations in several ways. First, our findings show that, although prior research has conceived of connection making as primarily a sense-making process, effective connection making involves fluency processes and therefore requires activities aimed at supporting sense making *and* activities aimed at supporting fluency. Second, we demonstrate that students need to be *active* in making connections between representations, and that a novel application of *worked examples* is effective in helping students to accomplish this difficult task. Third, the study provides insight into *the type of knowledge* for which connection-making support is beneficial. Connection-making support does not benefit students in learning to apply algorithms to solve procedural tasks, but it helps them acquire conceptual knowledge of domain principles. Finally, our findings extend the findings on the effectiveness of Cognitive Tutors to the younger population of elementary school students.

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