1

Define the language

$$S = \{\langle M \rangle \mid M \text{ is a DFA such that for all } w \in \Sigma^*,\ w \in L(M) \text{ implies } w^{\mathcal{R}} \in L(M)\}.$$

Show that S is decidable.

2

For this problem, we will define a variant of the Turing machine called a *enumerator*. Intuitively, an enumerator is a Turing machine with an attached printer, which it can use to print strings.
Formally, an enumerator is a tuple $E = (Q, \Sigma, \Gamma, \delta, q_0, q_{print}, q_{halt})$, where Q, Σ, Γ are finite sets and:

1. Q is the set of states
2. Σ is the work tape alphabet
3. Γ is the output tape alphabet
4. $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \times \Sigma_\epsilon$ is the transition function
5. $q_0 \in Q$ is the start state
6. q_{print} is the print state
7. q_{halt} is the halt state

The computation of E is defined the same as an ordinary Turing machine, but it has an extra tape, a *print tape*. The regular "work tape" and the print tape of E both start out blank, and the machine may write to either or both at each step. The head of the work tape moves as usual, while the head of the work tape moves right every time it is written to. When E reaches the state q_{print}, the machine is said to *print* the contents of its work tape, which is then reset to blank and the head is set to the left. The machine halts when q_{halt} is reached. (Note that an enumerator does not accept or reject, because it has no input!)

We say that an enumerator E enumerates a language L if L is exactly the set of strings w such that there is $t \in \mathbb{N}$ for which after t transitions, E reaches q_{print} with w in its print tape. The order in which the language is enumerated is the order defined by $w \prec v$ if the first time t_w at which w is printed is less than the first time t_v at which v is printed.

Show that a language is decidable if and only if some enumerator enumerates the language in the shortlex order.

(The shortlex order with respect to an ordering on Σ is defined by $w \prec v$ if $|w| < |v|$, or if $|w| = |v|$ and the first index i in which they differ has $w_i < v_i$.)

3

A Turing machine with *stay put instead of left* is defined like an ordinary Turing machine, but with transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{S, R\}$. At any point, it can move its head right, or leave its head in the same position; however, it cannot move its head left.

What class of languages is recognizable by Turing machines with stay put instead of left?

Hint: It's not the Turing-recognizable languages!

4

A *2-PDA* is a PDA with two stacks instead of one. It behaves like a normal PDA, except that a 2-PDA can pop from and push to either, both or neither of the stacks. The transition function of a 2-PDA is defined as

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \times \Gamma_\epsilon \to Q \times \Gamma_\epsilon \times \Gamma_\epsilon.$$

Show that a 2-PDA can simulate a Turing machine and vice versa.

(That is, given an arbitrary 2-PDA P, give a Turing machine that accepts the strings P accepts, and rejects or loops on the strings P rejects. Next, given an arbitrary Turing machine M, give a 2-PDA that accepts the strings M accepts, and rejects the strings M rejects or loops on.)

5

Include a References section. Cite all sources that you used and people, including yourself, that you collaborated with on this homework.