Patterns and Anomalies in $k$-Cores of Real-world Networks

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The $k$-core of an undirected graph $G$ is the maximal subgraph of $G$ where every node is adjacent to at least $k$ nodes. Introduced by Seidman in 1983 [1], $k$-cores have been utilized in many applications [2]. Examples include graph topology characterization, hierarchical structure analysis, graph visualization, protein function prediction, and community discovery. From $k$-cores, we have the following definitions: (1) the coreness of a node, which is the highest $k$ value such that the node is in the $k$-core; (2) the degeneracy of a graph, which is the highest value $k$ such that the $k$-core exists; and (3) the degeneracy-core of a graph, which is the $k$-core with $k$ equal to the degeneracy value. From a recent study on numerous social networks, Web graphs, Internet topology graphs and citation networks, we found the following patterns and anomalies [2].

**Mirror Pattern:** There is a strong correlation between a node’s degree and its coreness with a “Boomerang” shape distribution. Nodes that deviate from this pattern fall into two anomaly categories. **Loner Star Anomaly:** These are nodes who rank higher in terms of degree but relatively lower in terms of coreness such as the email account of a company’s CEO. **Lockstep Behavior Anomaly:** These are nodes who rank higher in terms of coreness but relatively lower in terms of degree; they are involved in a follower-boosting service, copy-and-paste behavior, or an isolated near-clique. **Core-triangle Pattern:** There is a power-law relationship between degeneracy and triangle count – namely, $\text{degeneracy} \propto (\# \text{triangles})^{1/3}$. **Structured-core Pattern:** Degeneracy-cores have non-trivial structures. In particular, we observe core-periphery and community structures in degeneracy-cores.

![Image](patterns.png)

Figure 1: (a) Mirror Pattern: Coreness and degree are strongly correlated. (b) Loner Star Anomaly (higher rank in degree and lower rank in coreness) in a Mirror Pattern. (c) Core-triangle Pattern: Degeneracy and the number of triangles in graphs obey a 3-to-1 power law. (d) Structured-core Pattern: Degeneracy-cores have structure, such as core-periphery and communities.

References
