Announcements

- Midterms graded back at the end of class
- Help session on Assignment 3 for last ~20 minutes of class

Scan Conversion

Overview of Rendering
Scan Conversion
    Drawing Lines
    Drawing Polygons

Watt 6.4 (Rasterization)

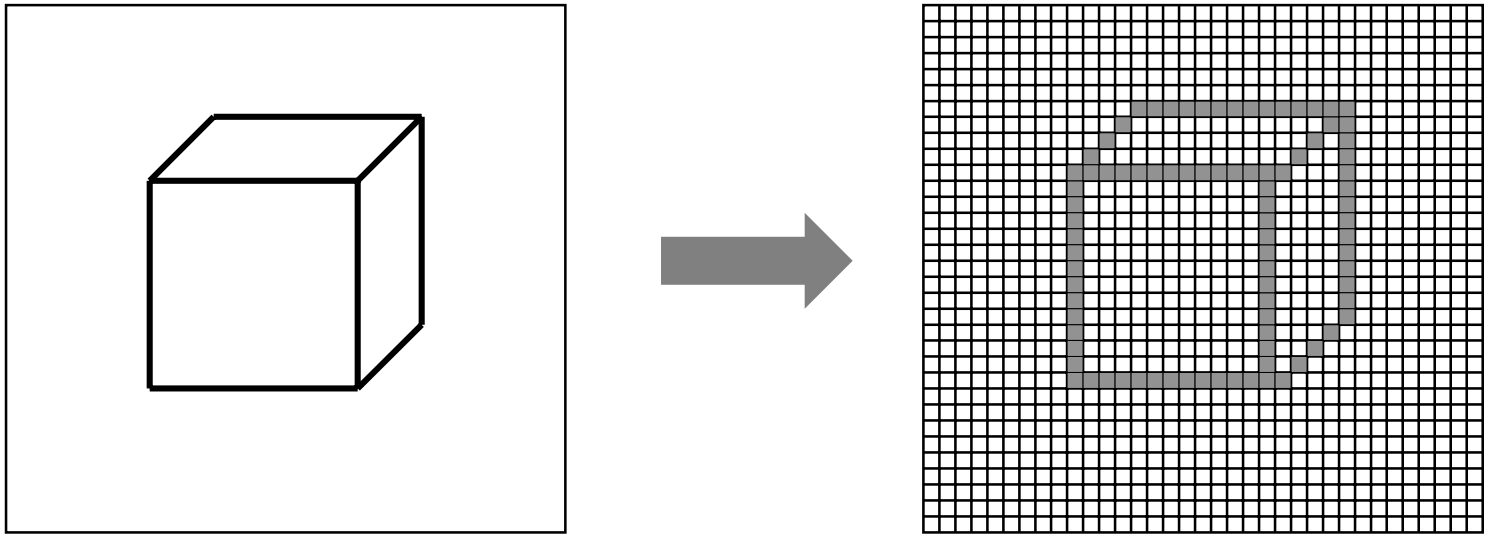
Rendering: creating images of our 3d models

Why is this hard?

- Must determine what's visible
- Must simulate how light flows through the environment
- Different approaches
  - Painter's Algorithm and Z-Buffer Algorithm (OpenGL): draw objects one by one (scan conversion to determine which pixels to write)
  - Ray Tracing: shoot rays out from light source/viewer
  - Radiosity: subdivide surfaces with mesh, solve linear system

- We will start with low-level, scan conversion, then move to raytracing and radiosity

Scan Conversion or Rasterization



Ideal Picture

Raster Representation

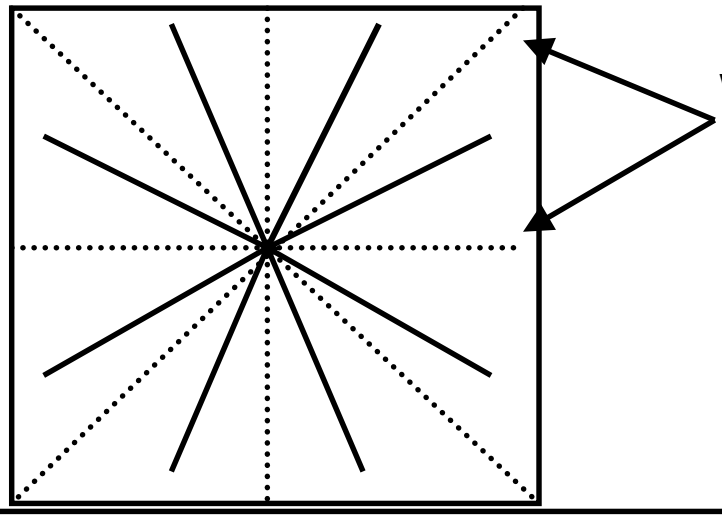
Scan Conversion: Process of converting ideal to raster

Scan Conversion Algorithms

- A discrete set of pixels can only approximate a continuous geometric object
- This means that scan conversion usually introduces error
- Properties of good scan conversion algorithms:
  - They should be as *accurate* as possible
  - They should be as *fast*
- Challenges
  - Modify all the right pixels
  - Modify only the right pixels
  - Calculate their values correctly
  - Do it quickly
- *So, start with a correct algorithm and optimize it*

Line Drawing, Cases by Octant

- Lines come with different slopes, algorithm varies according to which octant (45 degree sector) it lies in.
- We will talk about one octant only; the algorithms generalize to the other octants easily.
- The algorithms for drawing lines need to step along one pixel at a time in the “fast” direction, but which direction this is depends on the slope of the line
- We also have to worry about reversed end point order (drawing from large to small X, for example).



A Really Simple Line Algorithm

- Equation for a line: $y(x) = mx + b$
- Fill in one pixel per column
- So, just evaluate for each x
  - This requires a choice of quadrant, so x steps evenly
- Certainly correct, but slow:
  - integer add, cast to float, floating multiply and add, plus round every step.

```
void line (int x0, int y0, int x1, int y1){
    float m = whatever;
    float b = whatever;
    int x;
    for(x=x0;x<=x1;x++) {
        float y= m*x + b;
        draw_pixel(x,Round(y));
    }
}
```

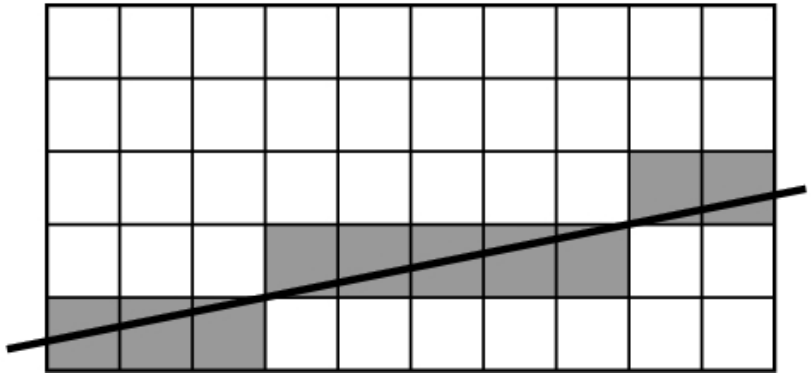
Lines: DDA Algorithm

- Optimize the previous to remove multiply from inner loop.
- If we know y(x), we can calculate y(x+1):
  - $y(x+1) = mx + m + b = y(x) + m$

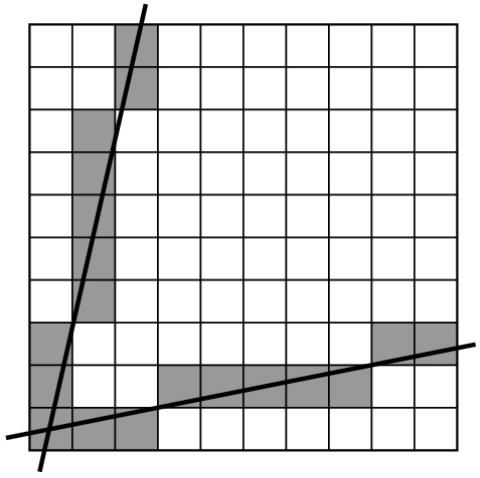
```
void line (int x0, int y0, int x1, int y1){
    float y = y0;
    float m = (y1 - y0)/ (float) (x1 - x0);
    int x;
    for(x=x0;x<=x1;x++) {
        draw_pixel(x,Round(y));
        y += m;
    }
}
```

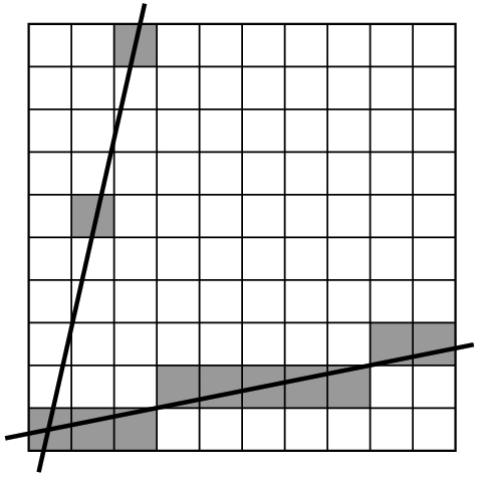
- This is called *Differential Digital Analyzer* (DDA) because it is solving the very simple differential equation dy/dx = m
- Problem: Floating-point add and rounds are expensive

What do we get from this algorithm?



Why did we limit the slope of the line?
So we can have this rather than that…





Bresenham's Algorithm

This does the right thing (same as DDA) at a cost of only 2 or 3 integer adds per point. (assumes sorted endpoints, 0<slope<1)

```
void draw_line(int x0, int y0, int x1, int y1) {
    int x, y = y0;
    int dx = 2*(x1-x0), dy = 2*(y1-y0);
    int dydx = dy-dx, F = (dy-dx)/2;

    for (x=x0 ; x<=x1 ; x++) {
        draw_pixel(x, y);
        if (F<0) F += dy;
        else {y++; F += dydx;}
    }
}
```

F is a decision variable—do you increment y on this iteration or not?

Why does this work?

Implicit Function for a Line

$Line\ \mathbf{L}\ from\ [x_0, y_0]\ to\ [x_1, y_1].$

$\mathbf{P}_0 = [x_0, y_0],$

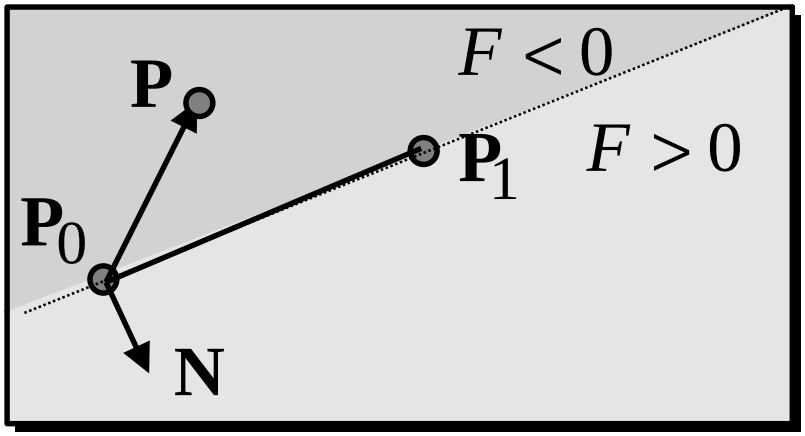
$\mathbf{P}_1 = [x_1, y_1].$

$dx = x_1 - x_0,\ dy = y_1 - y_0$

$\mathbf{N} = [dy, -dx]$

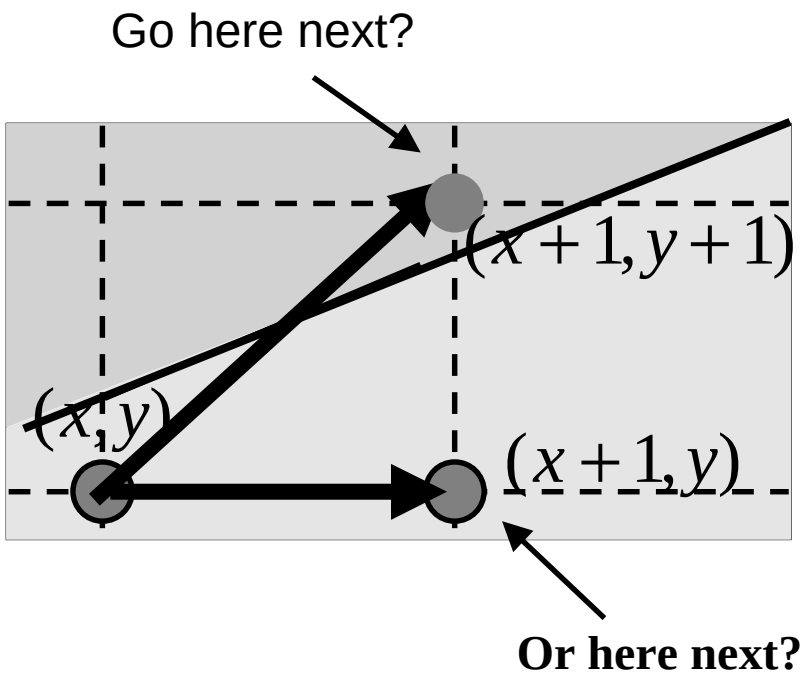
$implicit\ \ function: F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$

$F = 0 \rightarrow \mathbf{P}\ is\ on\ \mathbf{L}$



Why the factor of 2? Because we're going to divide by 2 later.

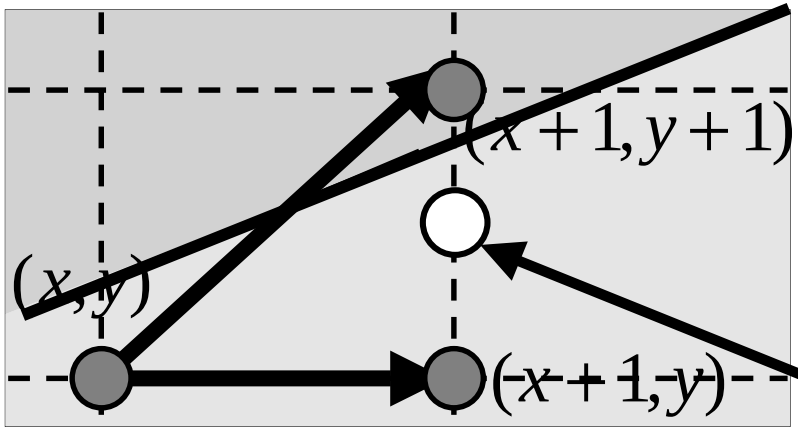
Line Drawing: Which Pixel is Next?



- Assume:
  - 0 < slope < 1
  - sorted endpoints, $x_0<x_1$
- At each step:
  - Current point is *(x,y)*
  - Next point is pixel (x+1,?) that's closest to the actual line
  - Do we increment *x* and *y* or only *x*?
- Use the implicit function to decide!

Use the Implicit Function

- Idea: Test the half-way point (x+1, y+1/2)



$F((x+1, y+1/2)) > 0?$

yes: *increment x and y*

no: *increment x*

Trick: Incrementally Update *F*

$$F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$$

$$F(\mathbf{P} + \Delta) = 2\mathbf{N} \cdot (\mathbf{P} + \Delta - \mathbf{P}_0)$$

$$= F(\mathbf{P}) + 2\mathbf{N} \cdot \Delta$$

- Computing *F*(P) requires a dot product:
  - 2 multiplications and 1 add
- But computing *F*(P+Δ) requires only 1 add
  - The 2N• Δ term is constant - it only needs to be calculated once
- Δ is [1,0] or [1,1]—x is incremented, y might or might not be

Decision Variable F

$$F_0 = F(\mathbf{P}_0 + [1,1/2])$$
$$= F(\mathbf{P}_0) + \mathbf{N} \cdot [2,1]$$
$$F' = F + 2\mathbf{N} \cdot \Delta$$

where

$$\Delta = [1,0] \; or \; [1,1]$$
$$\mathbf{N} = [dy, -dx]$$

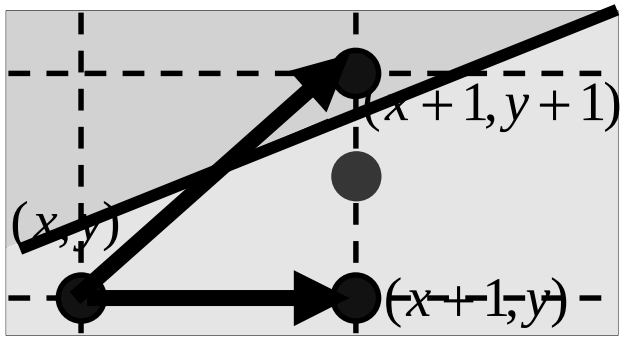
so

$$F_0 = F(P_0) + 2dy - dx$$
$$F' = F + 2dy$$

or

$$F' = F + 2dy - 2dx$$

- Initialize *x*, *y*, F
- Loop until end of line:
  - draw pixel *(x,y)*
  - increment *x*
  - if F>0, increment *y*
  - increment F according to whether Δ is [1,0] or [1,1]



Bresenham Line Algorithm—Code Snippet Again

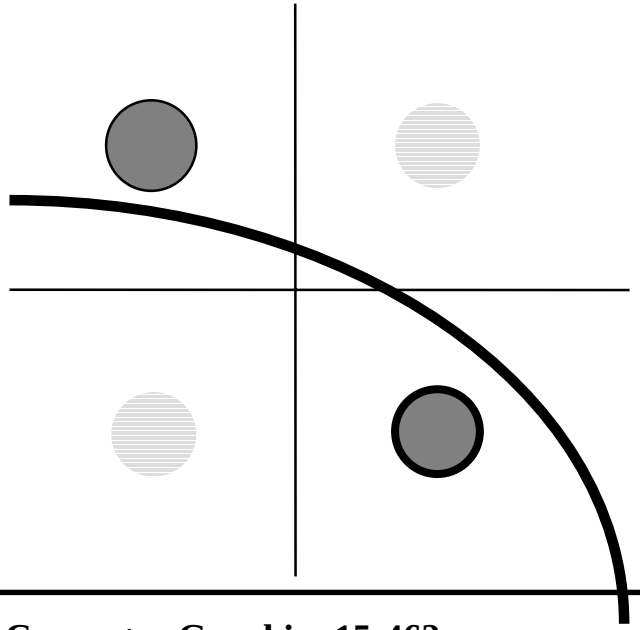
this does the right thing (same as DDA) at
a cost of only 2 or 3 integer adds per point.
(assumes sorted endpoints, 0<slope<1)

```
void draw_line(int x0, int y0, int x1, int y1) {
    int x, y = y0;
    int dx = 2*(x1-x0), dy = 2*(y1-y0);
    int dydx = dy-dx, F = (dy-dx)/2;

    for (x=x0; x<=x1; x++) {
        draw_pixel(x, y);
        if (F<0) F += dy;
        else {y++; F += dydx;}
    }
}
```

Bresenham Algorithm for Circles

- Same approach as line algorithm
  - use a decision variable formula derived for a circle ($F = x^2 + y^2 - r^2$)
- Eightfold symmetry
  - only compute the points for one octant - use sign flips to give the rest
- Extends to general conics (ellipses...)



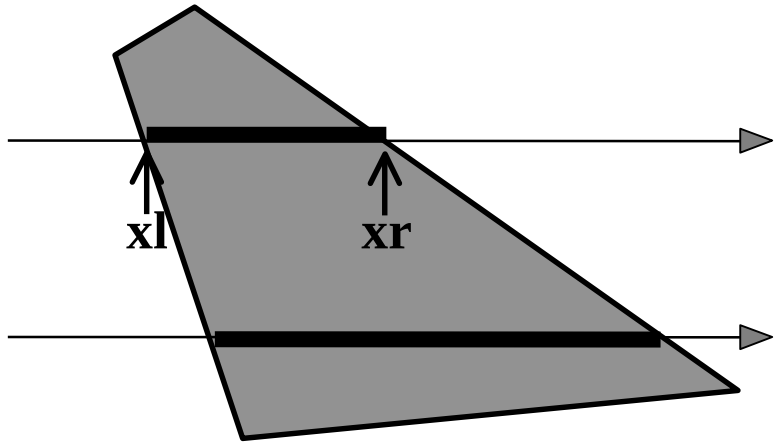
Bresenham Circle Algorithm

```
/* this draws a circle by calculating in one octant */
/* and re-using the resulting point 8 times */
void draw_circle(int radius) {
    int x = 0, y = radius;
    int d = 1-radius;
    while (y>x) {
        if (d<0) /* select East point next */
            d += 2*x + 3;
        else {   /* select South-East point next */
            d += 2*(x-y) + 5;
            y--;
        }
        x++;
        draw_8_pts(x,y); /* draws point in each octant */
    }
}
```

Scan Converting Filled, Convex Polygons: Basic Approach

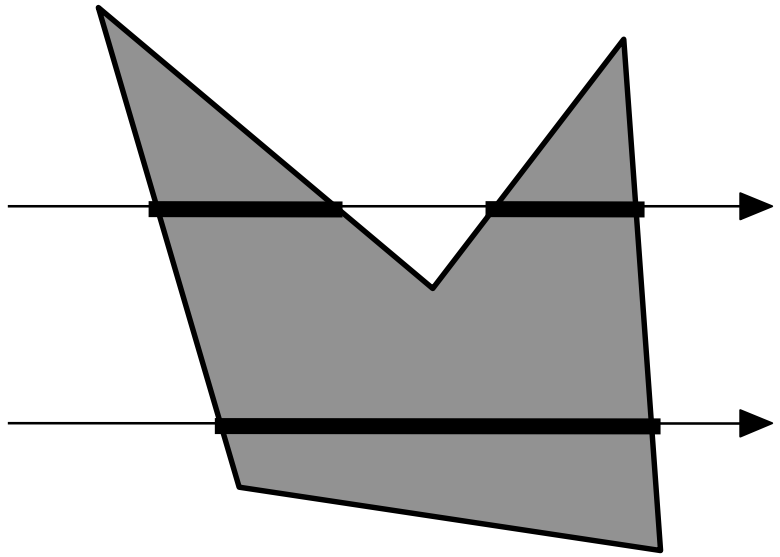
- Find top and bottom vertices
- Make list of edges along left and right sides
- For each scanline from top to bottom
  - *There's a single span to fill*
  - Find left & right endpoints of span, xl & xr
  - Fill pixels inbetween xl & xr
  - Can use Bresenham's algorithm to update xl & xr as you step from line to line

If you don't do all of the above carefully, cracks or overlaps between abutting polygons result!



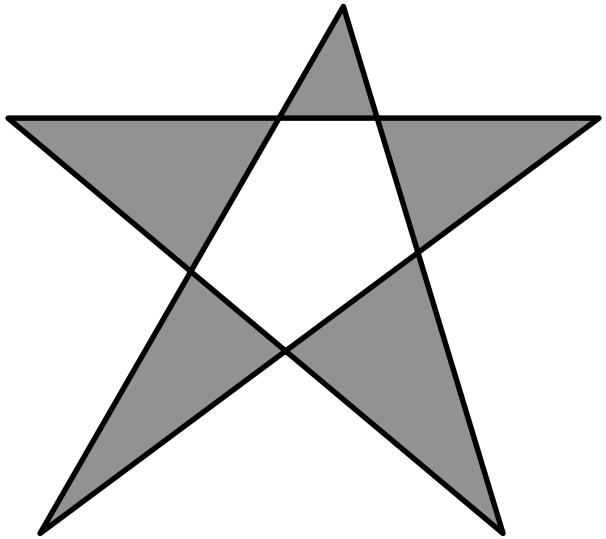
Scan Converting Filled, Concave Polygons: Basic Approach

- For each scanline
  - Find all the scanline/polygon intersections
  - Sort them left to right
  - Fill the interior *spans* between intersections
  - *Parity Rule:* odd ones are interior, even are exterior

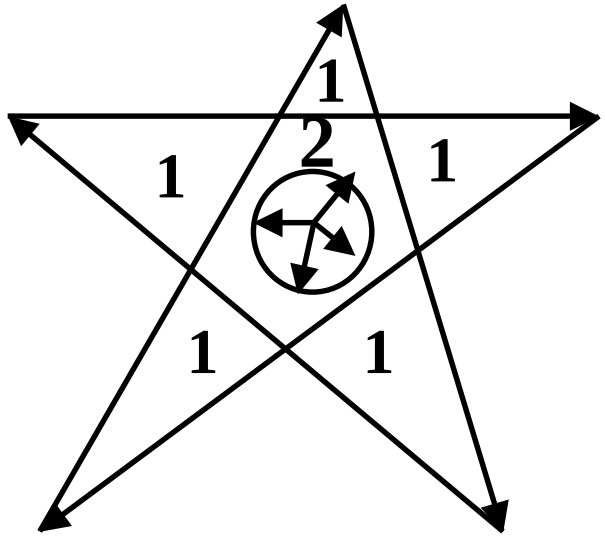


Inside/Outside Rules

- This even-odd rule is not the only option.
- An alternative would be the *winding rule:*
  - Pick a line to infinity (like the preceding part of the scan line)
  - Add up right-handed minus left-handed crossings
  - non-zero result means it's interior
- Only matters for self-intersecting primitives



Even-odd rule

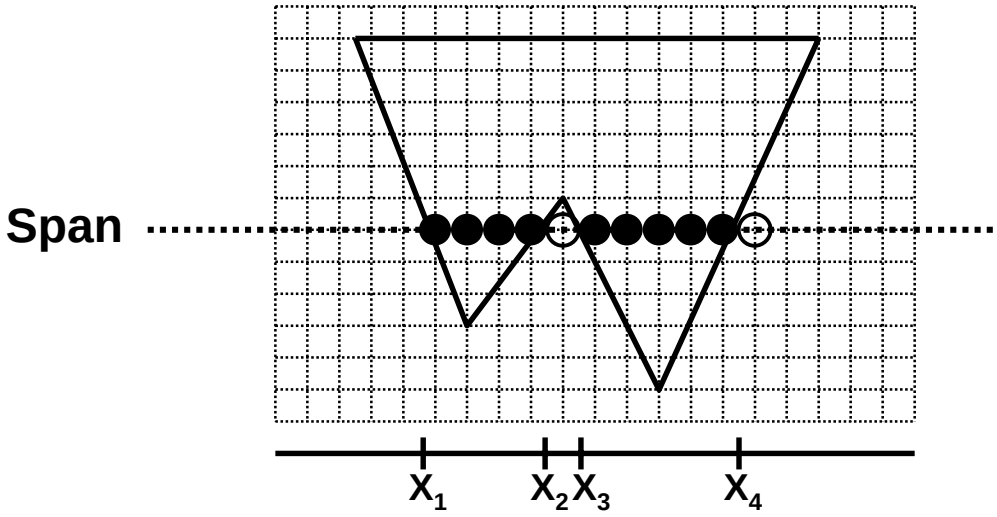


Winding rule

Span Filling

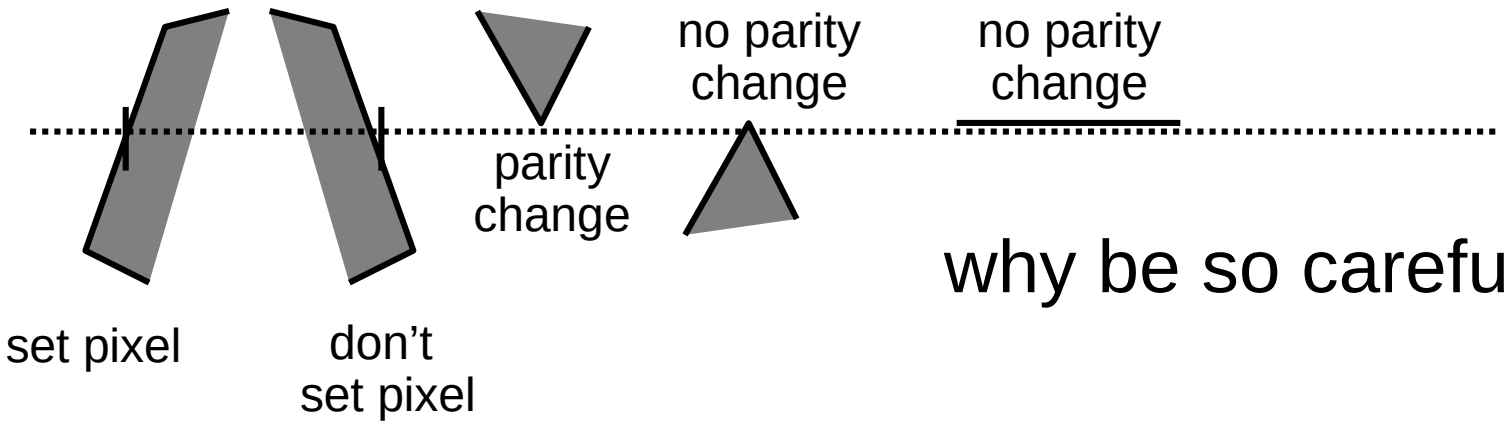
Given a sorted list of scanline intersections (X1,X2,...,Xn)

- Initialize: start at *ceiling*(X1), set *parity* = 1 (inside)
- While x < Xn
  - » if (parity == 1), pixel = fillcolor
  - » x = x + 1
  - » if you've passed an intersection, *parity* = (1 - *parity*)



Special Cases

- For an integer intersection - exactly on a pixel
  - set the pixel if it's the beginning of a span, not if it's the end
- Shared vertices (possible grazing contact)
  - count ymin for parity, not ymax
- Horizontal edges
  - don’t change parity



why be so careful?

Special Cases 2

- Q: Where did these inside/outside rules come from?
- A: Somebody made them up.

- You can resolve the special cases any way you want, as long as it's consistent:
  - Avoid drawing a pixel twice if the same edge is used for two adjacent polygons. (It's slightly slower, and leads to errors in XOR mode)
  - Degenerate or horizontal edges should not leave seams.
  - Make sure inside regions stay inside (and vice versa) for small shifts of the polygon

Computing Edge/Scanline Intersections

- How to compute intersection points (X1, ..., Xn)?
- Brute force method
  - Calculate intersection of each scanline with each edge
- Better method
  - Exploit *coherence* between scanlines
    - » intersections do not change much between scanlines
- Cache the intersection information in a table
  - *Edge Table* with all edges sorted by *ymin*
  - *Active Edge Table* containing the edges that intersect the current scanline, sorted by x-intersection, left to right
- Process the image from the smallest *ymin* up
- When you run out of edges you're done

Caching Edge/Scanline Intersections

- Cache the intersection information in a table
  - *Edge Table* (ET) with all edges sorted by *ymin*
  - *Active Edge Table* (AET) containing
    - » the edges that intersect the current scanline
    - » their points of intersection
    - » sorted by x-intersection, left to right