15-453

FORMAL LANGUAGES, AUTOMATA AND COMPUTABILITY

A_{TM} = { (M,w) | M is a TM that accepts string w }

$HALT_{TM}$ = { (M,w) | M is a TM that halts on string w }

E_{TM} = { M | M is a TM and L(M) = ∅ }

REG_{TM} = { M | M is a TM and L(M) is regular}

EQ_{TM} = {(M, N) | M, N are TMs and L(M) =L(N)}

ALL_{PDA} = { P | P is a PDA and L(P) = Σ* }

ALL UNDECIDABLE

Use Reductions to Prove

Which are SEMI-DECIDABLE?

THE POST CORRESPONDENCE PROBLEM

TUESDAY FEB 26

THE PCP GAME

ba	a	b	b
a	ab	bcb	a

THE PCP GAME

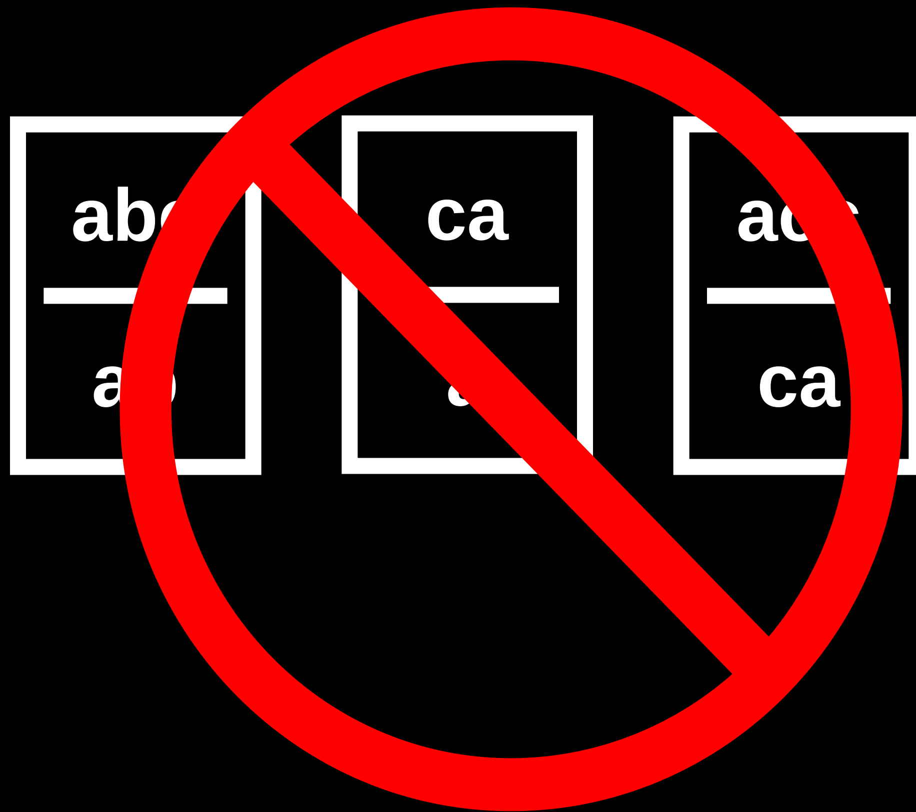
$$\frac{\mathbf{ba}}{\mathbf{a}} \quad \frac{\mathbf{a}}{\mathbf{ab}} \quad \frac{\mathbf{b}}{\mathbf{bcb}} \quad \frac{\mathbf{b}}{\mathbf{a}}$$

$$\frac{\mathbf{a}}{\mathbf{ab}} \quad \frac{\mathbf{ba}}{\mathbf{a}}$$

$$\left[\frac{aaa}{a}\right] \quad \left[\frac{a}{c}\right] \quad \left[\frac{a}{aa}\right] \quad \left[\frac{c}{a}\right]$$

b	a	ca	abc
ca	ab	a	c

abc	ca	acc
ab	a	ca


ca
ca

GENERAL RULE #1

If every top string is longer than the corresponding bottom one, there can’t be a match

caa	acc	b	aab	c
a	a	b	aa	a

GENERAL RULE #2

If there is a domino with the same string on the top and on the bottom, there is a match

POST CORRESPONDENCE PROBLEM

Given a collection of dominos, is there a match?

PCP = { P | P is a set of dominos with a match }

PCP is *undecidable*!

THE FPCP GAME

… is just like the PCP game except that a match has to start with the first domino

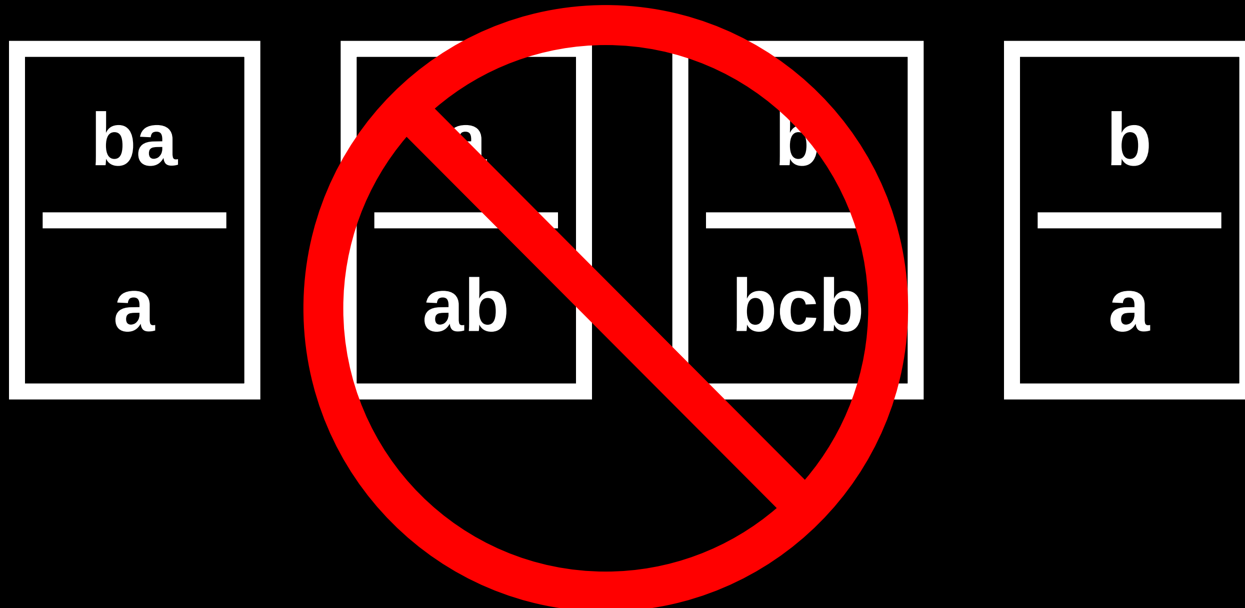
FPCP

$$\left[\frac{aaa}{a}\right] \quad \left[\frac{a}{c}\right] \quad \left[\frac{a}{aa}\right] \quad \left[\frac{c}{a}\right]$$

FPCP

ba	a	b	b
a	ab	bcb	a

FPCP



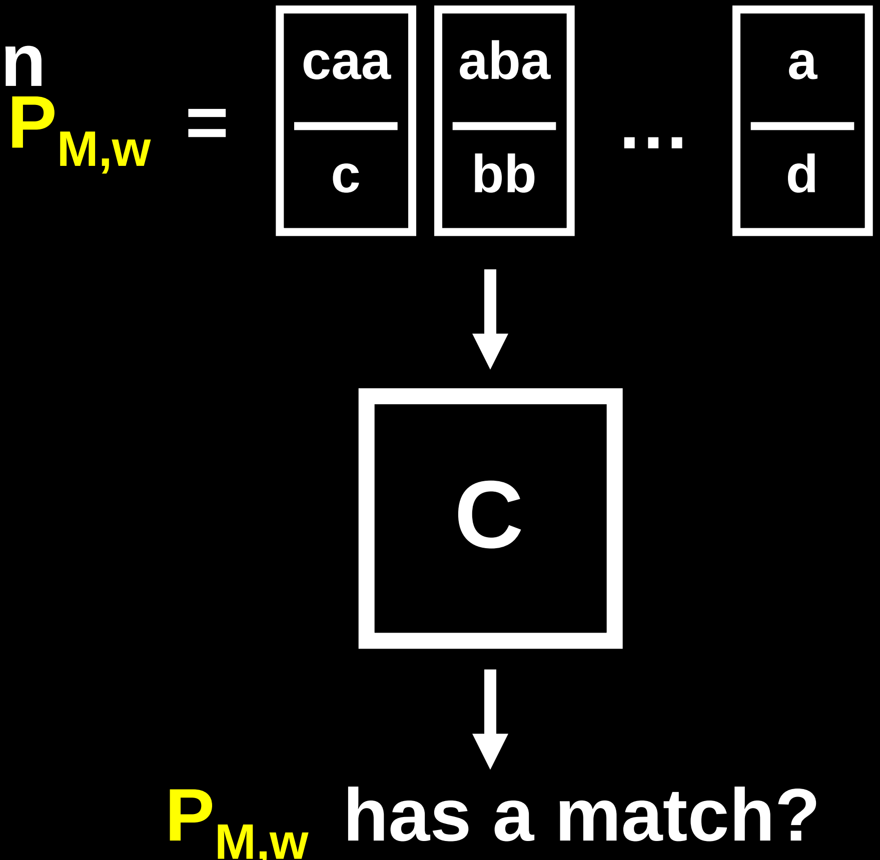
Theorem: FPCP is undecidable

Proof: Assume machine C decides FPCP

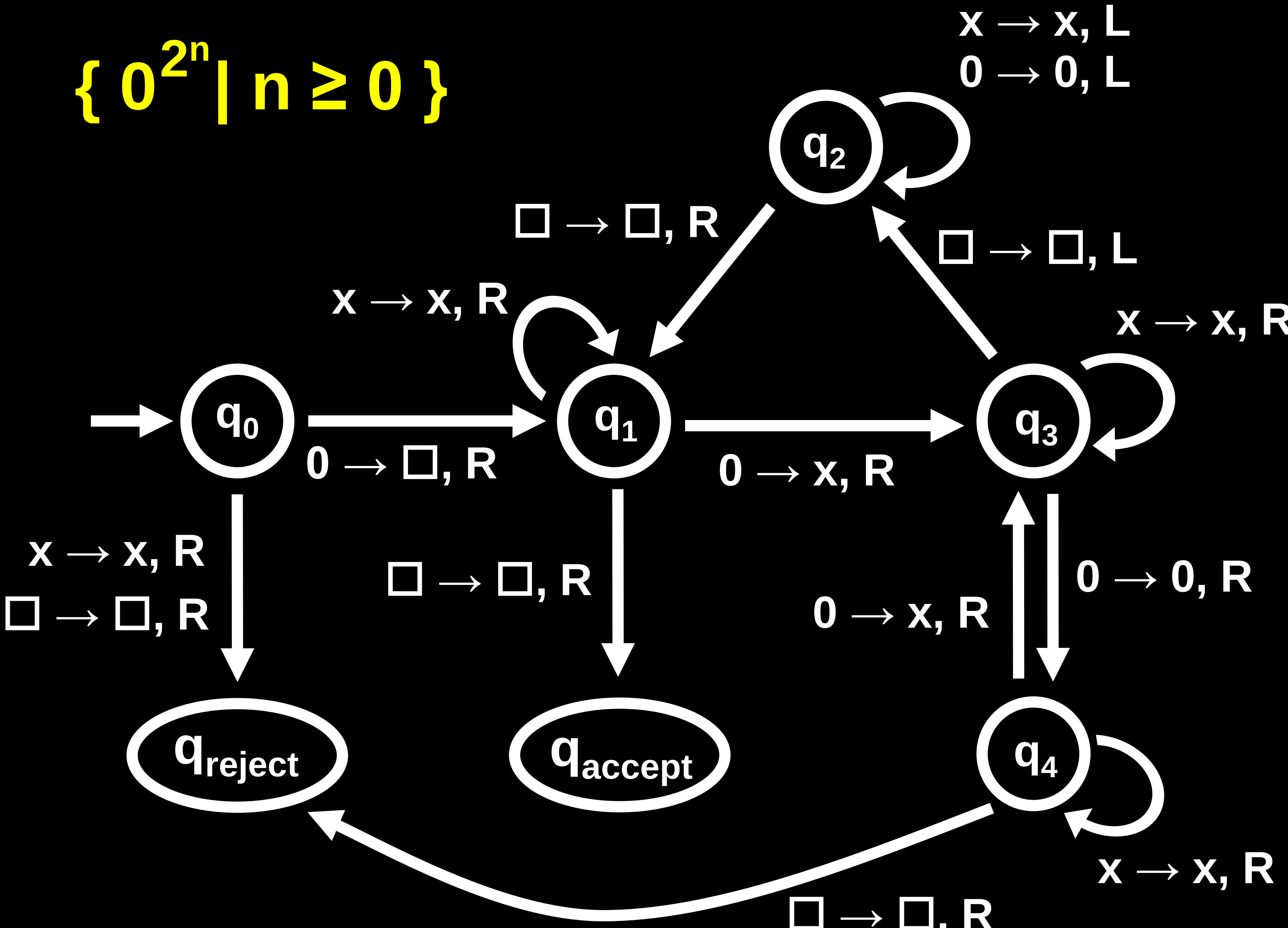
We will show how to use C to decide A_{TM}

Given (M,w)

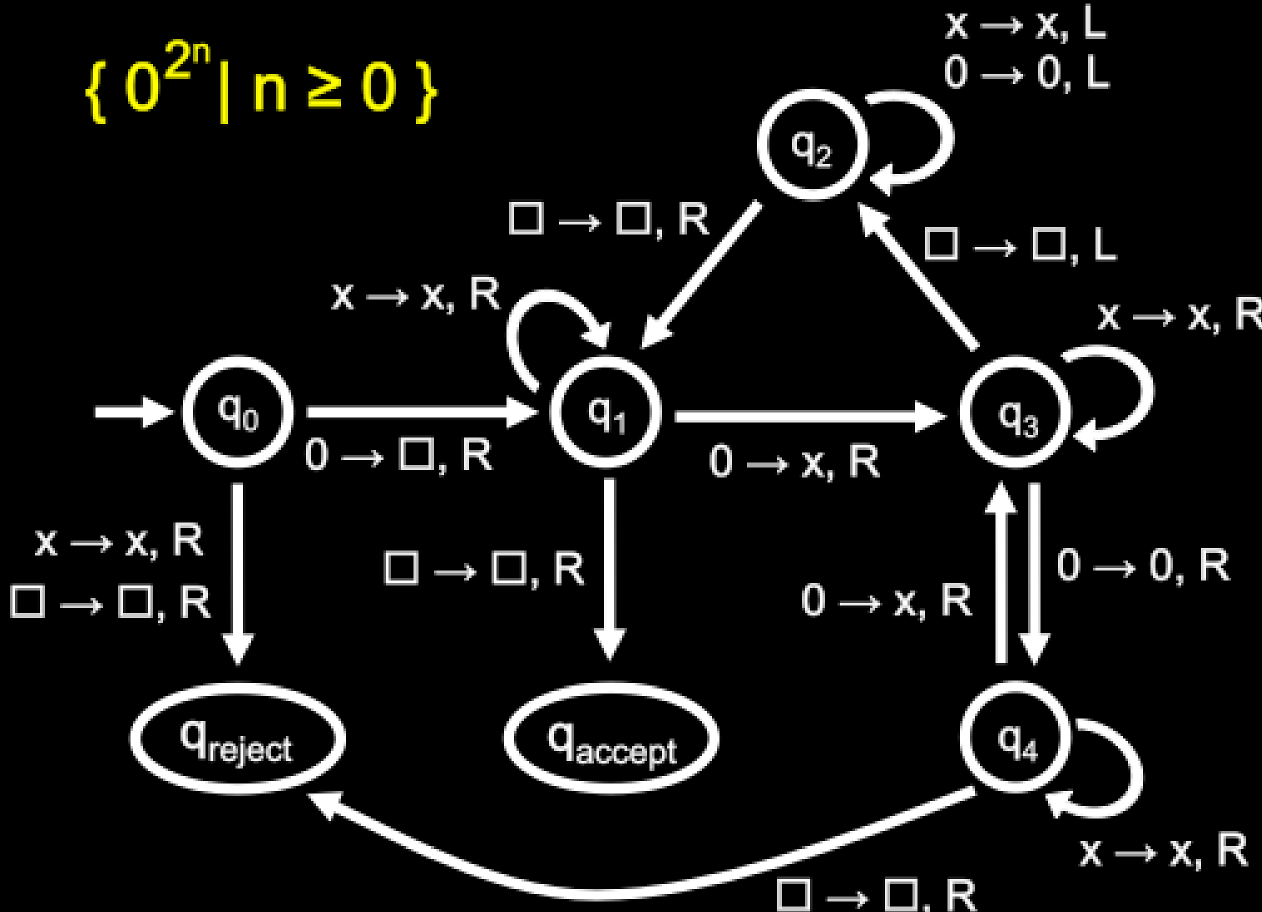
we will construct a set of dominos $P_{M,w}$ where a match is an accepting computation history for M on w



$\{ 0^{2^n} \mid n \geq 0 \}$



$\{ 0^{2^n} \mid n \geq 0 \}$



$q_0$0000

□$q_1$000

□x$q_3$00

□x0$q_4$0

□x0xq_3

□x0q_2x

□x$q_2$0x

□q_2x0x

q_2□x0x

:

#$q_0$0000#□$q_1$000#□x$q_3$00#□x0$q_4$0#□x0xq_3# ... #

Given (M,w), we will construct an instance P of FPCP in 7 steps

Assume M on w never attempts to move off left hand edge of tape

STEP 1

Put $\left[\dfrac{\#}{\#q_0w_1w_2\ldots w_n\#}\right]$ into P

For start configuration

START

STEP 2

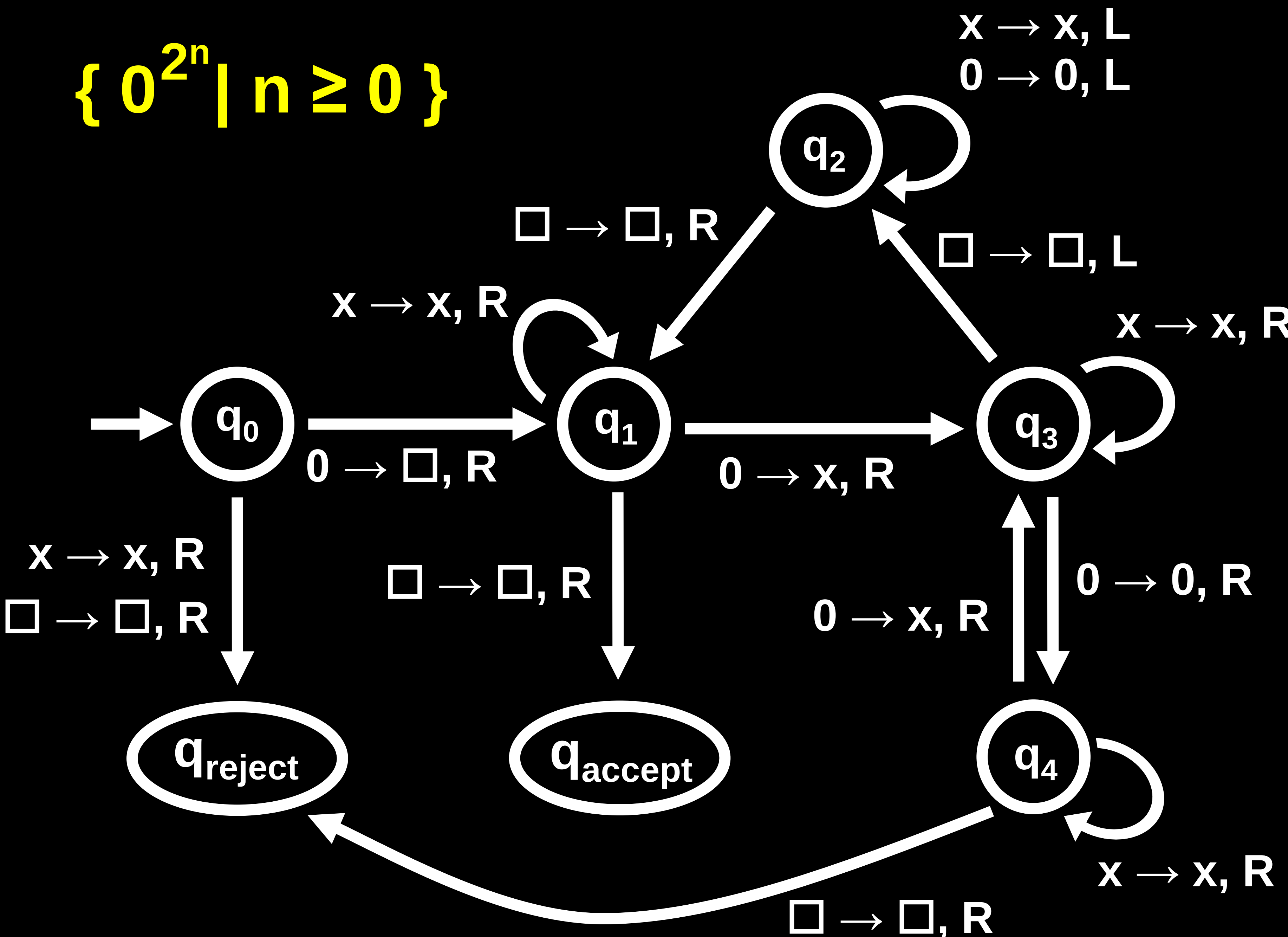
If $\delta(q,a) = (p,b,R)$ then add $\boxed{\dfrac{qa}{bp}}$

STEP 3

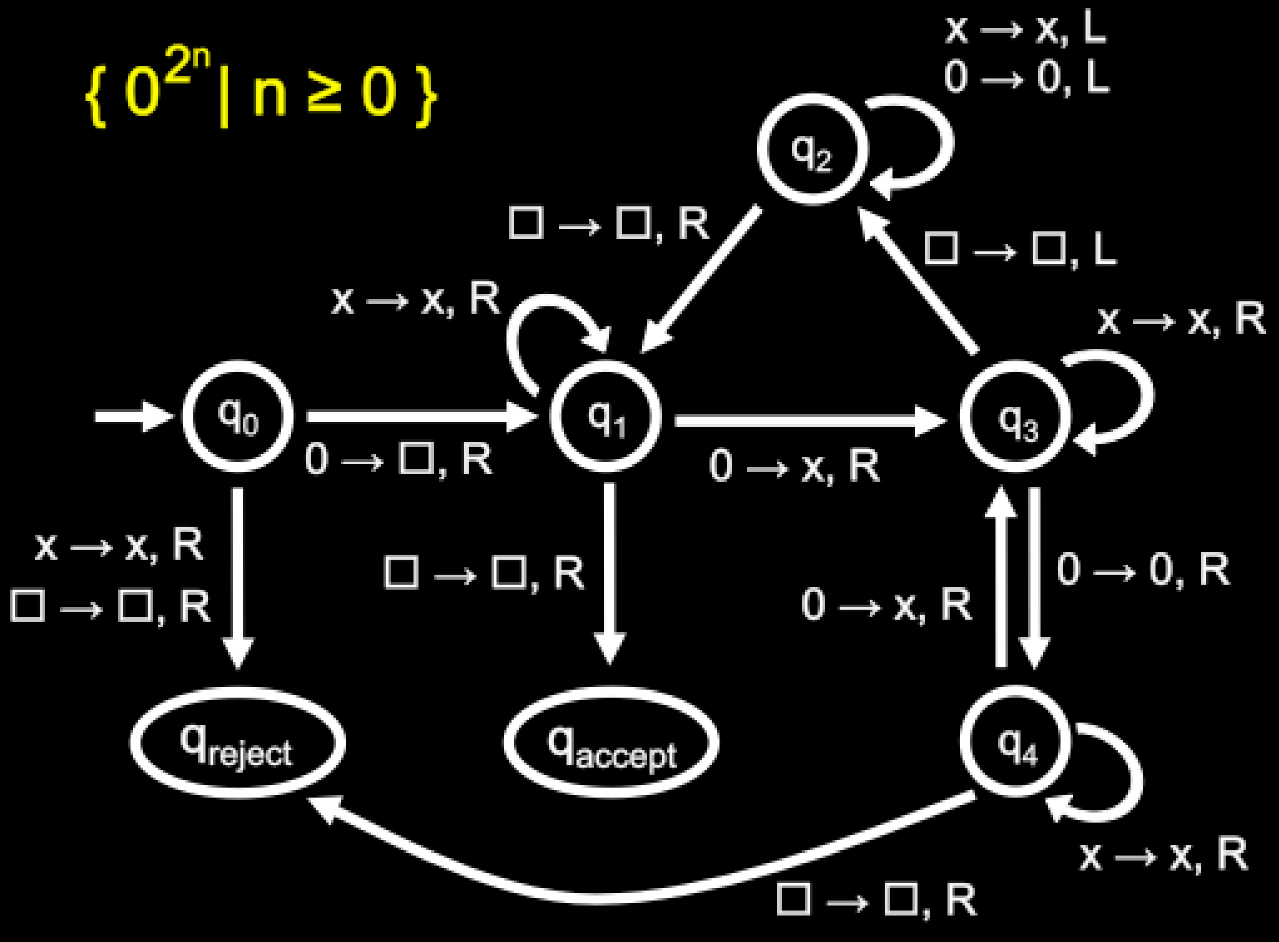
If $\delta(q,a) = (p,b,L)$ then add $\boxed{\dfrac{cqa}{pcb}}$ for all $c \in \Gamma$

RULES

$\{ 0^{2^n} \mid n \geq 0 \}$



$\{ 0^{2^n} \mid n \geq 0 \}$



$\left[\frac{0q_2 0}{q_2 00}\right]$ $\left[\frac{xq_2 0}{q_2 x0}\right]$ $\left[\frac{\square q_2 0}{q_2 \square 0}\right]$

$\left[\frac{\#}{\#q_0 0000\#}\right]$ $\left[\frac{q_0 0}{\square q_1}\right]$ $\left[\frac{q_1 0}{x q_3}\right]$ … $\left[\frac{0q_3 \square}{q_2 0 \square}\right]$ $\left[\frac{xq_3 \square}{q_2 x \square}\right]$ $\left[\frac{\square q_3 \square}{q_2 \square \square}\right]$

STEP 4

add $\boxed{\dfrac{a}{a}}$ for all $a \in \Gamma$

For tape cells not adjacent to head

STEP 5

add $\boxed{\dfrac{\#}{\#}}$ $\boxed{\dfrac{\#}{\square\#}}$

For configuration separator

To simulate the blanks on the right hand side of tape

CONTINUE

STEP 4

add $\left[\dfrac{a}{a}\right]$ for all $a \in \Gamma$

STEP 5

add $\left[\dfrac{\#}{\#}\right]$ $\left[\dfrac{\#}{\square\#}\right]$

STEP 6

add $\left[\dfrac{aq_{acc}}{q_{acc}}\right]$ $\left[\dfrac{q_{acc}a}{q_{acc}}\right]$ for all $a \in \Gamma$

Adds pseudo-steps after TM halts (catch up)

$$\left[\frac{\#}{\#q_0 0000\#}\right] \left[\frac{q_0 0}{\square q_1}\right] \left[\frac{q_1 0}{x q_3}\right] \left[\frac{q_1 x}{x q_1}\right] \left[\frac{q_0 x}{x q_r}\right] \left[\frac{q_0 \square}{\square q_r}\right] \left[\frac{q_1 \square}{\square q_a}\right] \left[\frac{q_2 \square}{\square q_1}\right]$$

$$\left[\frac{q_3 x}{x q_3}\right] \left[\frac{q_3 0}{0 q_4}\right] \left[\frac{q_4 0}{x q_3}\right] \left[\frac{q_4 x}{x q_4}\right] \left[\frac{q_4 \square}{\square q_r}\right] \left[\frac{0 q_2 0}{q_2 00}\right] \left[\frac{\square q_2 0}{q_2 \square 0}\right] \left[\frac{x q_2 0}{q_2 x 0}\right]$$

$$\left[\frac{0 q_3 \square}{q_2 0 \square}\right] \left[\frac{x q_3 \square}{q_2 x \square}\right] \left[\frac{\square q_3 \square}{q_2 \square \square}\right] \left[\frac{0 q_3 x}{q_2 0 x}\right] \left[\frac{x q_3 x}{q_2 x x}\right] \left[\frac{\square q_3 x}{q_2 \square x}\right] \left[\frac{x}{x}\right] \left[\frac{0}{0}\right] \left[\frac{\square}{\square}\right]$$

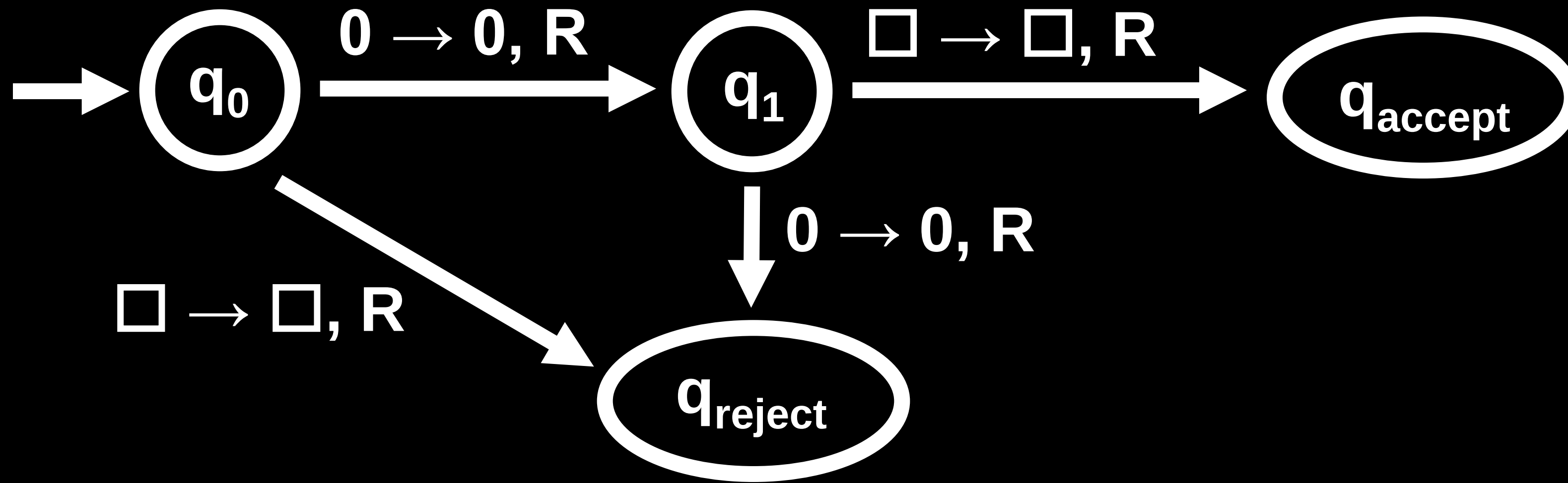
$$\left[\frac{\#}{\#}\right] \left[\frac{\#}{\square \#}\right] \left[\frac{0 q_{acc}}{q_{acc}}\right] \left[\frac{q_{acc} 0}{q_{acc}}\right] \left[\frac{x q_{acc}}{q_{acc}}\right] \left[\frac{q_{acc} x}{q_{acc}}\right] \left[\frac{\square q_{acc}}{q_{acc}}\right] \left[\frac{q_{acc} \square}{q_{acc}}\right]$$

$$\left[\frac{\#}{\#q_0 0000\#}\right] \left[\frac{q_0 0}{\square q_1}\right] \left[\frac{0}{0}\right] \left[\frac{0}{0}\right] \left[\frac{0}{0}\right] \left[\frac{\#}{\#}\right] \left[\frac{\square}{\square}\right] \left[\frac{q_1 0}{x q_3}\right] \left[\frac{0}{0}\right] \left[\frac{0}{0}\right]$$

STEP 7

add $\boxed{\dfrac{q_{acc}\#\#}{\#}}$

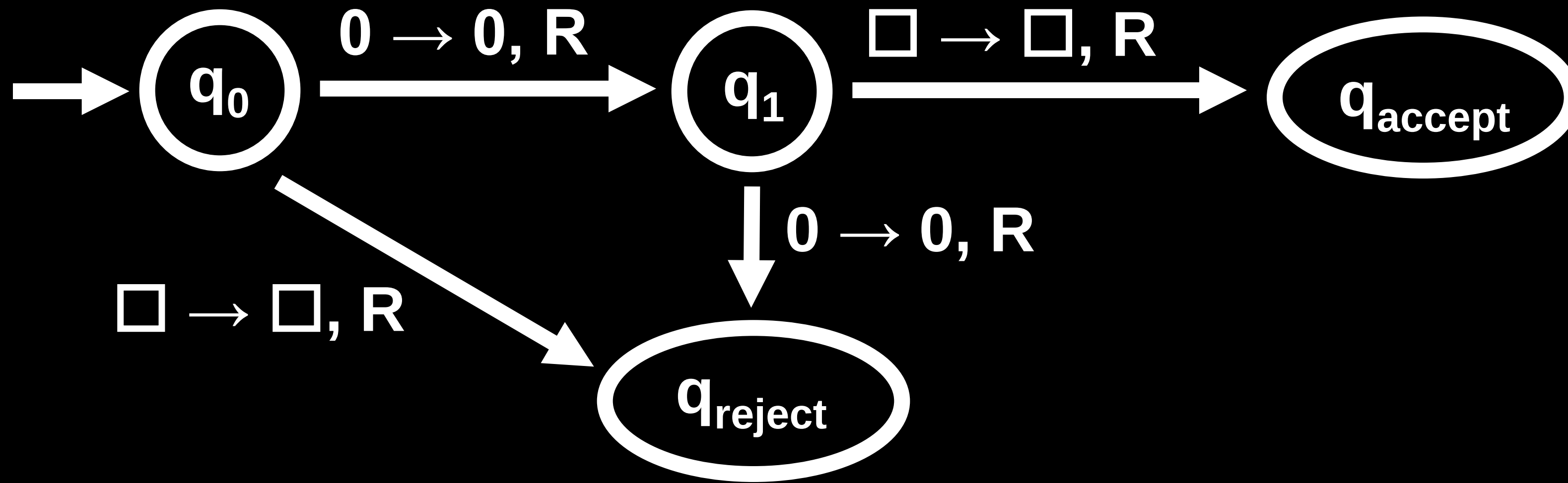
END



$$\left[\frac{\#}{\#q_0 0\#}\right] \left[\frac{q_0 0}{0 q_1}\right] \left[\frac{q_1 0}{0 q_{rej}}\right] \left[\frac{q_0 \square}{\square q_{rej}}\right] \left[\frac{q_1 \square}{\square q_{acc}}\right] \left[\frac{0}{0}\right] \left[\frac{\square}{\square}\right]$$

$$\left[\frac{\#}{\#}\right] \left[\frac{\#}{\square \#}\right] \left[\frac{\square q_{acc}}{q_{acc}}\right] \left[\frac{0 q_{acc}}{q_{acc}}\right] \left[\frac{q_{acc} \square}{q_{acc}}\right] \left[\frac{q_{acc} 0}{q_{acc}}\right] \left[\frac{q_{acc}\#\#}{\#}\right]$$

$$\frac{\#}{\#q_0 0\#} \; \frac{q_0 0}{0 q_1} \; \frac{\#}{\square \#} \; \frac{0}{0} \; \frac{q_1 \square}{\square q_{acc}} \; \frac{\#}{\#} \; \frac{0}{0} \; \frac{\square q_{acc}}{q_{acc}} \; \frac{\#}{\#} \; \frac{0 q_{acc}}{q_{acc}} \; \frac{\#}{\#}$$



$$\frac{\#}{\#q_0\square\#} \quad \frac{q_0 0}{0 q_1} \quad \frac{q_1 0}{0 q_{rej}} \quad \frac{q_0 \square}{\square q_{rej}} \quad \frac{q_1 \square}{\square q_{acc}} \quad \frac{0}{0} \quad \frac{\square}{\square}$$

$$\frac{\#}{\#} \quad \frac{\#}{\square\#} \quad \frac{\square q_{acc}}{q_{acc}} \quad \frac{0 q_{acc}}{q_{acc}} \quad \frac{q_{acc}\square}{q_{acc}} \quad \frac{q_{acc}0}{q_{acc}} \quad \frac{q_{acc}\#\#}{\#}$$

$$\frac{\#}{\#q_0\square\#} \quad \frac{q_0\square}{\square q_{rej}} \quad \frac{\#}{\square\#} \quad \frac{\square}{\square} \quad \frac{q_{rej}\square}{}$$

X

Given (M,w), we can construct an instance of FPCP that has a match if and only if M accepts w

Can convert an instance of FPCP into one of PCP:

Let $u = u_1u_2\ldots u_n$, define:

$$\star u \quad = * \, u_1 * u_2 * u_3 * \ldots * u_n$$

$$u\star \quad = u_1 * u_2 * u_3 * \ldots * u_n *$$

$$\star u \star \quad = * \, u_1 * u_2 * u_3 * \ldots * u_n *$$

FPCP: $\left[\frac{t_1}{b_1}\right] \left[\frac{t_2}{b_2}\right] \ldots \left[\frac{t_k}{b_k}\right]$

PCP: $\left[\frac{\star t_1}{\star b_1 \star}\right] \left[\frac{\star t_1}{b_1 \star}\right] \left[\frac{\star t_2}{b_2 \star}\right] \ldots \left[\frac{\star t_k}{b_k \star}\right] \left[\frac{* \blacklozenge}{\blacklozenge}\right]$

Given (M,w), we can construct an instance of PCP that has a match if and only if M accepts w

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Read Chapters 5.2 and 5.3 of the book for next time