Midterm I
15-453
Formal Languages, Automata and Computability (FLAC)
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Instructions:

1. Once the exam begins, **write your name on each sheet**.
2. This is a closed-book examination.
3. Do all your work on the attached sheets. Justify your answers.
4. There are **10** problems worth at total of **150** Points (**125** is a guaranteed A)
5. You have 80 minutes to answer the questions.
6. Read over the whole exam. Pace yourself. Check your work! Good luck!

Last name: _____ First name: _____

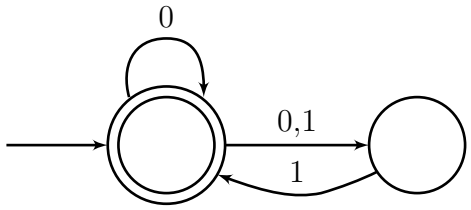
Signature: _____ Email address: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	20	
6	40	
7	10	
8	20	
9	10	
10	10	
Total:	150	

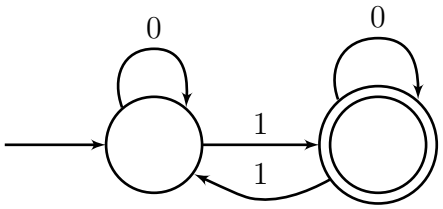
1. (10 points) Convert the following regular expression into an equivalent NFA:

$$0(001)^\star \cup 1$$

2. (10 points) Convert the following NFA to an equivalent DFA.



3. (10 points) Convert the following DFA to an equivalent Regular Expression.



4. (10 points) Explain what is wrong with the following proof that $L = \{w \mid w = w^R\}$ is *not* context free.

   Suppose L is context free and suppose G is a CFG such that $L(G) = L$. Let p be the pumping number of G. Let $w = 1^p01^p$. Clearly $w \in L$. By the *Pumping Lemma* for CFGs there is a string vxy in first 1^p part of w (with not both v and y empty) so that replacing vxy by v^2xy^2 in w produces another string in L. But this string is not in L. Contradiction. Therefore L is not context free.

5. Let G to be a context-free grammar with production rules:

$$S \to ASA \mid A \mid \varepsilon$$
$$A \to 00 \mid \varepsilon$$

(a) (10 points) Describe $L(G)$ in English.

(b) (10 points) Construct context-free grammar G' in Chomsky Normal Form that is equivalent to G.

6. (40 points) For each of the following languages, state which of the following classes they belong to: Regular, Context Free, Turing Recognizable, or None of the Above.

   Justify your answer.

   (Scoring: 10 points for each part, 5 of which is justification.)

   (a) The language of palindromes over $\{0, 1\}$ containing equal numbers of 0's and 1's.

   (b) $\{w \mid w$ contains and equal number of 0's and 1's$\}$

   (c) $A_n = \{0^k \mid k$ is a multiple of $n\}$

   (d) The complement of the language $\{0^n 1^n \mid n \geq 0\}$.

7. (10 points) Prove that a PDA with 2 stacks is more powerful than a PDA with 1 stack.

8. Define the following context-free language

$$L = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

(a) (10 points) Construct a CFG for L.

(b) (10 points) Construct a PDA for L.

9. (10 points) Prove or Disprove.

   If languages L_1 and L_2 are both not regular, then $L_1 \cup L_2$ is also not regular.

10. (10 points) For languages A and B, let the *shuffle* of A and B be the language

$$\{w \mid w = a_1 b_1 \dots a_k b_k \text{ where } a_1 \dots a_k \in A \text{ and } b_1 \dots b_k \in B \text{ and each } a_i, b_i \in \Sigma^\star\}$$

Prove that the class of regular languages is closed under *shuffle*.

This page left intentionally blank. We won't grade this sheet; it's just here in case you need extra space for scratchwork.