Polygon Meshes and Implicit Surfaces

Polygon Meshes
Parametric Surfaces
Implicit Surfaces
Constructive Solid Geometry

10/01/02

Modeling Complex Shapes

- We want to build models of very complicated objects
- An equation for a sphere is possible, but how about an equation for a telephone, or a face?
- Complexity is achieved using simple pieces
  - polygons, parametric surfaces, or implicit surfaces
- Goals
  - Model *anything* with arbitrary precision (in principle)
  - Easy to build and modify
  - Efficient computations (for rendering, collisions, etc.)
  - Easy to implement (a minor consideration...)

What do we need from shapes in Computer Graphics?

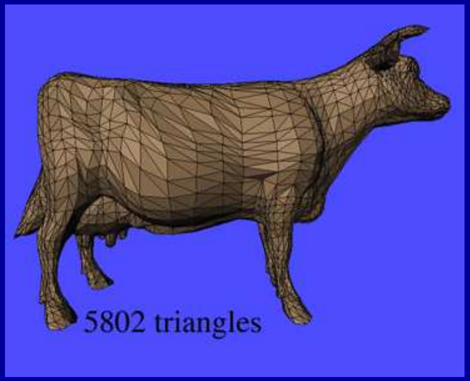
- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

No one technique solves all problems

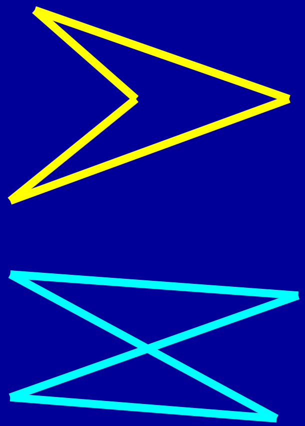
Curve Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

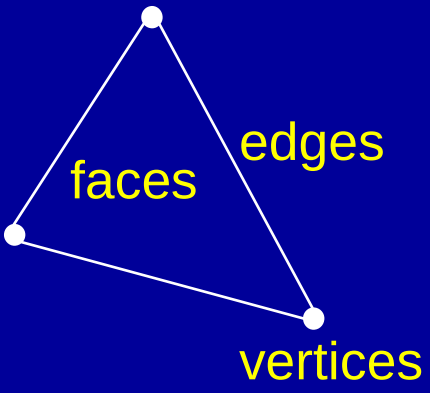
Polygon Meshes



- Any shape can be modeled out of polygons
  - if you use enough of them…
- Polygons with how many sides?
  - Can use triangles, quadrilaterals, pentagons, … n-gons
  - Triangles are most common.
  - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.



- Polygon meshes are built out of
  - *vertices* (points)
  - *edges* (line segments between vertices)
  - *faces* (polygons bounded by edges)



Polygon Models in OpenGL

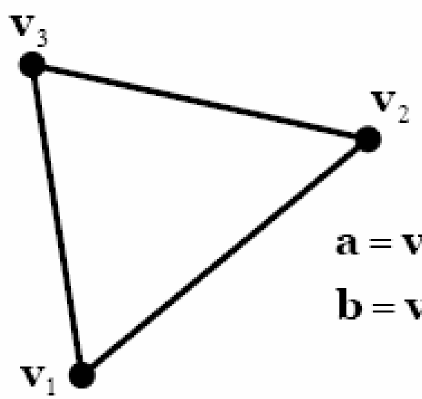
- for faceted shading

```
glNormal3fv(n);
glBegin(GL_POLYGONS);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();
```

- for smooth shading

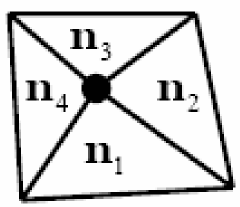
```
glBegin(GL_POLYGONS);
glNormal3fv(normal1);
glVertex3fv(vert1);
glNormal3fv(normal2);
glVertex3fv(vert2);
glNormal3fv(normal3);
glVertex3fv(vert3);
glEnd();
```

Normals



$$\mathbf{a} = \mathbf{v}_2 - \mathbf{v}_1$$

$$\mathbf{b} = \mathbf{v}_3 - \mathbf{v}_1$$



Triangle defines unique plane

- can easily compute normal

$$\mathbf{n} = \frac{\mathbf{a} \times \mathbf{b}}{\|\mathbf{a} \times \mathbf{b}\|}$$

- depends on vertex orientation!
- clockwise order gives

$$\mathbf{n}' = -\mathbf{n}$$

Vertex normals less well defined

- can average face normals
- works for smooth surfaces
- but not at sharp corners
  - think of a cube

Where Meshes Come From

- Specify manually
  - Write out all polygons
  - Write some code to generate them
  - Interactive editing: move vertices in space
- Acquisition from real objects
  - Laser scanners, vision systems
  - Generate set of points on the surface
  - Need to convert to polygons

Data Structures for Polygon Meshes

- Simplest (but dumb)
  - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  - redundant: each vertex stored multiple times
- Vertex List, Face List
  - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

  Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.
- Fancier schemes:

  Store more topological info so adjacency queries can be answered in O(1) time.

  Winged-edge data structure – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

A File Format for Polygon Models: OBJ

```
# OBJ file for a 2x2x2 cube
v -1.0  1.0  1.0   - vertex 1
v -1.0 -1.0  1.0   - vertex 2
v  1.0 -1.0  1.0   - vertex 3
v  1.0  1.0  1.0   - …
v -1.0  1.0 -1.0
v -1.0 -1.0 -1.0
v  1.0 -1.0 -1.0
v  1.0  1.0 -1.0
f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3
```

Syntax:

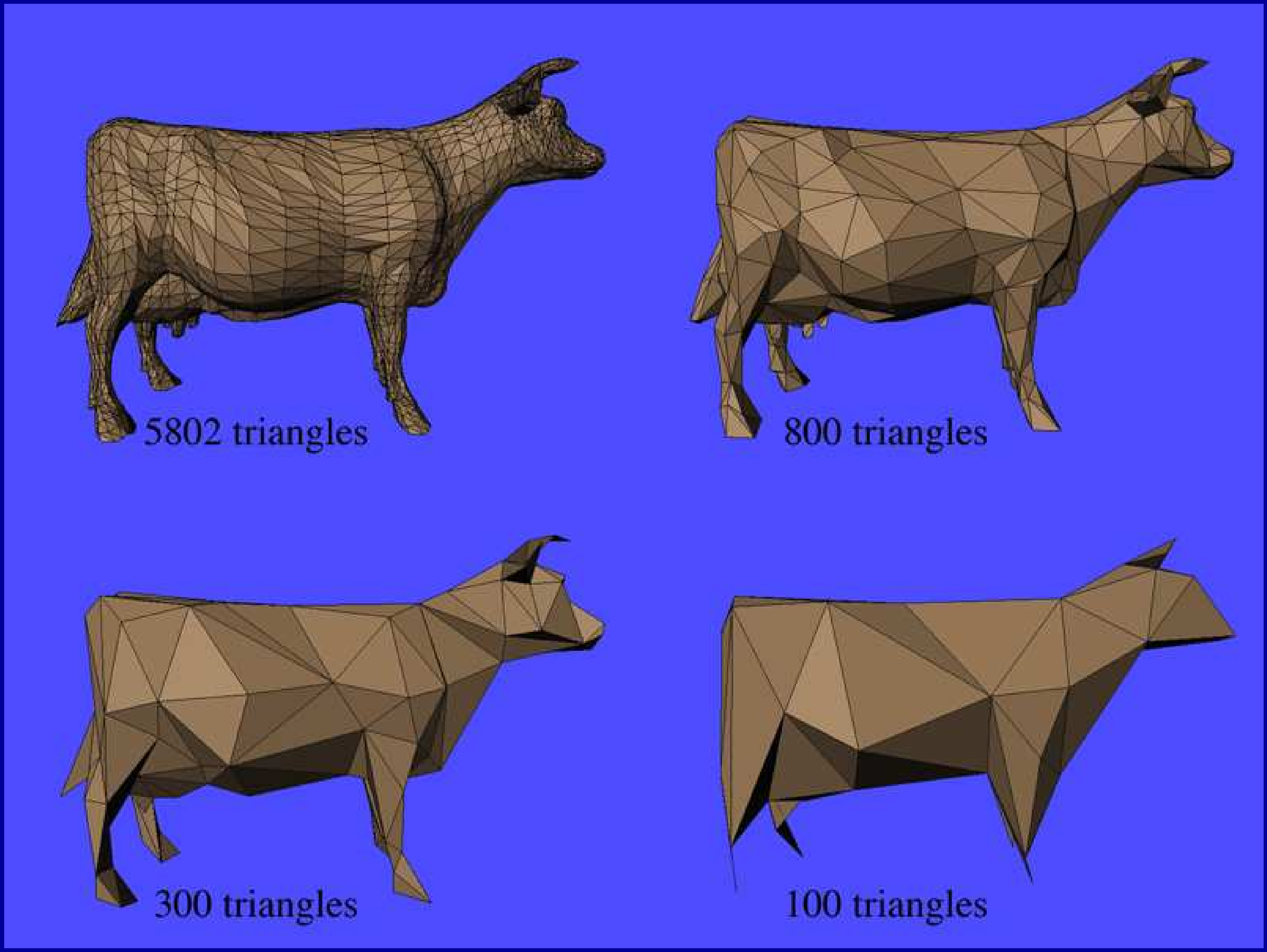
v *x y z* - *a vertex at (x,y,z)*

f v_1 v_2 … v_n

a face with vertices $v_1, v_2, \ldots v_n$

*anything* - *comment*

How Many Polygons to Use?



Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without

- losing key features
- getting artifacts in the silhouette
- popping

Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes

- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?

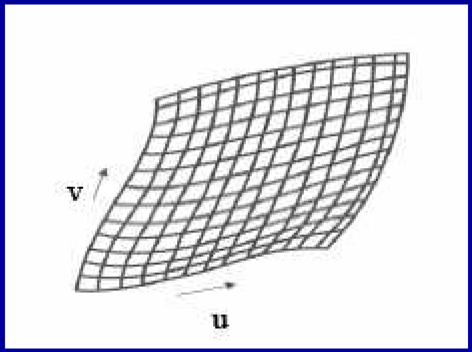
Curve Representations

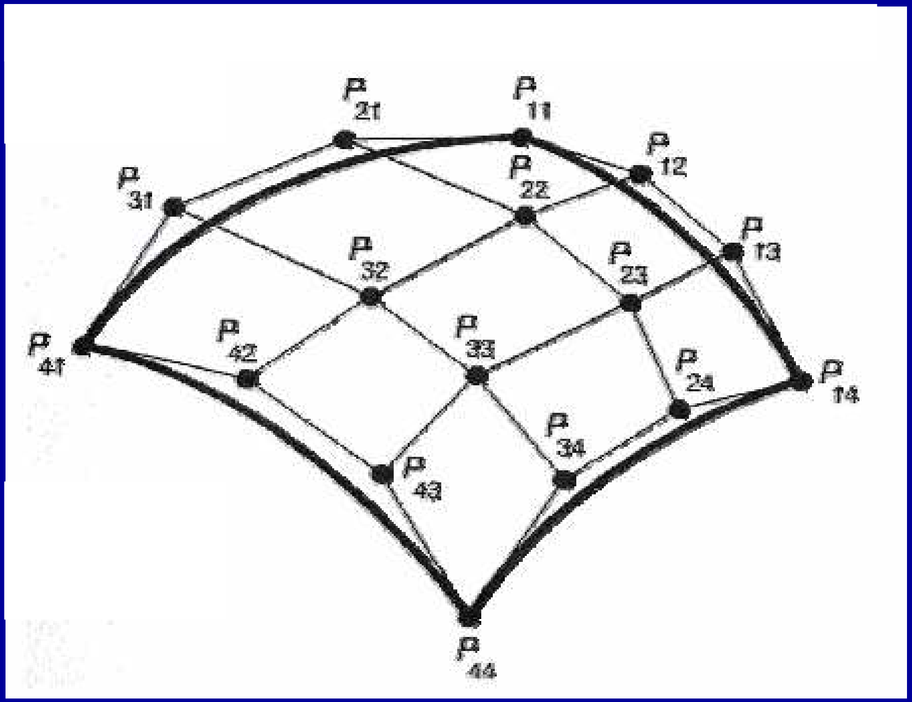
Polygon Meshes
Parametric Surfaces
Implicit Surfaces

Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

– e.g. plane, cylinder, bicubic surface, swept surface



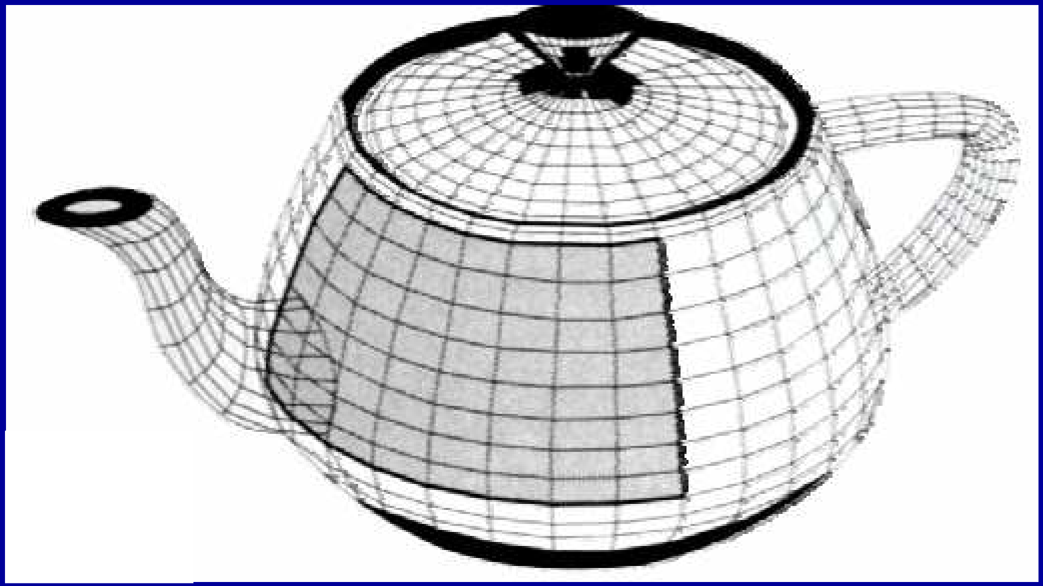


bezier patch

Parametric Surfaces

$p(u,v) = [x(u,v), y(u,v), z(u,v)]$

– e.g. plane, cylinder, bicubic surface, swept surface



Parametric Surfaces

Why better than polygon meshes?

– Much more compact
– More convenient to control --- just edit control points
– Easy to construct from control points

What are the problems?

– Work well for smooth surfaces
– Must still split surfaces into discrete number of patches
– Rendering times are higher than for polygons
– Intersection test? Inside/outside test?

Curve Representations

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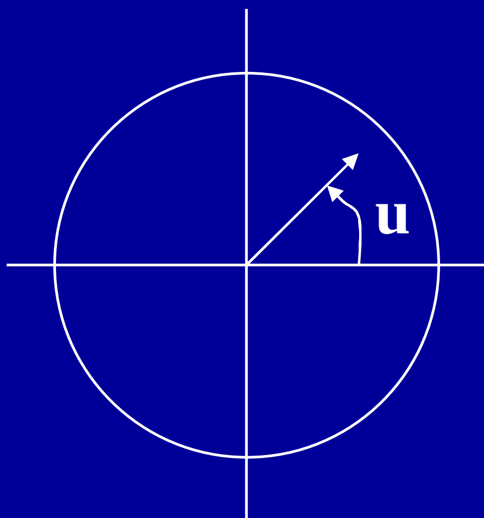
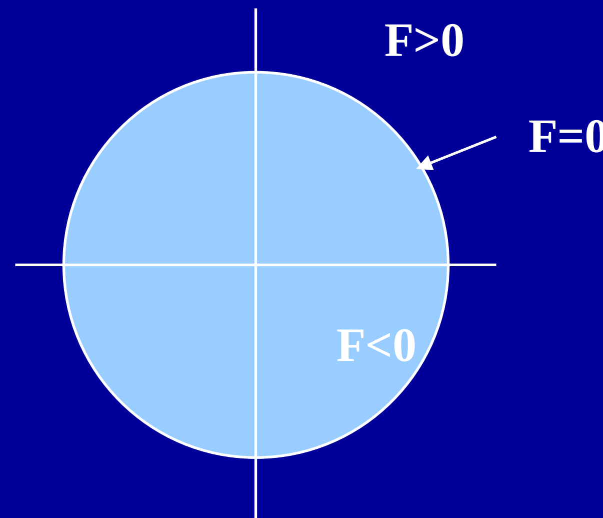
Two Ways to Define a Circle

Parametric



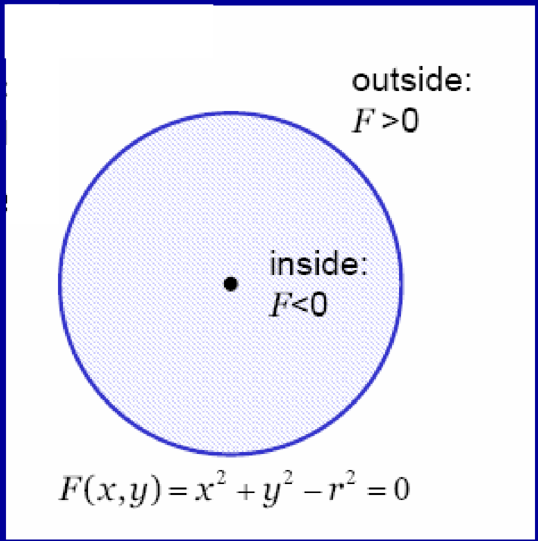
$$x = f(u) = r \cos(u)$$
$$y = g(u) = r \sin(u)$$

Implicit



$$F(x,y) = x^2 + y^2 - r^2$$

Surface Representations



well defined inside/outside
polygons and splines do not have this information

Computing is hard:

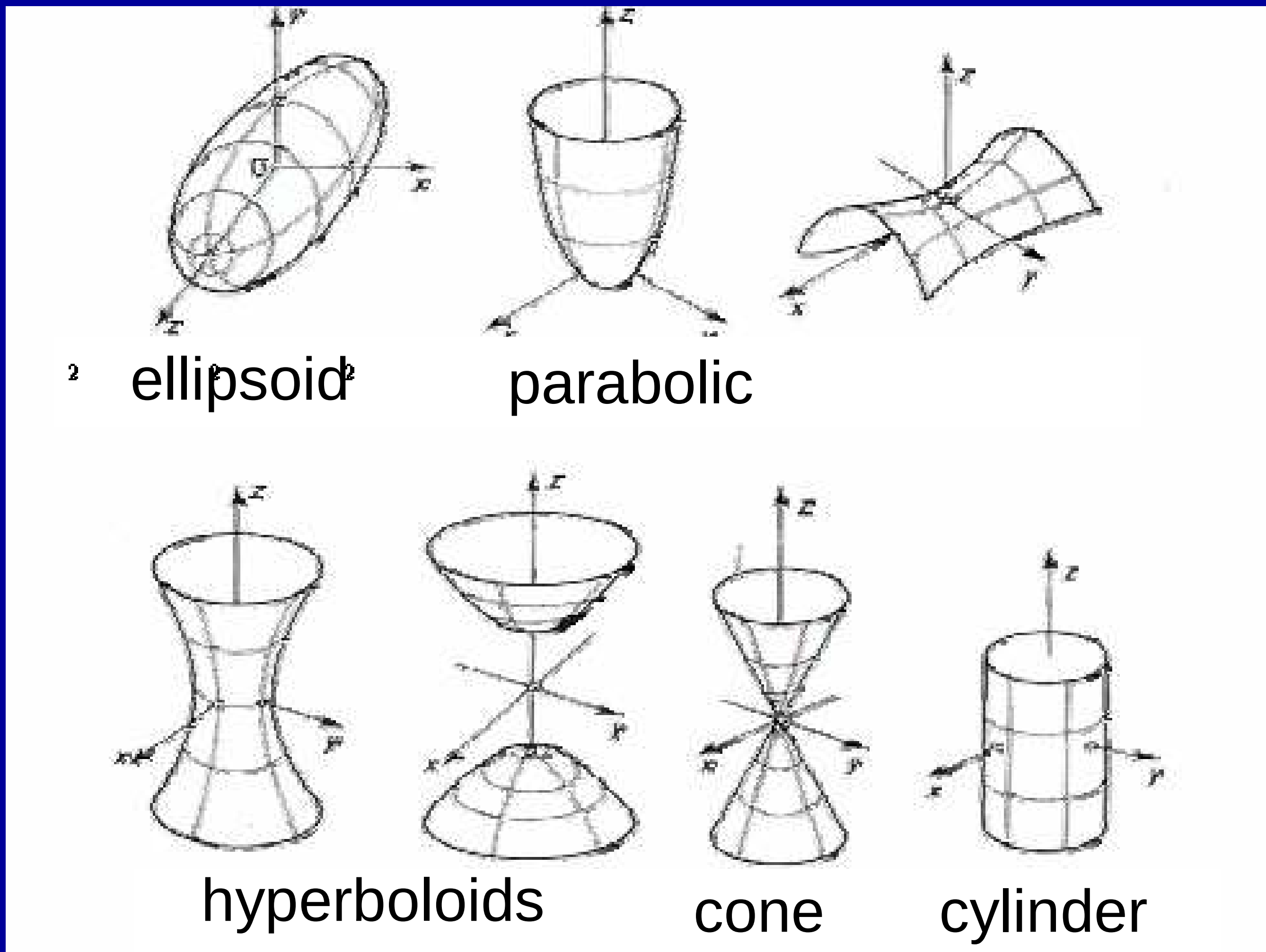
implicit functions for a cube?

telephone?

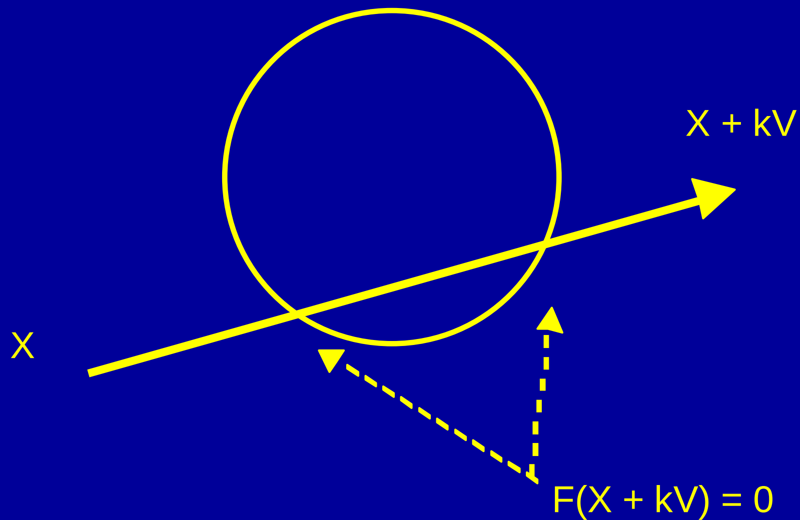
- Implicit surface: F(x,y,z) = 0
  - e.g. plane, sphere, cylinder, quadric, torus, blobby models
    sphere with radius *r* : $F(x,y,z) = x^2+y^2+z^2-r = 0$
  - terrible for iterating over the surface
  - great for intersections, inside/outside test

Quadric Classes

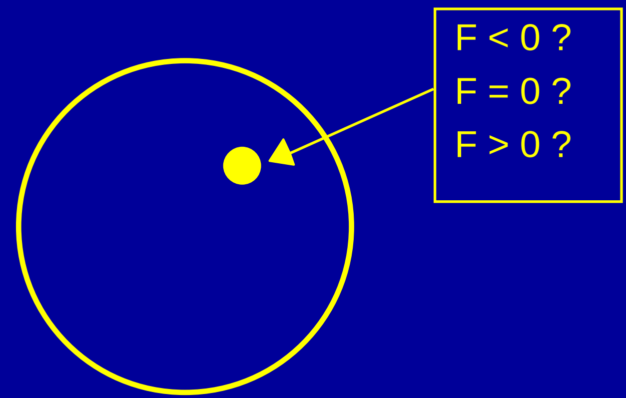
$$F(x,y,z) = ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2px+2qy+2rz+d=0$$



What Implicit Functions are Good For



Ray - Surface Intersection Test



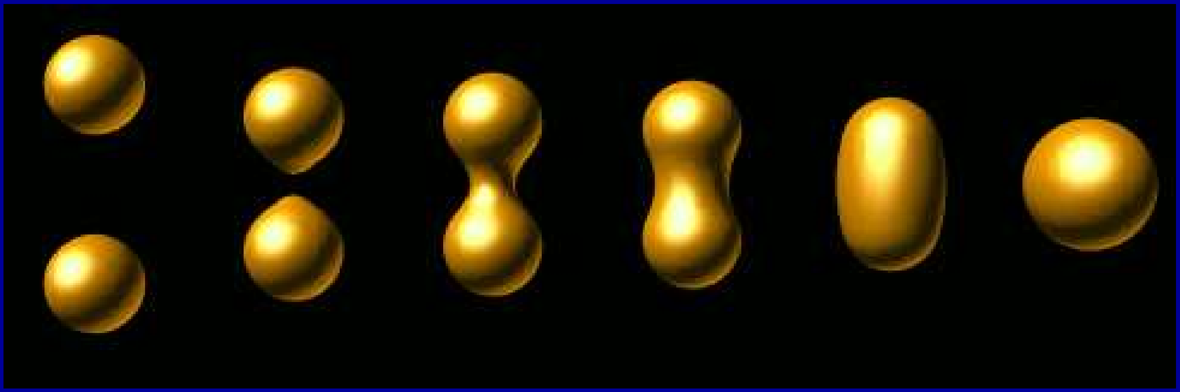
Inside/Outside Test

Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
  - constant value surfaces
  - level sets
  - isosurfaces

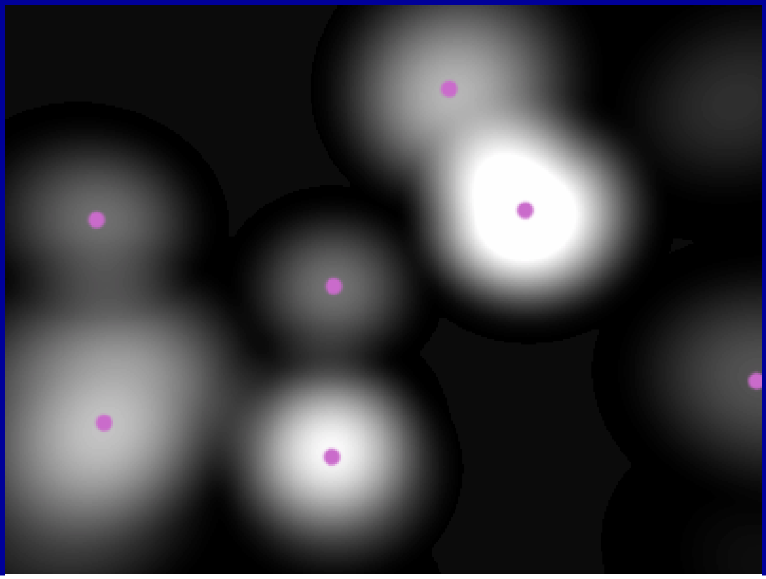
- Nice Feature: you can add them! (and other tricks)
  - this merges the shapes
  - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs,* or *Soft Objects*. Great for modeling animals.

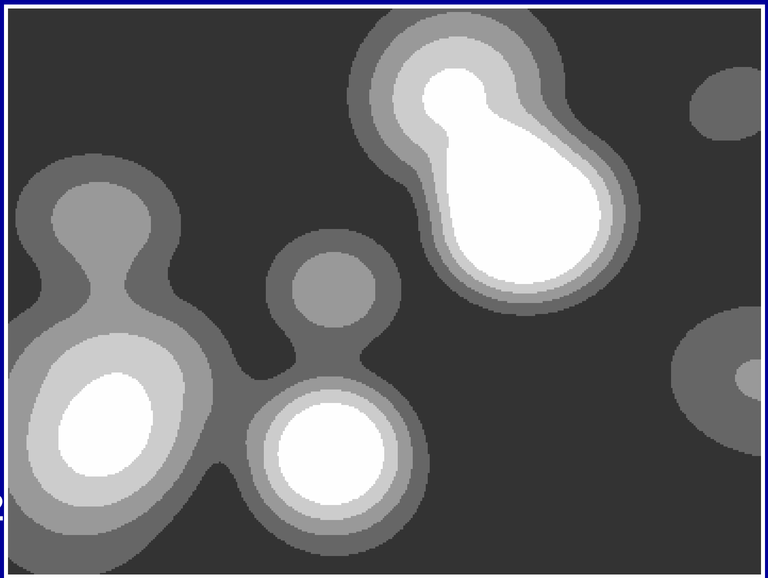
Blobby Models



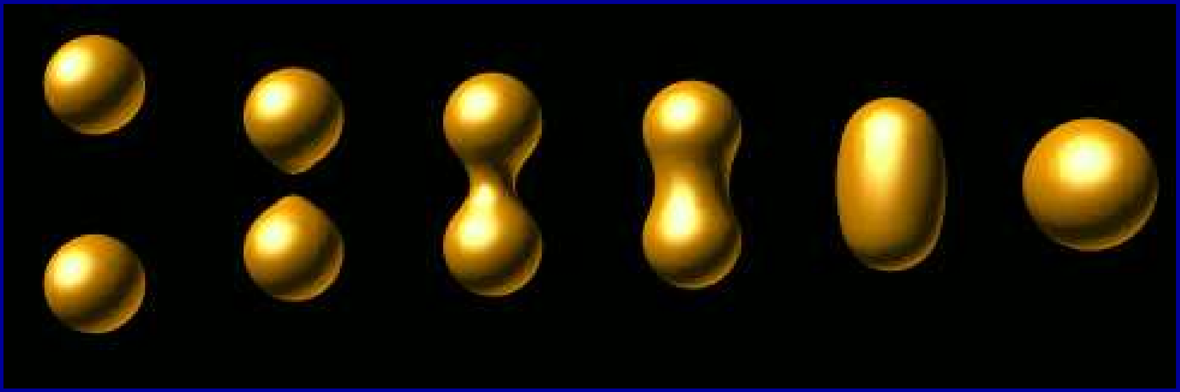
[f(x,y,z) = 1.0 / (x^2 + y^2 + z^2)]   graph for 1/r^2

form blobs if close





Blobby Models



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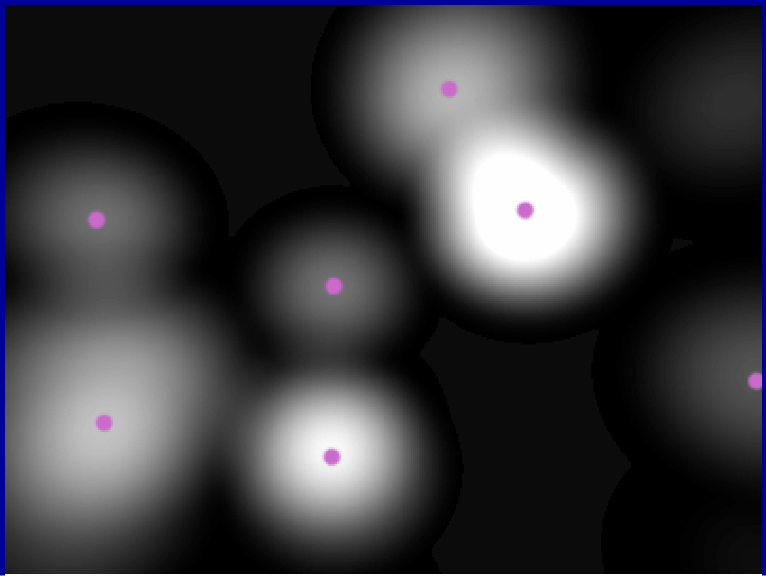
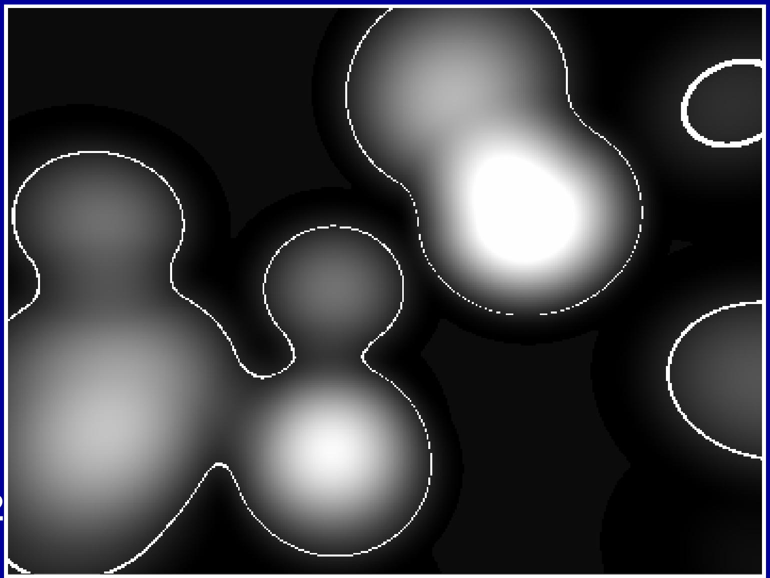
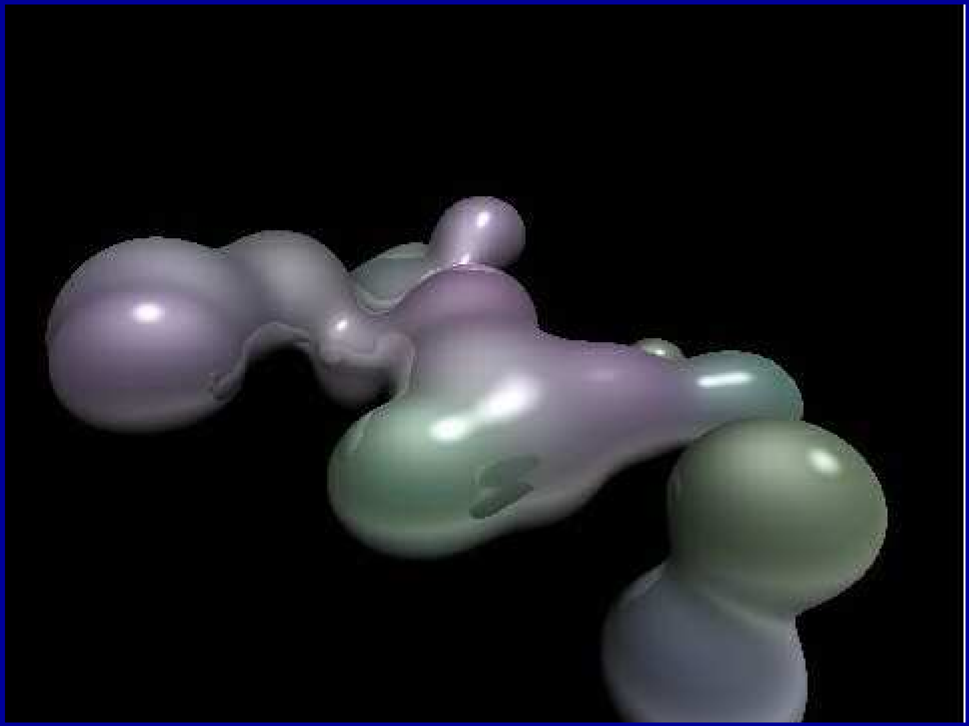
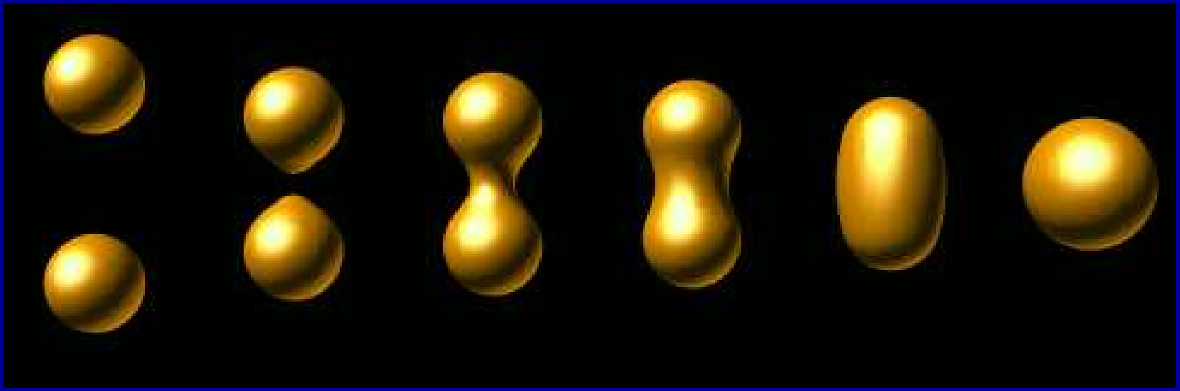




Blobby Models



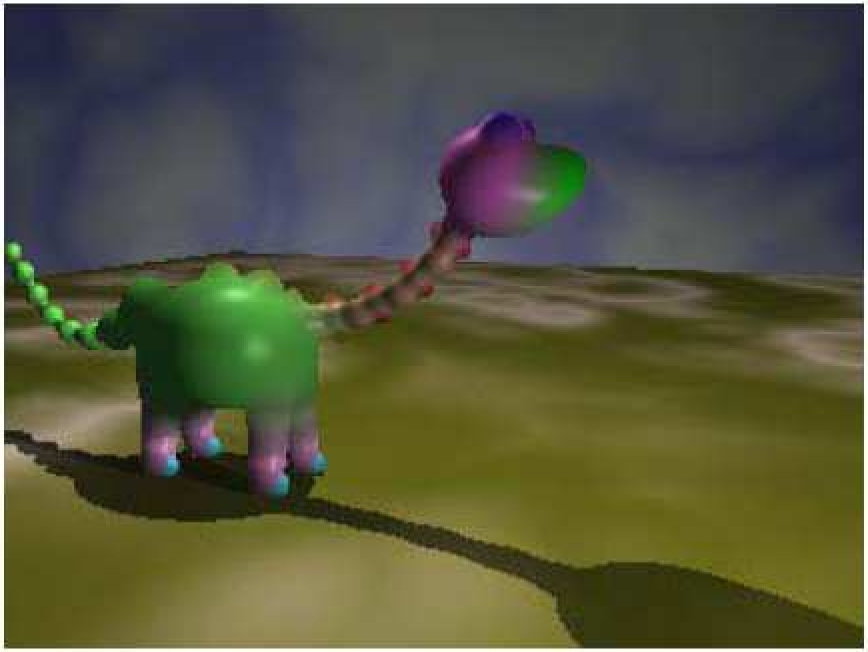
Blobby Models



- Implicit function is the sum of Gaussians centered at several points in space, minus a threshold

- varying the standard deviations of the Gaussians makes each blob bigger
- varying the threshold makes blobs merge or separate

Blobby Models



by Brian Wyvill, http://www.cpsc.ucalgary.ca/~blob/

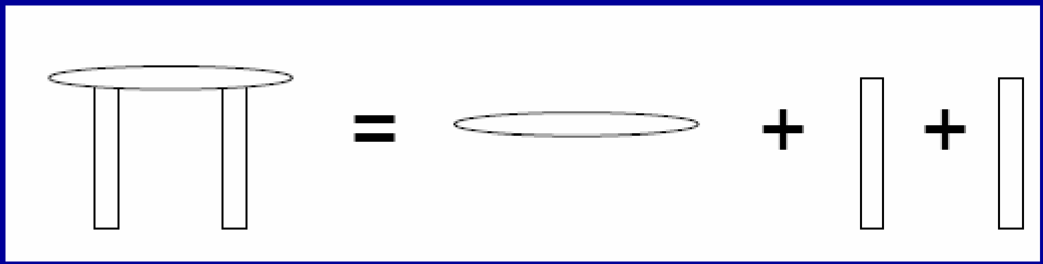
How to draw implicit surfaces?

- It's easy to ray trace implicit surfaces
  - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
  - Divide space into cubes
  - Evaluate implicit function at each cube vertex
  - Do root finding or linear interpolation along each edge
  - Polygonize on a cube-by-cube basis

Constructive Solid Geometry (CSG)

Generate complex shapes with basic building blocks

- machine an object - saw parts off, drill holes
- glue pieces together

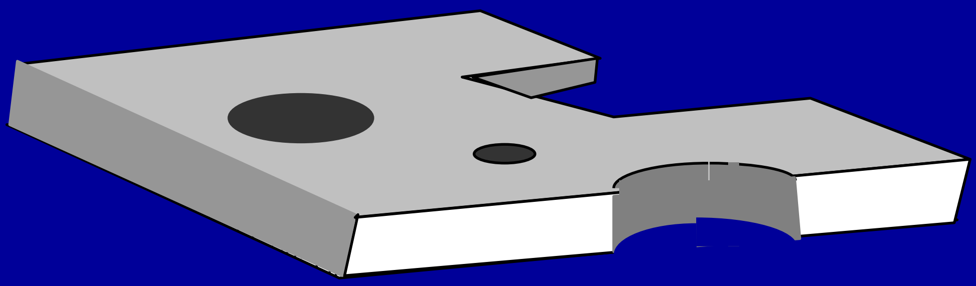


Constructive Solid Geometry (CSG)

Generate complex shapes with basic building blocks

- machine an object - saw parts off, drill holes
- glue pieces together

This is sensible for objects that are actually made that way (human-made, particularly machined objects)



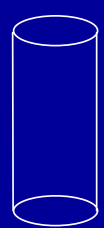
A CSG Train

Brian Wyvill & students, Univ. of Calgary

Negative Objects

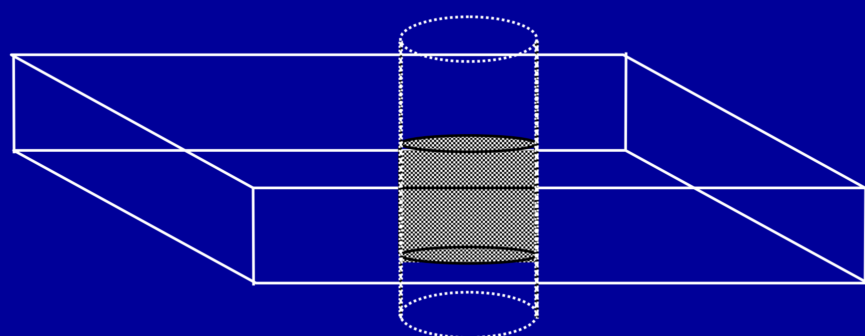
- Use point-by-point boolean functions
  - remove a volume by using a negative object
  - e.g. drill a hole by subtracting a cylinder

Subtract



From

To get



Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))

Set Operations

- UNION: Inside(A) || Inside(B)
  — Join A and B

- INTERSECTION: Inside(A) && Inside(B)
  — Chop off any part of A that sticks out of B.

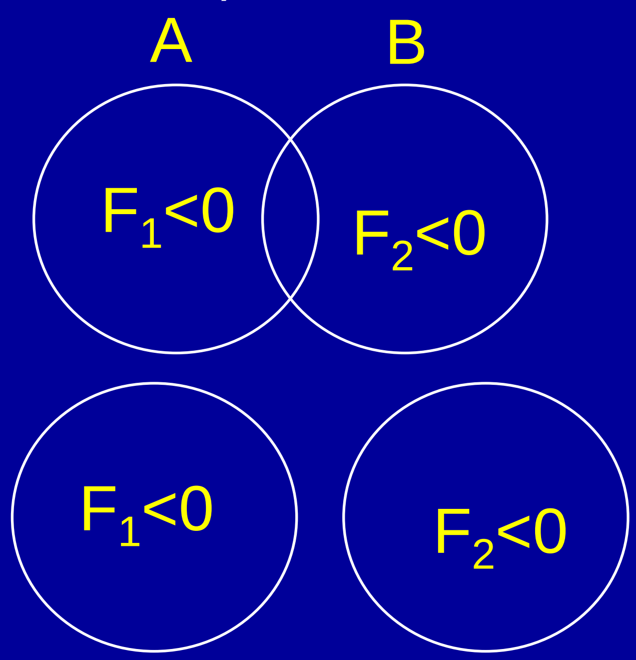
- SUBTRACTION: Inside(A) && (! Inside(B))
  — Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

Implicit Functions for Booleans

- Recall the implicit function for a solid: $F(x,y,z)<0$
- Boolean operations are replaced by arithmetic:
  - MAX replaces AND (intersection)
  - MIN replaces OR (union)
  - MINUS replaces NOT(unary subtraction)



- Thus
  - F(Intersect(A,B)) = MAX(F(A),F(B))
  - F(Union(A,B)) = MIN(F(A),F(B))
  - F(Subtract(A,B)) = MAX(F(A), -F(B))

Implicit Surfaces

– Good for smoothly blending multiple components
– Clearly defined solid along with its boundary
– Intersection test and Inside/outside test are easy

– Need to polygonaze to render --- expensive
– Interactive control is not easy
– Fitting to real world data is not easy
– Always smooth

Announcements

Graded:

Written Assignment – **Joel**

Michael is out this week

Written part of the second programming assignment is due Today before the class or Friday before 9am in Jessica's mailbox