Name: ______________________ Email: ____________

Midterm

Computer Graphics I (15-462), Spring 1999

This midterm is closed book. In order to get full credit on the problems, you *must* show your work. If you need more space for your answers, you may use the backs of the pages so long as they are clearly marked. You should glance over the whole midterm first before attacking individual problems.

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1. Image Processing (2.5 Points)

Suppose that an image **A** was corrupted by "salt-and-pepper noise" (isolated black or white pixels), producing image **B** (see below). The changed pixels are shown in bold face. Your task is to remove this noise using a combination of filtering and point processing. Specifically, you will apply a linear shift-invariant filter **H** and a real-valued point processing function $f:\Re \Rightarrow \Re$, such that convolving **B** with **H** and then applying f to the result will reproduce the original image **A**.

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	1	1	1	1	1	0	0
0	1	1	1	1	1	0	0
0	1	1	1	1	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0

A

0	0	0	0	0	0	0	0
0	**1**	0	0	0	0	**1**	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	1	1	**0**	1	1	0	0
0	1	1	1	1	1	0	0
0	1	1	1	1	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0

B

(a) Apply the filter **H** to the non-shaded 4x3 section of **B** specified below (you need only to fill in the corresponding 4x3 region of **H⊗B**).

1	1	1
1	9	1
1	1	1

H

0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	1	1	0	1	1	0	0
0	1	1	1	1	1	0	0
0	1	1	1	1	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0

B

	9	1	0				
	2	3	3				
	3	12	13				
	13	15	8				

H⊗B

(c) Design a point processing function $f : \Re \Rightarrow \Re$ such that applying f to each pixel of **H⊗B** produces an image that is identical to **A**.

$$f(x) = \begin{cases} 1 & x = 8 \text{ or } x > 9 \\ 0 & otherwise \end{cases}$$

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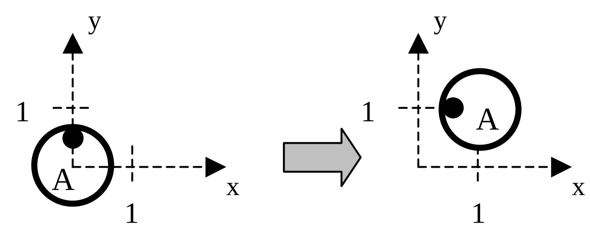
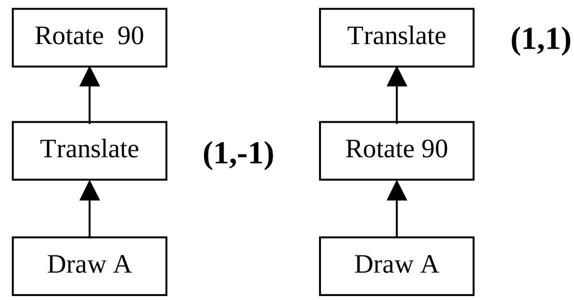
2. Hierarchies (3 Points)

Suppose we want to transform a circular object as shown below. Below are two possible hierarchies for accomplishing this transformation. Assume that rotations always occur about the origin of the world coordinate frame (shown in the figures).



Before Transformation **After Transformation**



Hierarchy 1 **Hierarchy 2**

(a) Write a 3x3 homogeneous matrix that performs the following translation:
x = x + a, y = y + b. Also give the 3x3 matrix that performs a (counter-clockwise) rotation of 90 degrees about the origin.

Translation:

$$T=\begin{bmatrix}1 & 0 & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}$$

Rotation:

$$R=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

(b) For each of the matrix transformations in part (a), prove whether it is linear or nonlinear.

They are both linear functions in homogeneous (3D) coordinates because they are matrix transformations. If you interpret these instead as 2D transformations, T(x,y) is nonlinear and R(x,y) is linear:

T is **nonlinear** since T ((x,y)+(p,q)) = T(x+p,y+q) = (x+p+a, y+q+b)
!= (x+p+2a, y+q+2b) = T(x,y) + T(p,q)
R is **linear** since R ((x,y)+(p,q)) = R(x+p,y+q) = (-y-q, x+p)
= (-y,x) + (-q,p) = R(x,y) + R(p,q)
R(kx,ky) = (-ky,-kx) = k (-y,-x) = kR(x,y)

(c) Write the x and y translation next to the Translate nodes in both hierarchies needed to accomplish the transformation shown above.

(d) Give the matrix at the top of the stack when A is drawn in both hierarchies The black point on A is at coordinates (0, 0.5) before the transformation. Use these matrices to compute the location of the point after the transformations.

For hierarchy 1:

$$\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$$

For hierarchy 2: both send (0, 0.5) to (0.5, 1)

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$$

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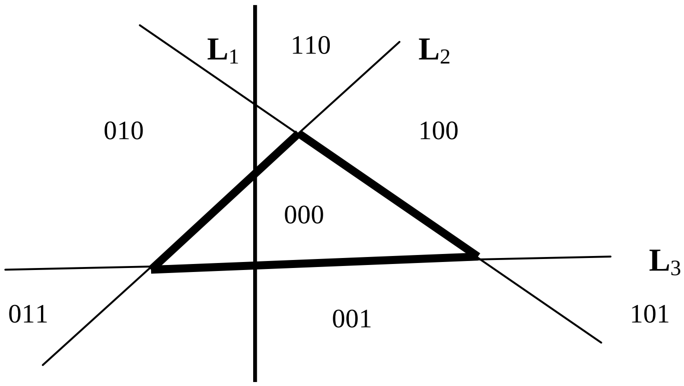
3. Clipping (2.5 Points)

The Cohen-Sutherland line clipping algorithm can be generalized to clip against any convex polygon, not only rectangles.

(a) Give the outcode bits and the half-plane tests that you would need to clip a line to a triangle. The triangle is bounded by three lines, shown below.



outcode = $b_1b_2b_3$
half-plane tests:
b_1 = 1 if point is above L1, else 0
b_2 = 1 if point is above L2, else 0
b_3 = 1 if point is below L3, else 0

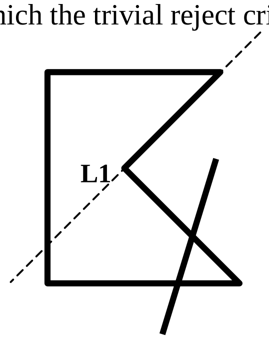
(b) What are the trivial accept and trivial reject checks in terms of the outcodes from part (a)? Make sure you define any symbols that you use (e.g., logical or bitwise operators).

Trivial accept: outcode1 == outcode2 == 0
Trivial reject: outcode1 & outcode2 > 0 (& is bitwise AND operator)

(c) Given a line segment, what is the maximum number of subdivide steps that the Cohen Sutherland algorithm will perform? Draw a line segment on the triangle above that requires this maximum number of subdivisions. Label the line endpoints, and specify the outcodes.

The line above requires 3 subdivide steps, assuming we clip against L1, then L2, then L3. The outcodes of the original line endpoints are 110 and 001.

(d) The technique used in parts (a-c) will in fact work for any convex polygon, by adding new half-plane test and outcodes for each edge. However this approach will not work for concave polygons, such as the one shown below. For this polygon, give an example of a line segment for which the trivial reject criterion will give an incorrect result. Why does it fail for this case?



The line segment shown below is partially inside the polygon, but will be trivially rejected since both endpoints are outside L1

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4. Splines (3 Points)

(a) Write the equation for a cubic spline segment in terms of a Basis matrix **B**, a control vector **p**.

$X(u) = u\,\mathbf{B}\,\mathbf{p} \qquad u = [u^3\;\; u^2\;\; u\;\; 1]$

(b) What are the components of the control vector $\mathbf{p}_H$ for a Hermite spline?

$\mathbf{p}_H = [P_1\;\; P_2\;\; T_1\;\; T_2]^T$

P_1, P_2 are points on the curve. T_1, T_2 are the tangents at P_1, P_2, respectively.

(c) Derive the Bezier basis matrix $\mathbf{B}_B$. You may use the following facts about *Bezier* splines: the Bezier control vector is $\mathbf{p}_B = [P_1\;\; P_2\;\; P_3\;\; P_4]^T$, where: $x(0) = P_1$, $x(1) = P_4$, $x'(0) = 3*(P_2 - P_1)$, $x'(1) = 3*(P_4 - P_3)$. The Hermite basis matrix is given by:

$$\mathbf{B}_H = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}\mathbf{B}_B\mathbf{p}_B &= \mathbf{B}_H\mathbf{M}_{B\to H}\mathbf{p}_B \\ &= \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \\ &= \boxed{\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}\end{aligned}$$

(d) Suppose we have two Bezier spline segments defined by control vectors $[P_1\;\; P_2\;\; P_3\;\; P_4]^T$ and $[Q_1\;\; Q_2\;\; Q_3\;\; Q_4]^T$, respectively. What conditions must we place on these parameters in order to obtain C1 continuity?

We need: $P_4 = Q_1$ and $P_4 - P_3 == Q_2 - Q_1$.

(e) Give the four basis functions for the Hermite spline. Express x(u) as a linear blend of these four functions (be sure to define the coefficients).

$U_1 = 2u^3 - 3u^2 + 1$
$U_2 = -2u^3 + 3u^2$
$U_3 = u^3 - 2u^2 + u$
$U_4 = u^3 - u^2$
$x(u) = P_1*U_1 + P_2*U_2 + T_1*U_3 + T_2*U_4$

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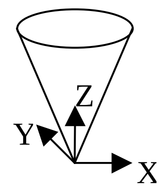
5. Modeling Edible Geometry (2 Points)

In this question you will be asked to create the equation for an ice-cream cone.

(a) Given an implicit F(X,Y,Z) function for an object, how do we determine whether a point (X,Y,Z) is inside, outside, or on the object?

$F < 0 \rightarrow$ inside
$F = 0 \rightarrow$ on
$F > 0 \rightarrow$ outside

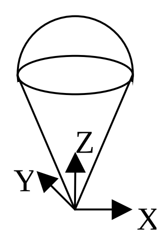


(b) Give an implicit equation F for a cone with apex at the origin and whose radius increases linearly in the positive Z direction (to infinity).

$$F(X,Y,Z) = X^2 + Y^2 - Z^2$$

(c) Suppose you want to "clip" the infinite cone F so that all points in the cone have Z values less than or equal to 1. Give an implicit equation for the "clipped" cone G *in terms of F*. You should be able to do parts (b-c) even if you didn't get part (a), just assume the existence of a cone function F.

$$G(X,Y,Z) = \max(\max(F(X,Y,Z), -Z), Z-1)$$



(d) Now suppose you want to place a sphere on top of the cylinder, yielding the ice-cream cone as shown at right. Give the implicit equations H for the sphere and I for the cone.

$$H(X,Y,Z) = X^2 + Y^2 + (Z-1)^2 - 1$$

$$I(X,Y,Z) = \min(\max(H(X,Y,Z), 1-Z), G(X,Y,Z))$$