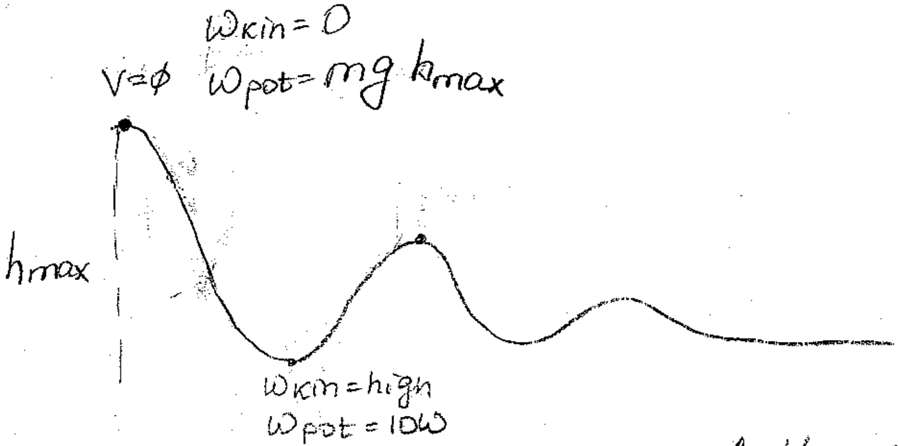
Physics of the roller-coaster

- Mass point moving on the spline



V - tangential direction of the curve

hmax - reached when $|V| = \emptyset$

h - the current vertical coordinate

Conservation of energy:

$W_{kin} + W_{pot} = const = mgh_{max}$

$W_{kin} = \frac{1}{2}mv^2$ => energy of motion

$W_{pot} = mgh$ => energy of position

$\frac{1}{2}mv^2 + mgh = mgh_{max}$

$V^2 = \frac{\cancel{m}gh_{max} - \cancel{m}gh}{1/2\,\cancel{m}} \Rightarrow |V| = \sqrt{2g(h_{max} - h)}$

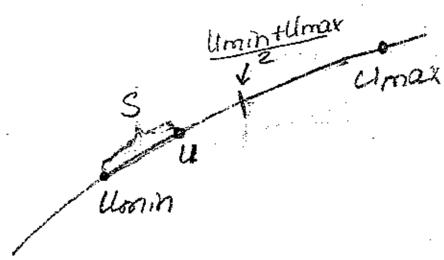
- given heigh h => we can compute $|V|$

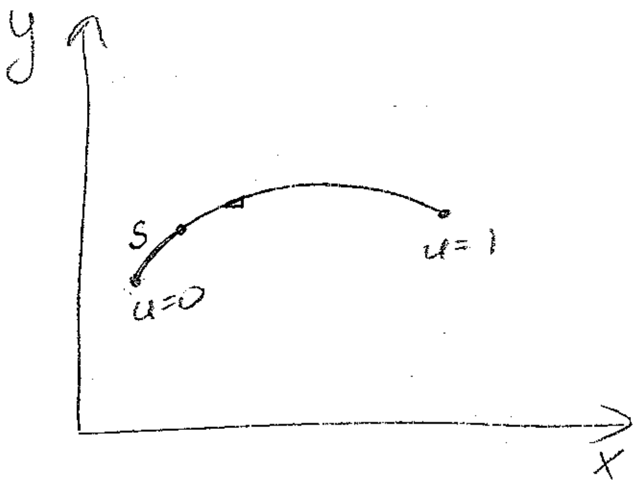
Inverse problem

- given arclength s, determine parameter u
- because $s = s(u)$ is monotonically increasing func so is its inverse
- There is no exact formula for inverse
- use Bisection method

Bisection (umin, umax, s)

```
forver
{
    u = (umin + umax) / 2
    if |s(u)-s| < epsilon
        Return u;
    if s(u) > s
        umax = u
    Else
        umin = u
}
```





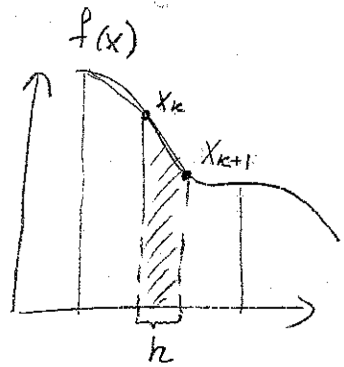
$S(u)$ = the length of the curve from start to $P(u)$

$$S(u) = \int_0^u \sqrt{x'(u)^2 + y'(u)^2}\, du$$

- can not evaluate this integral analytically
- Have to evaluate numerically
- Numerical Integration:

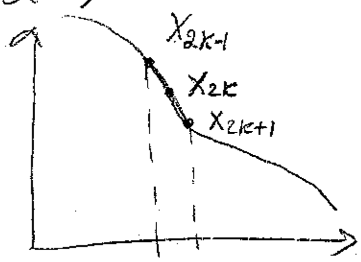
$$\int_a^b f(x)\,dx = \sum_{k=1}^{(n-1)} \frac{h}{2}\left(f(x_k) + f(x_{k+1})\right) + O(h^3)$$

Trapezoidal rule



n = corresponds to the number of intervals

$$\int_a^b f(x)\,dx = \sum_{k=1}^{(n-1)/2} \frac{h}{3}\left[f(x_{2k-1}) + 4f(x_{2k}) + f(x_{2k+1})\right] + O(h^5)$$



$n > 3$, n must be odd