Multiple Interactive Representations for Fractions Learning

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Abstract. Multiple External Representations (MERs) have been used successfully in instructional activities, including fractions. However, students often have difficulties making the connections between the MERs spontaneously. We argue that interactive fraction representations may help students in discovering relevant features and relating the MERs to one another. Support for guiding student interaction is provided by example-tracing tutors.

Keywords: Interactive fraction representations, virtual manipulatives

1. Introduction

“Oh fractions! I know there are lots of rules but I can’t remember any of them and I never understood them to start with.” This utterance, by a middle-school student performing fraction operations, exemplifies the difficulties students have with understanding rational numbers [1]. Indeed, fractions are considered the most challenging topic in the elementary school curriculum [2].

Part of the challenge is the different conceptual interpretations of rational numbers: part-whole, percentage, ratio, measurement, division, decimal, etc. [3]. Using multiple external representations (circles, number lines, rectangles) besides the mathematical symbols, leads to deeper understanding of fractions [3].

The positive effect of instructional activities that combine MERs has been widely acknowledged [4][5]. Learners must detect relevant structures within representations and relate the different representations to each other. However students often do not make these connections spontaneously [6]. Accordingly, to facilitate learning with multiple fraction representations it seems to be important to support students in dealing with the requirements of connection making.

In a prior study on fractions learning conducted by our research group, students learned better with MERs, compared to working with a single graphical representation of fractions, when prompted to self-explain how each graphical representation relates to the standard fraction notation [7]. The representations in this study were presented as static graphics. In the current study, we want students to gain an even better understanding of the fraction concepts, by allowing them to actively manipulate the representations. These *interactive fraction representations* encourage exploration of the representational features, by requiring student action on the underlying concepts (e.g. drag-and-drop equivalent fractions on top of each other, to signal the importance of equal proportion to the whole (see Fig. 1)). By doing so, student attention is directed to the relevant structures, supporting connection making between the different representations.

However, when learners explore interactive environments they are often not able to interact with them in a systematic and goal-oriented way [6][8][9]. To support and guide students in interaction, many solutions have been tried, such as an integration phase with static representations [8], integrating representations [6], step-by-step introduction of interactivity and dynamic linking of representations [9]. We will add additional support for interaction by embedding the interactive representations within a technology proven to improve student learning: *example-tracing tutors* [10], providing hints and feedback at every problem solving step and directing student attention by both visual cues and step-by-step instruction.

Reimer et al. [11] explored the use of interactive fraction representations (referred to as *virtual manipulatives*) in a classroom and found a positive learning effect. They did not however, compare its effect against static representations.

2. Research questions and Method

The goal of this study is to determine how students can best be supported in making connections between the MERs. More specifically: *1. Are interactive representations more effective in supporting robust fraction learning compared to static representations?* And *2. Should learners explicitly relate the representations?*

The experiment will have a 2x2 factorial design with four experimental conditions. One factor we differ is the graphical representations: interactive representations compared to static representations. The other factor is explicit connection making activities (multiple different representations in the same problem) versus implicit connection making (single representations only).

We expect that interactive representations will lead to better performance in reproduction and transfer items. Also, we expect tasks where students explicitly relating representations to lead to increased robust fractions learning. We believe that students assigned to the interactive and explicit conditions will show more learning than students in the static, implicit condition.

The study will be carried out in three elementary schools in the Penn-Trafford school district with 312 4th and 5th grade students. Students will be randomly assigned to the four experimental conditions within each of the participating classrooms. The web-based example-tracing tutors will cover a part of the fraction curriculum, including naming and constructing (graphical) fractions, fraction equivalence and ordering fractions. The (interactive) fraction representations include circles, rectangles and number lines. The pre-test and post-test will contain reproduction items and transfer-items (e.g. future learning items) in equal proportion.

The fractions tutors will be delivered over the web, and used as an adjunct to regular 4th or 5th-grade fractions instruction. Students will work on the tutor starting March 22nd 2010 for a total of five consecutive days, with on each day a one-hour session.

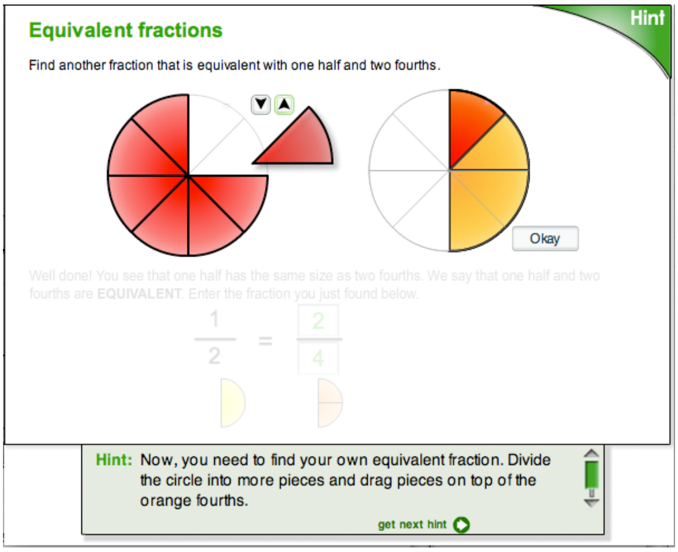

Fig. 1. Example-tracing tutors with interactive representations

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