15-453: Formal Languages, Automata and Computability
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Homework # 3 **Due: February 4, 2014**

1
Prove that for any m, there exists an NFA with m states such that any equivalent DFA has at least 2^{m-1} states.

2

a) Consider the language of all binary strings with twice as many 0s as 1s. Give a CFG and a PDA for this language.

b) Prove that the following lanuage is Context-free:
$\{s_1 s_2 \dots s_n t_1 t_2 \dots t_n | s_i \in L_1, t_i \in L_2, n \in \mathbb{N}\}$ where L_1 and L_2 are Context-free languages.

3

Prove that the following languages are not context free by using the pumping lemma for context free grammars:

a) $\{a^{2^n} | n \in \mathbb{N}\}$

b) Set of all binary strings with a prime number of 1s.

4

Say that a language is prefix-closed if the prefix of any string in the language is also in the language. Let C be an infinite, prefix-closed, context-free language. Show that C contains an infinite regular subset.

5
Include a References section. Cite all sources and people, including yourself, that you collaborated with on this assignment.