15-453: Formal Languages, Automata and Computability **L.Blum**
Homework # 2 Solutions

1

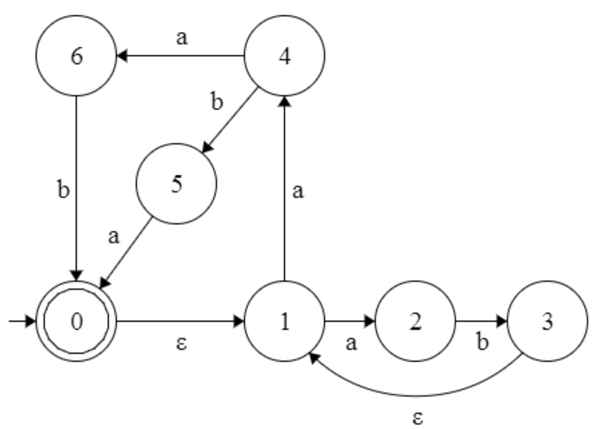
Solution:
(a) $(a \cup b \cup c)^* cab (a \cup b \cup c)^*$ OR $.^* cab .^*$ where . matches any character in the alphabet.

(b) $(a \cup c)^*(b(a \cup c)^* b(a \cup c)^*)^*$

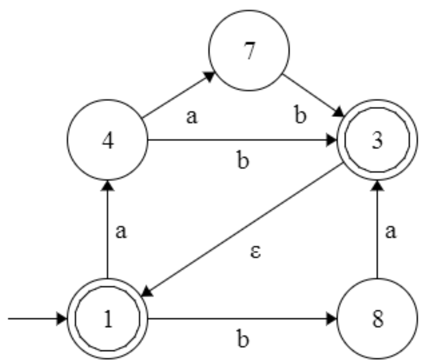
(c) $(b \cup c \cup (ab))^* c$

2

a)



b)



Here, states 3 and 1 can be combined.

3

First we draw our table, distinguishing only accept from reject states.

	A	B	C	D	E	F	G
B							
C	D	D					
D			D				
E			D				
F			D				
G			D				
H			D				

Now D and F are indistinguishable, since they both lead to C on 0 and G on 1. Similarly, B and H are indistinguishable, since they both lead to D on C on 1 and G on 0. B and F are distinguishable, since B accepts the string 1 while F rejects it. This also means B and D are distinguishable, F and H are distinguishable, and H are distinguishable. We put these entries into the table:

	A	B	C	D	E	F	G
B							
C	D	D					
D		D	D				
E			D				
F		D	D	I			
G			D				
H		I	D	D		D	

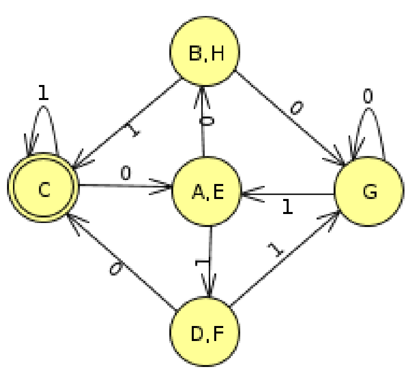
Now A and E have the same transition for 1; for 0, they transition to B and H, which are indistinguishable. Thus A and E are indistinguishable.

On the other hand, all of B,C,D,H have transitions to accept states, while neither A,E, nor G does. Thus A,E, and G are distinguishable from any of B,C,D, or H.

Finally, the string 0 takes A to B and G to G. B and G are distinguishable, so A and G are distinguishable. Hence E and G are also distinguishable.

	A	B	C	D	E	F	G
B	D						
C	D	D					
D	D	D	D				
E	I	D	D	D			
F	D	D	D	I	D		
G	D	D	D	D	D	D	
H	D	I	D	D	D	D	D

Finally, we draw the minimized DFA.



4

(a) AFSOC that the given language is regular. Let P be the pumping length for the given language. Consider the string $s = 0^{P+3}1^{P}$. Clearly, this string is in the language.
Let s be of the form xyz where $|xy| \leq P$ and $|y| \geq 1$. By the pumping lemma, xy^iz is in the language as well for all $i \geq 1$.
Clearly xy consists of all 0s and hence y consists of only 0s. Therefore, xy^2z will add at least one more 0 to s while adding no more 1s. Hence, the new string now has at least 4 more 0s than 1s and hence should not be in the language. However, by the pumping lemma, it is in the language.
This is a contradiction. Hence the language is not regular.

(b) AFSOC that the given language is regular. Let P be the pumping length for the given language. Consider the string $s = 0^{P}1^{2P}2^{P}$. Clearly, this string is in the language.
Let s be of the form xyz where $|xy| \leq P$ and $|y| \geq 1$. By the pumping lemma, xy^iz is in the language as well for all $i \geq 1$.
Clearly xy consists of all 0s and hence y consists of only 0s. Therefore, xy^2z will add at least one more 0 to s while adding no more 1s or 2s. Hence, the new string is now of the form $s = 0^{P+k}1^{2P}2^{P}$ where k is non zero. Clearly this should not be in the language. However, by the pumping lemma, it is in the language.
This is a contradiction. Hence the language is not regular.