

1

Consider the game of minesweeper played on an arbitrary graph. Some unknown set of vertices are mines. At any given time during gameplay, the player has revealed some number of vertices, and knows the number of mines adjacent to each of these revealed vertices. The player's goal is to reveal all of the non-mines without revealing any of the mines.

In the "mine consistency problem," you are given an arbitrary graph, along with numerical labels on some of its vertices. You wish to know if there is a placement of mines on the remaining vertices such that each labeled vertex is adjacent to exactly as many mines as its label suggests.

Formulate this problem as a formal language. Then show that it is NP-complete.

2

In the proof of the Cook-Levin theorem, a window is a 2×3 rectangle of cells.

Show why the proof would have failed if we had used 2×2 windows instead.

3

Show that any PSPACE-hard language is also NP-hard.

4

Consider $EQ_{REG} = \{\langle R, S \rangle \mid R \text{ and } S \text{ are equivalent regular expressions}\}$. Show that $EQ_{REG} \in PSPACE$.