(a) One possible solution is to modify the DFA from part (a) by adding a third set of nonfinal states identical to \{q_0, q_1, q_2\} or \{q_3, q_4, q_5\}. This would recognize strings with “cab” at least three times. Then by making every state except for the last one final, we make a DFA that recognizes the complement of this language – the strings with “cab” at most twice.

(b) One possible solution is to modify the DFA from part (a) by adding a third set of nonfinal states identical to \{q_0, q_1, q_2\} or \{q_3, q_4, q_5\}. This would recognize strings with “cab” at least three times. Then by making every state except for the last one final, we make a DFA that recognizes the complement of this language – the strings with “cab” at most twice.

(c) The most obvious solution is a DFA with ten states \{q_0, \ldots, q_9\} arranged in a circle, with each transitioning to the next for all symbols (i.e. \(\delta(q_i, \sigma) = q_{i+1 \mod 10}\)). The initial state is \(q_0\) and states which are even or divisible by 5 are final.

(d)
We claim by induction on $k$ that after $k$ steps, this DFA is in state $q_{s(k)}$, where $s(k)$ is the sum of the first $k$ characters of the inputs mod 5.

Suppose this claim is true for $k$. By construction for each $j = 1, 2, \text{or } 3$, $\delta(q_0, j) = q_{i+j} \mod 5$. Hence after $k+1$ steps it is at $q_{s(k)+j} = q_{s(k+1)}$.

Thus it accepts only if the sum of the entire input string is $0 \mod 5$.

3

Consider a DFA $M$ for regular language $A$, given by $M = (Q, \Sigma, \delta, q_0, F)$.

Let $F' \subseteq F$ be the set of states $q$ in $F$ from which there is no path from $q$ to any other final state.

Define $M'$ to be the DFA $M' = (Q, \Sigma, \delta, q_0, F)$.

If $M'$ accepts $x$, then $x$ takes $M$ to a final state $q$ so $x \in A$, and there are no paths to final states from $q$, so no strings with $x$ as a prefix are accepted by $M$. Thus $x \in \text{Max}(A)$.

If $M'$ does not accept $x$, then either the state $q_0$ which $x$ takes $M'$ to is a nonfinal state, so $x \notin A$, or there is a path from $q_0$ to a final state. If the path is $q_0q_1 \ldots q_n$ and $\delta(q_{i-1}, w_i) = q_i$, then the string $xw_1 \ldots w_n$ takes $M$ to a final state and has $x$ as a proper prefix, so $x \notin \text{Max}(A)$.

4

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing $L_1$. For each state $q_i$, let $M_i$ be the DFA $(Q, \Sigma, \delta, q_i, F_i)$. Define $f_i = 1$ if $L(M_i) \cap L_2$ is nonempty, and $f_i = 0$ otherwise. Define $F' = \{q_i \in Q \mid f_i = 1\}$, and $M' = (Q, \Sigma, \delta, q_0, F')$. The strings which are accepted by $M'$ are exactly those which take the DFA to a state from which some string in $L_2$ takes the DFA to a final state of $M$, which are exactly $L_1/L_2$. 