Announcements

Movie from Assignment 1
Grades out soon

3D Viewing & Clipping

Where do geometries come from?
Pin-hole camera
Perspective projection
Viewing transformation

Clipping lines & polygons

Watt 5.2 and 6.1

COMPUTER GRAPHICS

15-462

12 Sept 2001

Where do geometries come from?

- Build them with 3D modelers
- Digitize or scan them
- Results of simulation/physically based modeling
- Combinations:
  - Edit a digitized model
  - Simplify a scanned model
  - "Evolve" a model
- Often, need multiple models at different complexity

Getting Geometry on the Screen

Given geometry in the world coordinate system, how do we get it to the display?

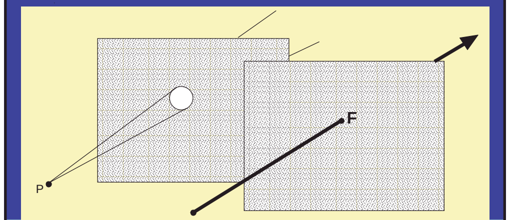
- Transform to camera coordinate system
- Transform (warp) into canonical view volume
- Clip
- Project to display coordinates
- (Rasterize)

Viewing and Projection

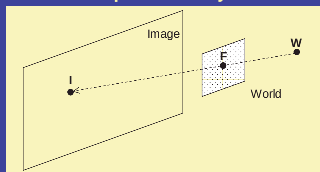
- Our eyes collapse 3-D world to 2-D retinal image (brain then has to reconstruct 3D)
- In CG, this process occurs by *projection*
- Projection has two parts:
  - *Viewing transformations:* camera position and direction
  - *Perspective/orthographic transformation:* reduces 3-D to 2-D
- Use homogeneous transformations
- As you learned in Assignment 1, camera can be animated by changing these transformations—the root of the hierarchy

Pinhole Optics

- Stand at point P, and look through the hole - anything within the cone is visible, and nothing else is
- Reduce the hole to a point - the cone becomes a *ray*
- Pin hole is the *focal point, eye point* or *center of projection.*



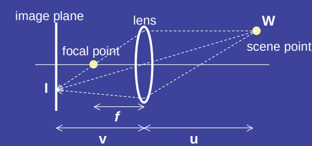
Perspective Projection of a Point



- *View plane* or *image plane* - a plane behind the pinhole on which the image is formed
  - point *I* sees anything on the line (ray) through the pinhole *F*
  - a point *W* projects along the ray through *F* to appear at *I* (intersection of WF with image plane)

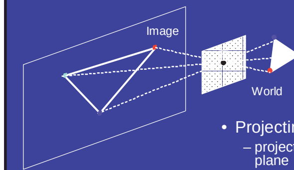
Problems with Pinholes

- Correct optics requires infinitely small pinhole
  - No light gets through
  - Diffraction
- Solution: Lens with finite aperture

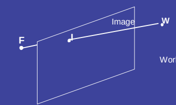


Lens Law: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Image Formation



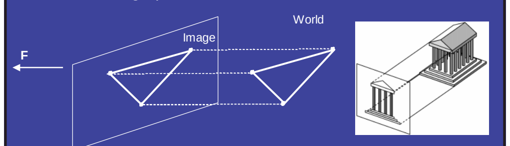
- Projecting a shape
  - project each point onto the image plane
  - lines are projected by projecting end points only



Note: Since we don't want the image to be inverted, from now on we'll put F behind the image plane.

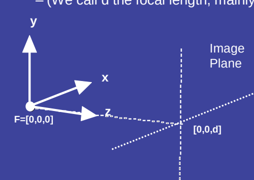
Orthographic Projection

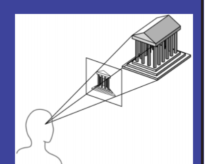
- when the focal point is at infinity the rays are parallel and orthogonal to the image plane
- good model for telephoto lens. No perspective effects.
- when *xy*-plane is the image plane (x,y,z) -> (x,y,0) front orthographic view



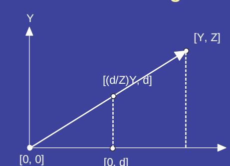
A Simple Perspective Camera

- Canonical case:
  - camera looks along the *z*-axis
  - focal point is the origin
  - image plane is parallel to the *xy*-plane at distance *d*
  - (We call d the focal length, mainly for historical reasons)





Similar Triangles



- *vup:* a vector that is pointing straight up in the image usually want world "up" direction
- Diagram shows *y*-coordinate, *x*-coordinate is similar
- Using similar triangles
  - point [x,y,z] projects to [(d/z)x, (d/z)y, d]

A Perspective Projection Matrix

•Projection using homogeneous coordinates:

– transform [x, y, z] to [(d/z)x, (d/z)y, d]

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx & dy & dz & z \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{d}{z}x & \frac{d}{z}y & \mathrm{d} \end{bmatrix}$$

Divide by 4th coordinate
(the "w" coordinate)

- 2-D image point:
  - discard third coordinate
  - apply viewport transformation to obtain physical pixel coordinates

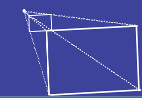
Wait, there's more!

Perspective transformation can also

- map rectangle in the image plane to the viewport
- specify near and far clipping planes
  - instead of mapping z to d, transform *z* between *znear* and *zfar* on to a fixed range
  - used for *z*-buffer hidden surface removal
- specify field-of-view (fov) angle

The View Volume

- Pyramid in space defined by focal point and window in the image plane (assume window mapped to viewport)
- Defines visible region of space
- Pyramid edges are clipping planes
- *Frustum* = truncated pyramid with near and far clipping planes
  - Why near plane? Prevent points behind the camera being seen
  - Why far plane? Allows *z* to be scaled to a limited fixed-point value (*z*-buffering)



But wait...

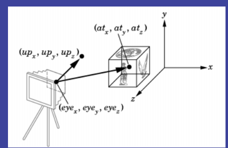
- What if we want the camera somewhere other than the canonical location?
- Alternative #1: derive a general projection matrix. (*hard*)
- Alternative #2: transform the world so that the camera is in canonical position and orientation (*much simpler*)
- These transformations are *viewing transformations*
- They can be specified in many ways - some more sensible than others (beware of Foley, Angel and Watt are ok)

Camera Control Values

- All we need is a single translation and angle-axis rotation (orientation), but...
- Good animation requires good camera control--we need better control knobs
- Translation knob - move to the *lookfrom* point
- Orientation can be specified in several ways:
  - specify camera rotations
  - specify a *lookat* point (solve for camera rotations)

A Popular View Specification Approach

- Focal length, image size/shape and clipping planes are in the perspective transformation
- In addition:
  - *lookfrom:* where the focal point (camera) is
  - *lookat:* the world point to be centered in the image
- Also specify camera orientation about the *lookat-lookfrom* axis



Implementation

Implementing the *lookat/lookfrom/vup* viewing scheme

(1) Translate by *-lookfrom*, bring focal point to origin

(2) Rotate *lookat-lookfrom* to the *z*-axis with matrix R:

» v = (*lookat-lookfrom*) (normalized) and z = [0,0,1]

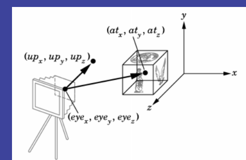
» rotation axis: a = (v×z)/|v×z|

» rotation angle: cosθ = v•z and sinθ = |v×z|

glRotate(θ, a_x, a_y, a_z)

(3) Rotate about *z*-axis to get *vup* parallel to the y-axis

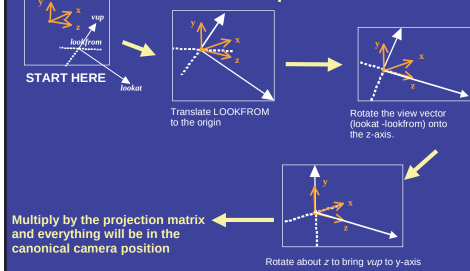
The Whole Picture



LOOKFROM:	Where the camera is
LOOKAT:	A point that should be centered in the image
VUP:	A vector that will be pointing straight up in the image
FOV:	Field-of-view angle.
d:	focal length

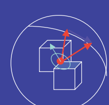
WORLD COORDINATES

It's not so complicated…



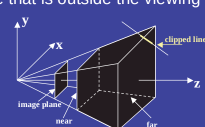
Virtual Trackballs

- Imagine world contained in crystal ball, rotates about center
- Spin the ball (and the world) with the mouse
- Given old and new mouse positions
  - project screen points onto the sphere surface
  - rotation axis is normal to plane of points and sphere center
  - angle is the angle between the radii
- There are other methods to map screen coordinates to rotations



Clipping

- There is something missing between projection and viewing...
- Before projecting, we need to eliminate the portion of scene that is outside the viewing frustum

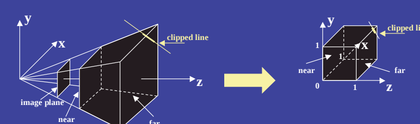


•Need to clip objects to the frustum (truncated pyramid)

•Now in a canonical position but it still seems kind of tricky...

Normalizing the Viewing Frustum

- Solution: transform frustum to a cube before clipping



- Converts perspective frustum to orthographic frustum
- This is yet another homogeneous transform!

The Normalized Frustum

- OpenGL uses -1<=x<=1, -1<=y<=1, -1<=z<=1
- But it doesn't really matter… we can clip against any such cube.
  - Or, we can translate normalizing transformations by applying the appropriate trans.
- Must clip in homogeneous coordinates:
  - w>0: -w<=x<=w, -w<=y<=w, -w<=z<=w
  - w<0: -w>=x>=w, -w>=y>=w, -w>=z>=w

But wait! Divide by zero?

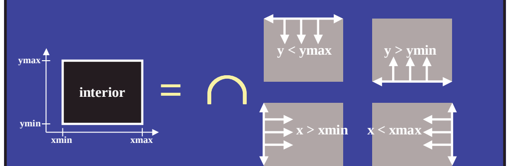
- But doesn't projection require dividing by the z coordinate? If -1<=z<=1, won't we get divide by 0?
- Ah, but it's really the w coordinate we divide by, and it's positive definite!
  - The original perspective transformation puts a vertex's z value in w
  - Since hither<=z<=yon for vertices that don't get clipped, w is positive definite (modulo sign convention for hither and yon)
- Hence, no worries on that front. All the z=0 vertices will get clipped before we divide out the homogeneous coordinate.

Clipping to a Cube

- Determine which parts of the scene lie within cube
- We will consider the 2D version: clip to rectangle
- This has its own uses (viewport clipping)
- Two approaches:
  - clip during scan conversion (rasterization) - check per pixel or end-point
  - clip before scan conversion
- We will cover
  - clip to rectangular viewport before scan conversion

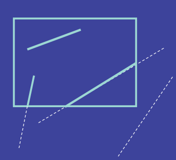
Line Clipping

- Modify endpoints of lines to lie in rectangle
- How to define "interior" of rectangle?
- Convenient definition: intersection of 4 half-planes
  - Nice way to decompose the problem
  - Generalizes easily to 3D (intersection of 6 half-planes)



Line Clipping

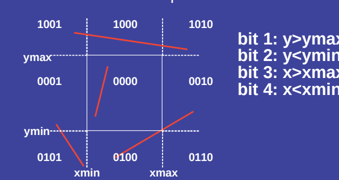
- Modify end points of lines to lie in rectangle
- Method:
  - Is end-point inside the clip region? - half-plane tests
  - If outside, calculate intersection between the line and the clipping rectangle and make this the new end point



- Both endpoints inside: trivial accept
- One inside: find intersection and clip
- Both outside: either clip or reject (tricky case)

Cohen-Sutherland Algorithm

- Uses *outcodes* to encode the half-plane tests results



bit 1: y>ymax
bit 2: y<ymin
bit 3: x>xmax
bit 4: x<xmin

- **Rules:**
  - **Trivial accept: outcode(end1) and outcode(end2) both *zero***
  - **Trivial reject: outcode(end1) & (bitwise and) outcode(end2) *nonzero***
    - **Else subdivide**

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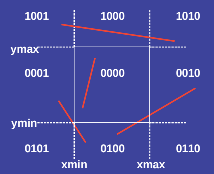

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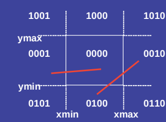
    - **Else subdivide**

Cohen-Sutherland Algorithm: Subdivision

- If neither trivial accept nor reject:
  - Pick an outside endpoint (with nonzero outcode)
  - Pick an edge that is crossed (nonzero bit of outcode)
  - Find line's intersection with that edge
  - Replace outside endpoint with intersection point
  - Repeat until trivial accept or reject



bit 1: y>ymax
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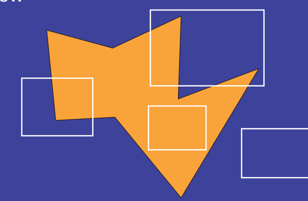
bit 3: x>xmax
bit 4: x<xmin

Polygon Clipping

Convert a polygon into one *or more* polygons that form the intersection of the original with the clip window



Sutherland-Hodgman Polygon Clipping Algorithm

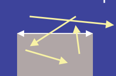
- Subproblem:
  - clip a polygon (vertex list) against a single clip plane
  - output the vertex list(s) for the resulting clipped polygon(s)
- Clip against all four planes
  - generalizes to 3D (6 planes)
  - generalizes to any convex clip polygon/polyhedron

Sutherland-Hodgman Polygon Clipping Algorithm (Cont.)

To clip vertex list against one half-plane:

- if first vertex is inside - output it
- loop through list testing inside/outside transition - output depends on transition:

> in-to-in:	**output vertex**
> out-to-out:	**no output**
> in-to-out:	**output intersection**
> out-to-in:	**output intersection and vertex**



Cleaning Up

- Post-processing is required when clipping creates multiple polygons
- As external vertices are clipped away, one is left with edges running along the boundary of the clip region.
- Sometimes those edges dead-end, hitting a vertex on the boundary and doubling back
  - Need to prune back those edges
- Sometimes the edges form infinitely-thin bridges between polygons
  - Need to cut those polygons apart