THEOREM

For every regular language L, there exists a UNIQUE (up to re-labeling of the states) minimal DFA M_{mim} such that L = L(M_{mim})

Minimal means wrt number of states

PROOF

1. Let M be a DFA for L (wlog, assume no inaccessible states)
2. For pairs of states (p,q) define:

p *distinguishable* from q and
p *indistinguisable* from q (p~q).
3. Table-filling algorithm: first distinguish final from non-final states and then work backwards to distinguish more pairs.
4. What's left over are exactly the indistinguishable pairs, ie ~ related pairs. Needs proof.

PROOF

5. ~ is an equivalence relation so partitions the states into equivalence classes, E_M
6. Define M_{min}

Define: $M_{MIN} = (Q_{MIN}, \Sigma, \delta_{MIN}, q_{0\ MIN}, F_{MIN})$

$Q_{MIN} = E_M$, $q_{0\ MIN} = [q_0]$, $F_{MIN} = \{ [q] \mid q \in F \}$

$\delta_{MIN}([q], \sigma) = [\delta(q, \sigma)]$ show well defined

Claim: $\hat{\delta}_{MIN}([q], w) = [\hat{\delta}(q, w)], w \in \Sigma^*$

So: $\hat{\delta}_{MIN}([q_0], w) = [\hat{\delta}(q_0, w)], w \in \Sigma^*$

Follows: $M_{MIN} \equiv M$

PROOF

But is M_{min} unique minimum?

Yes, because if M' ≡ M and minimum then M' has no inaccesible states and is irreducible and ….

Theorem. M_{min} is isomorphic to any M' with the above properties
(need to give mapping and prove it has all the needed properties: everywhere defined , well defined, 1-1, onto, preserves transitions, and {final states} map onto {final states})

So M_{min} is isomorphic to *any* minimum M' ≡ M