DISCRETE CHANNEL APODIZATION METHOD
FOR THE ANALYSIS OF HIGH-ENERGY X-RAY DATA.

by

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ABSTRACT

The discrete channel apodization method to unfold detected x-ray energy spectra is derived for a detector with a Gaussian response function. Other processes required to determine the true source spectrum at the top of the atmosphere are described. A successful computer implementation, with sample results of the spectral determination process, including the discrete channel apodization method, is presented.

Thesis Supervisor: Dr. Anton Scheepmaker
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INTRODUCTION

X-ray astronomy is a new and rapidly developing branch of Astronomy. X-ray telescopes, which must be lifted to stratospheric heights or beyond because of the opacity of the atmosphere to x-rays, provide the means of observing celestial x-ray sources. The analysis of the x-ray observation data frequently culminates in the determination of the x-ray energy spectrum for the observed celestial source. The determination of the spectrum is essential to the theoretical study and modeling of the natural phenomena which produce the x-rays.

In order to determine the x-ray spectrum, the observation mechanism must be well understood, and all distorting effects must be fully accounted for. X-ray detectors have a response function which is convoluted (i.e., folded) with the energy spectrum in the detection process. The logical way to determine the spectrum impinging on the detector is to unfold the detected spectrum; unfortunately, this is no simple process. The general problem of reversing the effects of a response function is known as apodization. The theme of this dissertation is the development of an apodization algorithm which may be applied to the phoswich x-ray detector system used by the M.I.T. x-ray balloon group. The discrete channel apodization
method is such an algorithm, generally applicable to the class of detector systems which have Gaussian response functions and discrete channels.

The high energy x-ray telescope system cited and described in this investigation was flown to 130,000 feet on a stratospheric balloon by the x-ray balloon group of the M.I.T. Center for Space Research on June, 1974. The telescope system consists primarily of two detector banks of phoswich type x-ray detectors, and associated electronics.

The observed x-ray sources were the Crab Nebula and the Coma and Perseus clusters of galaxies. The sources were observed by the drift-scan method.
METHODS OF DETERMINING THE ENERGY SPECTRUM OF X-RAY SOURCES

The determination of the true spectrum of a celestial x-ray source involves several areas of investigation. For a balloon borne x-ray telescope these areas may be categorized as follows: The measured source spectrum must be determined; i.e., the source and background x-ray fluxes as functions of energy must be separated. Secondly, non-linear efficiency effects of the electronic pulse height analysis must be accounted for in order to determine the spectrum at the detector level. Thirdly, the convolution of the response function with the spectrum impinging on the detector, known as the folding process, must be considered. Finally, there is energy dependent atmospheric attenuation and some attenuating effects in the telescope system which must be taken into account.

The drift scan method of observing an x-ray source facilitates the separation of source and background fluxes. Since the diurnal motion of the Earth causes the celestial sphere to rotate at a constant rate with respect to Earth based coordinates, the telescope can be aimed just ahead of the x-ray source which will drift through the field of view. This observation method is known as a drift scan. If the aspect of the balloon borne system is known, the increase in x-ray count rates, pro-
**Figure 1.1 - 2 Crab Nebula Drift Scans**

**Top:** Fraction of detector area exposed to the Crab

**Bottom:** Detected count rate for 3° x 3° FWHM detector
portional to the increase in detector area exposed to the source, can be used to calculate the source intensity (Fig. 1.1). A straight line least squares fit to the x-ray count rate as a function of the detector area may be applied to different energy ranges. Extrapolating the lines, if necessary, to full exposure and to zero exposure yields the full source plus background, and the background count rates respectively (Ryckman, 1974). The background can be independently determined by extending the scan to include a section where no part of the detector is exposed to the source.

The other areas of investigation constitute the determination of the unperturbed source spectrum from the detected spectrum. If all of the attenuating and perturbing effects of the atmospheric absorption of the x-rays, folding in the phoswich detectors, and electronic pulse height analysis could be sequentially reversed, then the true spectrum at the top of the atmosphere could be easily determined. This is not the case because the spectrum impinging on the NaI crystal of the phoswich detectors is folded with the detector response function, a process which is not directly reversible.

Most of the X-ray Astronomy research groups use a repeated trial method to converge on a function which closely approximates the true source spectrum. This process requires the critical assumption that the yet unknown
source spectrum is best approximated by the theoretically predicted spectral functions. The three types of functions most often postulated by theoretical models of x-ray emitting mechanisms are:

\[ \frac{dN}{dE} = \beta E^{-\alpha} \quad \text{for the power law - synchrotron radiation} \]

\[ \frac{dN}{dE} = \frac{Ce^{-E/KT}}{E} \quad \text{for the exponential - bremsstrahlung} \]

\[ \frac{dN}{dE} = \frac{CE^2}{e^{-E/KT} - 1} \quad \text{for black body radiation} \]

In the repeated trial method one function is selected and the free parameters (e.g., \(\alpha\) and \(\beta\) for the power law spectrum) are estimated to generate a trial spectrum. The attenuating and folding effects are applied to the trial spectrum in order to compare it to the detected spectrum. The closeness of the match is usually evaluated under a \(\chi^2\) criterion. Next, the free parameters are altered and the spectrum generation process is reiterated in order to minimize \(\chi^2\). Often, after many iterations to find the best parameters to fit one function, the entire process is repeated for other theoretically feasible functions.

The repeated trial method has two clear disadvantages over the direct determination of the spectrum at the top
of the atmosphere by apodization of the response function and reversing the attenuating processes. Repeated trials are computationally inefficient, and the number of trials required to find an optimal fit to a given function explodes combinatorically as the number of free parameters increases. There are algorithms to generate reasonable guesses for the new values of the free parameters for subsequent trials given the results of previous trials, but these algorithms are computationally costly and dependent on the form of the spectral function being approximated. The other major disadvantage of the repeated trial method is that the choice of approximating function is constrained to simple, theoretically predicted, trial spectra. It is conceivable that more than one mechanism, including the possibility of some absorption mechanism, may be operating simultaneously to generate the observed spectrum.

An apodization method that can be implemented and used efficiently avoids the aforementioned difficulties; it avoids the problem of guessing trial spectra and the combinatorial inefficiency of repeated trials with a moderate number of free parameters. The apodization method for discrete channels will be discussed in detail after the different attenuation and perturbation effects are presented.
STUDY OF THE ABSORPTION, FOLDING AND EFFICIENCY EFFECTS OF THE X-RAY DETECTION PROCESS

This section will investigate each process in the sequence of events which a primary x-ray undergoes on its way to the detector, in the detection process, and in the subsequent pulse height analysis. A brief description of the x-ray telescope system for the June 1974 flight should put these processes in their proper perspective.

A) Description of the Detector.

The high-energy x-ray telescope system consists of two detector banks. Each bank consists of four phoswich type x-ray counters behind a slat collimator. For the June flight, one collimator had a 6° x 6° full width at half maximum (FWHM) field of view, and the other a 3° x 3° FWHM field of view. The phoswich detectors have a primary 3mm thick NaI crystal coupled to a 1.6" thick CsI secondary crystal. A plastic scintillation veto counter surrounds the detector banks to reject charged particles. There is an on-board pulse height analysis and telemetry system.5

B) Atmospheric Absorption.

The detector system was lifted above 99% of the Earth’s atmosphere by a stratospheric balloon, since the opacity of the atmosphere to x-rays prevents them from penetrating substantially deeper. Even in the tenuous stratosphere, x-rays are absorbed as a function of x-ray
energy and air thickness traversed. The probability that an x-ray will not be absorbed, called the transmission probability, is given by:

\[ P_{TR-AIR}(\mu_A, E) = e^{-[5.30\left(\frac{10}{E}\right)^{2.90} + .16] \mu_A} \]  

(2.1)

where \( E \) = x-ray energy in KeV and \( \mu_A \) = thickness of air traversed measured in \( \text{gm/cm}^2 \). Air thickness in the zenith direction is a tabulated function of altitude. To calculate the air thickness in the observation direction, the zenith air thickness is multiplied by the co-sine of the zenith angle of the collimator x-ray axis.

C) Styrofoam Absorption.

There is a protective styrofoam layer above the detectors which absorbs a small fraction of the x-ray flux as a function of energy. The transmission probability function is simpler than the one for air because of the macroscopically homogeneous nature of styrofoam. The transmission probability is:

\[ P_{IR-INS}(E) = e^{-\left(\frac{8.6}{E}\right)^{2.69}} \]  

(2.2)

D) Detection Efficiency.

Since the NaI crystal has finite thickness, there is a probability that some x-rays will penetrate the full thickness of the crystal without being detected. In this
case the probability that the x-ray is not lost (i.e., detected) is the absorption probability given by

\[ P_{AB-NaI}(\mu_{NaI}, E) = 1 - e^{-A(E)\mu_{NaI}(\frac{33}{E})^{2.65}} \]

where \( \mu_{NaI} = \) thickness of the NaI crystal \( = 1.17 \, \text{gm/cm}^2 \) for a 3mm NaI crystal, and

\[
A(E) = \begin{cases} 
5.8 & \text{if } E \leq 33 \, \text{KeV} \\
28.0 & \text{if } E > 33 \, \text{KeV}
\end{cases}
\]

The difference in values for \( A(E) \) occurs because of the K absorption edge of Iodine at 33 KeV.

E) Escape Probability.

An impinging x-ray whose energy is greater than 33 KeV may excite a K electron in Iodine, giving up 33 KeV's of energy. X-rays are re-emitted when an electron, usually an L state electron, falls into the empty K state. Since x-rays are re-emitted isotropically, there is a theoretical probability that some may escape through the front surface of the NaI crystal. The vast majority of x-rays are detected near the front surface of the 3mm crystal; hence, the probability of escape through the back surface is negligible. The average energy of the re-emitted
x-rays is 29.2 KeV. The escape probability as a function of x-ray energy is given in Figure 2.1.

(Fig 2.1) Theoretical probability that an x-ray impinging on a 3 mm NaI crystal will produce an Iodine K-flourescent escape x-ray.

F) Detector Response Function Folding.

In the process of detection in a phosphor which type detector, the impinging x-ray energy spectrum is folded with the response function of the detector to create the detected pulse height spectrum. Let \( S(E) \) and \( S'(E) \) represent the x-ray energy spectrum, and the pulse height spectrum respectively. \( S'(E) = S'(h(H)) \) where \( h(H) \) is the calibration function that assigns to each pulse height the corresponding energy it represents. The general folding process takes the form:

\[
S'(E) = \frac{d}{dE} \int S(E) G(E) dE
\]

where \( G(E) \) is the response function. Since
(2.5) \[ S(E) = \frac{dN(E)}{dE}, \quad S'(E) = \frac{dN'(E)}{dE}, \]
equation (2.5) can be expressed as:

(2.6) \[ N'(E) = \int E(E) dN(E) \]

where \( N(E) \) is the number of x-ray counts of energy \( E \) per \( \text{cm}^2 \text{sec KeV} \) (i.e., the x-ray flux as a function of energy). The response function for a phoswich type detector is a Gaussian, therefore the probability that an x-ray, whose impinging energy is \( E_0 \), is detected as having energy between \( E_1 \) and \( E_2 \) is given by:

(2.7) \[ P_d(E_1 < E < E_2) = \frac{1}{\sqrt{2\pi} \sigma(E_0)} \int_{E_1}^{E_2} e^{-\frac{(E-E_0)^2}{2\sigma(E_0)^2}} dE \]

where \( E_0 \) is the mean and \( \sigma(E_0) \) is the standard deviation (Figure 2.2).

(Fig 2.2) Folding effect. X-ray with impinging energy \( E_0 \) (6 function) has a probability of being detected between \( E_1 \) and \( E_2 \) = shaded area under Gaussian response function.
G) Pulse Shape Discriminator Efficiency.

The pulse shape discriminator (PSD) is an electronic system which selects pulses according to the rise time and pulse height. Its efficiency in admitting the appropriate pulses varied (June 1974) as a function of temperature and pulse height during the flight. Therefore, the efficiency had to be determined by an analysis of source calibrations taken during the flight (Scheepmaker, 1974). A source calibration consists of exposing the detectors to an x-ray source of known intensity for a few seconds. Figure 2.3 is the best determination of the efficiency function of the PSD for the June, 1974 flight.

(Fig 2.3) PSD efficiency as a function of time for each energy channel.
H) Pulse Height Analyzer.

The pulse height analyzer (PHA) bins the detected x-rays according to energy into discrete pulse height channels. Hence, the spectral data consists of x-ray count rates per pulse height channel. If some of the channel boundaries in the PHA are not well defined, $dN'(E)/dE$ is folded with the boundary resolution functions.

From the mathematical models presented in this section it is evident that all the processes, except for folding, are easily reversible. The following section will analyze an approximation method to reverse the folding process.
DISCRETE CHANNEL APODIZATION METHOD

Discrete channel apodization is the process of unfolding the effects of a response function on a finite number of discrete channels whose energy width is greater than the minimum resolution of the detection system. In this section a mathematical formulation of the folding process is examined and an algorithm for inverting the process is derived. This algorithm, implemented in a spectrum determination program, has proven successful in unfolding detected continuous spectra.

The uncertainty factor in the analysis of discrete channel spectral representations is that no direct determination can be made of the x-ray energy distribution within a single channel. Therefore, for an arbitrary impinging spectrum, a uniform distribution within each channel is assumed, giving the spectrum a characteristic step function. For a system with N energy channels (Fig. 3.1) let \((E_i, E_{i+1})\) represent the \(i^{th}\) channel, i.e., the energy range \(\{E \mid E_i < E < E_{i+1}\}\). \(\bar{E}_i\) is the energy at the center of the \(i^{th}\) channel; \(\bar{E}_i = (E_i + E_{i+1})/2\). The indices \(i\) and \(j\) are assumed to run from 1 to \(N\) (\(N=7\) for the June 1974 balloon flight system). \(S(\bar{E}_i)\) and \(S'(\bar{E}_i)\) represent, respectively, the values of the impinging and detected (unfolded and folded) spectra in
of the $i^{th}$ channel.

(Fig. 3.1) Folding of discrete channel x-ray spectrum. X-ray flux represented by shaded area in (a) is folded into shaded area in (b) by Gaussian response function.

The response function of the phoswich x-ray detector is a typical Gaussian distribution:

$$G(E_j, E) = e^{-\frac{(E - E_j)^2}{2\sigma(E_j)^2}}$$  \hspace{1cm} (3.1)$$

where

$$\int_{E=-\infty}^{\infty} G(E_j, E) dE = \sqrt{2\pi} \sigma(E_j).$$  \hspace{1cm} (3.2)$$
$E_j$ is the mean of the distribution and $\sigma(E_j)$ the standard deviation. $\sigma(E_j)$ is calculated empirically from calibration data taken from x-ray line emission sources before the balloon flight (Scheepmaker, 1974). The form of $\sigma(E_j)$ may be estimated to within the limits of experimental accuracy by:

$$\sigma(E_j) = A \cdot E_j + B \cdot \sqrt{E_j} + C$$

where $A$, $B$ and $C$ are constants.

Normalizing (3.1) gives a probability distribution function similar to equation (2.7):

$$P_d(E_i < E_j < E_{i+1}) = \frac{1}{\sqrt{2\pi} \sigma(E_j)} \int_{E_i}^{E_{i+1}} G(E_j, E) dE$$

This formulation allows the detected spectrum to be expressed as a step function of the impinging spectrum and the response function for each channel:

$$S'(E_i) = \sum_{j=1}^{N} S(E_j) \frac{1}{\sqrt{2\pi} \sigma(E_j)} \int_{E_i}^{E_{i+1}} G(E_j, E) dE$$

$$i = 1, 2, \ldots, N$$
The goal of this section is to derive \( S(\overline{E}_i) \) from \( S'(\overline{E}_i) \), hence reversing the folding process of the response function \( G(\overline{E}_j,E) \). The apodization method is usually an approximating process converging to a best approximation of the impinging spectrum. In the present apodization scheme the algorithm simply involves solving a set of \( N \) linear equations in \( N \) unknowns. Since \( G(\overline{E}_j,E) \) is directly computable, (3.5) yields a system of linear equations with unknowns \( S(\overline{E}_j) \) for a given set of \( S'(\overline{E}_i) \). Letting

\[
(3.6) \quad [a_{ij}]_{i=1}^{E_{i+1}} = \left[ \frac{1}{\sqrt{2\pi} \sigma(\overline{E}_j)} \int_{E_i}^{E_{i+1}} G(\overline{E}_j,E) dE \right]
\]

be the coefficient matrix, (3.5) takes the form:

\[
(3.7) \quad [a_{ij}] [S(\overline{E}_j)] = [S'(\overline{E}_i)].
\]

Since \([a_{ij}]\) is nonsingular and diagonally dominant, there exists a straightforward solution procedure for the \( S(\overline{E}_j) \).

The aforementioned apodization algorithm makes four simplifying assumptions to the general apodization problem. Two of the assumptions are imposed by the detector system:
the finite number of discrete channels and the Gaussian response function. The other assumptions are approximations to simplify the mathematical analysis. The uniform distribution within each channel is an assumption which does not introduce significant inaccuracies. The development of an apodization method which does not require this assumption will be discussed at the end of this section. The fourth assumption is implicit in (3.4) where the probability of detection is calculated to be a single Gaussian distribution centered about the mean $E_j$. This assumes that the uniform distribution inside the $j^{th}$ channel may be considered a delta function at the mean, an assumption which is good only if the channel width is small with respect to the standard deviation, 

$$\sigma(E_j) > > E_{j+1} - E_j.$$ 

For the June 1974 detector system the channels are wide with respect to the standard deviation. Given a uniform x-ray flux for the $i^{th}$ channel, the exact form of the convolution with the Gaussian response function for the $j^{th}$ channel is:

$$P_j(E) = \frac{E_{j+1}}{E_j} \int_{E_j}^{E_{j+1}} U(E_j', E_{j+1}) G(E, E') dE'$$

$$= \frac{1}{(E_{j+1} - E_j) \sqrt{2\pi} \sigma(E_j)} \int_{E_j}^{E_{j+1}} e^{-\frac{(E-E')^2}{2\sigma(E_j)^2}} \ dE'.$$
Therefore, the probability that an x-ray impinging on the $j$th channel is detected at the $i$th channel is the definite integral of (3.8) over the $i$th channel:

$$P_{d_{ij}}(E_i < E < E_{i+1}) = \int_{E_i}^{E_{i+1}} P_j(E) dE = \frac{1}{(E_{j+1} - E_j)^{1/2} \sigma(E_j)}$$

$$= \int_{E_i}^{E_j} \int_{E_j}^{E_{j+1}} e^{-\frac{(E''-E)^2}{2\sigma(E_j)^2}} dE' dE$$

(3.9)

The exact expression for the total flux detected in the $i$th channel is:

$$S'(E_i) = \sum_{j=1}^{N} \frac{S(E_j)}{(E_{j+1} - E_j)^{1/2} \sigma(E_j)}$$

$$= \int_{E_i}^{E_j} \int_{E_j}^{E_{j+1}} e^{-\frac{(E''-E)^2}{2\sigma(E_j)^2}} dE' dE$$

(3.10)

which is a system of linear equations. (3.10) may be represented in the same form as (3.7) and solved by inverting the coefficient matrix. In the limit as $(E_{j+1} - E_j) \to 0$ (3.9) is equivalent to (3.4); hence, as previously stated, the first apodization algorithm is valid for very narrow energy channels.
Solving the system of equations (3.10) proved to be impractical in terms of computer time required to generate the coefficient matrix. The numerical evaluation of

\[ D_{ij} = \int_{E_i}^{E_{i+1}} \int_{E_j}^{E_{j+1}} e^{-\left(E' E\right)/2\sigma(E_j)^2} \, dE' \, dE \]  

by iterative application of numerical integration techniques is somewhat costly, and there are \( N^2 \) (\( N^2 = 49 \) for the present system) \( D_{ij} \) to evaluate. A more efficient method of calculating the \( D_{ij} \) has recently been found, after an approximating system of equations equivalent to (3.10) was programmed and used in the data analysis.

(Fig 3.2) Folded channel function for a) exact form of Gaussian response convoluted with uniform distribution, b) close numerical approximation to (a), c) single Gaussian, considering x-ray flux to be δ function at center of channel.
Equation (3.9) may be approximated by minimizing the $L_1$ norm of the concatenation of two half Gaussians and a constant function (Fig. 3.2). The probability that an x-ray impinging in the $j^{th}$ channel is detected at the $i^{th}$ channel becomes:

$$P_{dj}(E_i < E < E_{i+1}) = \begin{cases} 
\frac{1}{\sqrt{2\pi} \sigma(E_j) + (E_{i+1} - E_i) P_{\alpha i}} \\
\frac{1}{\sqrt{2\pi} \sigma(E_j) + (E_{i+1} - E_i) P_{\alpha i}} \times \int_{E_i}^{E_{i+1}} e^{-\frac{(E-E_j)^2}{2\sigma(E_j)^2}} dE, & \text{if } j < i
\end{cases}$$

(3.12)

For $i = 1, 2, \ldots, N$

where $P_{\alpha j} = 1 - \sum_{j \neq i} P_{dj}(E_i < E < E_{i+1})$ is the probability that an x-ray impinging on the $j^{th}$ channel is detected in the $j^{th}$ channel. In this approximation the respective means of the Gaussians are at the channel boundaries, making the derivative of the approximating function well de-
fined and everywhere continuous.

The detected spectrum on the \(i\)th channel can be expressed as a sum of the impinging spectrum multiplied by the respective detection probabilities, as in the previous methods:

\[
S'(E_i) = \sum_{j=1}^{N} S(E_j) P_d(E < E < E_{i+1})
\]

(3.10)

\[i = 1, 2, \ldots, N\]

The system of linear equations may then be solved for \(S_{\alpha}(E_j) = S(E_j) P_{\alpha j}\), which yields \(S(E_j)\) directly. \(S_{\alpha}(E_j)\) is called the alpha spectrum; it is the fraction of the spectrum in each channel not carried to a different channel by the response function.

(Fig 3.3) Extrapolation of detected spectrum to estimate the fraction of the flux folded into the extreme channels from x rays whose impinging energy lies beyond the threshold of the extreme channel boundaries.
There is a non-negligible probability that x-rays impinging with slightly less energy than the lowest energy channel boundary, or with energy slightly higher than the highest channel boundary, will be folded into the respective extreme channel by the response function. Likewise, a fraction of the impinging flux at the extreme channels is never detected; it is carried out of detection range by the Gaussian response (Fig. 3.3). In order to minimize errors caused by ignoring this effect, the detected spectrum is extrapolated beyond the energy range of the extreme channels. A reasonable extrapolation will yield correction estimates for the x-ray fluxes carried across the extreme boundaries.

At the time of this writing the development of a more accurate, but also more complicated, method is being investigated. This method determines a smooth, best $L_2$-approximate to the detected spectrum. The process assigns to each $\bar{E}_i$ a tentative $\frac{d}{dE} S'(E) \bigg|_{E=E_i}$ and a somewhat more tentative $\frac{d^2}{dE^2} S'(E) \bigg|_{E=E_i}$ in order to give a reasonable approximation to the x-ray distribution within each channel. The approximating functions under investigation are interpolating cubic splines, where the number of splined sections is a function of the number of energy channels. The splining method will give a more accurate
determination of the impingent spectrum in systems incorporating a somewhat larger number of energy channels.
IMPLEMENTATION AND RESULTS OF THE APODIZATION ALGORITHM

SPECTRA is a Fortran IV computer program which lifts an x-ray spectrum from the detected count rates in each channel to the true x-ray source spectrum at the top of the atmosphere. An implementation of the discrete channel apodization algorithm lies at the heart of SPECTRA in a subprogram called GAUS.

SPECTRA applies the inverse process of each attenuation or perturbation, previously described, in reverse order from the detection process. Since the apodization algorithm inverts the folding process, a single program run will yield the best values of the source spectrum at a small computational cost. SPECTRA takes as input the set of detector system parameters and the detected count rates per energy channel for each detector bank. The detector parameters are: air thickness, NaI crystal thickness, detector area at full source exposure for each detector bank, time in the flight when the detected count rates were accumulated, PSD efficiency table with efficiency values for different x-ray energies and different times during the flight, and PHA channel boundaries in terms of energy. (Pulse height is directly proportional to energy.)

The count rate in each channel is converted to units of counts/cm²sec KeV. The PSD efficiency table is in-
terpolated to calculate the closest value for each energy channel at the given time in the flight, and the efficiency correction is applied to the spectrum. SPECTRA calls GAUS with the efficiency corrected spectrum to apply the discrete channel apodization algorithm. GAUS, in turn, calls several functions and subroutines, including RSIMQ to solve the linear matrix equation (3.13). (RSIMQ was developed by the Information Processing Center at M.I.T.). After the escape correction is applied to the unfolded spectrum, there follows a sequential application of the corrections for NaI crystal transmission, styrofoam layer absorption, and atmospheric absorption. The resultant step function is the best discrete determination of the continuous x-ray source spectrum at the top of the atmosphere. The spectral step function can be easily \( \chi^2 \) fitted to a theoretically predicted spectrum (e.g., power law).

SPECTRA tabulates the results at each step in lifting a detected spectrum to the top of the atmosphere (Figure 4.1). GAUS prints out the probability coefficient matrix,

\[
(4.1) \quad [a_{ij}] = P_{dj}(E_i < E < E_{i+1})/P_{ai}
\]

and the unfolding of the energy spectrum (Figure 4.2).

In order to test the accuracy of SPECTRA, a close approximation to the Crab Nebula power law spectrum, \(^8\)
UNITs = cts/(cp**2*sec*kev)

<table>
<thead>
<tr>
<th>ENERGY BIN</th>
<th>MEASURED SP</th>
<th>EFFEL CORR</th>
<th>EFFPS CORR</th>
<th>NAI UNFOLDING</th>
<th>ESCAPE CORR</th>
<th>NAI ABSORPTION</th>
<th>TRINS</th>
<th>TRAIR_FINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.50 - 34.50</td>
<td>1.391E-03</td>
<td>1.437E-03</td>
<td>2.404E-03</td>
<td>2.492E-03</td>
<td>2.416E-03</td>
<td>2.586E-03</td>
<td>1.875E-02</td>
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<tr>
<td>34.50 - 47.00</td>
<td>1.196E-03</td>
<td>1.272E-03</td>
<td>1.864E-03</td>
<td>1.871E-03</td>
<td>2.314E-03</td>
<td>2.351E-03</td>
<td>5.436E-13</td>
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<tr>
<td>47.00 - 64.00</td>
<td>9.524E-04</td>
<td>9.068E-04</td>
<td>1.254E-03</td>
<td>1.276E-03</td>
<td>1.436E-03</td>
<td>1.447E-03</td>
<td>2.797E-13</td>
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<tr>
<td>64.00 - 80.00</td>
<td>5.429E-04</td>
<td>5.776E-04</td>
<td>7.673E-04</td>
<td>7.481E-04</td>
<td>7.966E-04</td>
<td>8.094E-04</td>
<td>8.124E-14</td>
<td></td>
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<tr>
<td>80.00 - 115.00</td>
<td>2.748E-04</td>
<td>2.923E-04</td>
<td>3.674E-04</td>
<td>3.721E-04</td>
<td>3.837E-04</td>
<td>4.548E-04</td>
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</tr>
<tr>
<td>115.00 - 146.00</td>
<td>1.048E-04</td>
<td>1.115E-04</td>
<td>1.334E-04</td>
<td>1.215E-04</td>
<td>1.281E-04</td>
<td>2.226E-04</td>
<td>2.227E-14</td>
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</tr>
</tbody>
</table>

(Total Integrated Flux Between 20.00 and 146.00 KeV In Units of X-rays/(cp**2*sec) is 3.84525E-01)

(Fig 4.1) Tabulation of the results printed by SPECTRA at all stages in the determination of the discrete representation of the x-ray source spectrum at top of the Atmosphere. The results presented herein and graphed in Figure 4.3 are the recovery of a power law spectrum (α=-2.25, β=22) after modeled detector folding, attenuation and efficiency processes were applied.
UNFOLDING OF SPECTRUM NUMBER 2

AIR THICKNESS = 3.350 GRAMS/CM SC
RESPONSE FUNCTION = 0.630 * ENERGY
NAI THICKNESS = 1.170 GRAMS/CM SC
+ 0.420 * SQRT(ENERGY) + 0.230

PROBABILITY CONVOLUTION MATRIX

<table>
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<tr>
<th></th>
<th>1.000</th>
<th>0.257</th>
<th>0.300</th>
<th>0.000</th>
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<th>0.0</th>
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<td>0.189</td>
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<td>0.261</td>
<td>0.000</td>
<td>0.000</td>
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<td>1.000</td>
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<tr>
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<td>0.000</td>
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<td>0.262</td>
<td>0.000</td>
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<td>0.000</td>
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ENERGY BIN   FELDEC SPECT   ALPHA SPECT   UNFOLDED SPECT

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
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<tr>
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<td>27.50</td>
<td>2.2293E-03</td>
<td>1.8346E-02</td>
<td>2.4826E-03</td>
<td>2.7511E-03</td>
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<td>27.50</td>
<td>34.50</td>
<td>2.6318E-03</td>
<td>1.9066E-03</td>
<td>2.7511E-03</td>
<td>2.7511E-03</td>
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<td>34.50</td>
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<td>3.7214E-04</td>
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<td>1.215C-04</td>
<td>1.215C-04</td>
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</table>

(Fig 4.2) Discrete Channel Apodization computational results. GAUS prints the $[a_{ij}]$ coefficient probability matrix and the unfolded spectrum. Results tabulated in this run are from the recovery of the power law mentioned in the previous figure.
(Fig 4.3) Comparison of original power law spectrum, folded, attenuated spectrum, and recovered spectrum by SPECTRA.
$$\frac{dN}{dE} = 22E^{-2.25} \text{ (photons/cm}^2\text{sec KeV)}$$

was folded with the detector response function, and all other absorption and efficiency effects were applied (Laros, 1973). The resultant spectrum, generated to simulate a detected spectrum, was given to SPECTRA, which recovered the original spectrum at the top of the atmosphere with small computational errors (Fig. 4.1 and 4.3). The highest and lowest energy channels have somewhat larger errors than the central channels due to the extrapolation of the "detected" spectrum necessary to calculate the effects of the response function at the boundaries of the spectrum.

SPECTRA was applied to detected spectra from Crab Nebula drift scans. The result of one computer run, for the second Crab scan in the June, 1974 flight, is presented in Figures 4.4 and 4.5. The second and fourth energy channels are respectively too low and too high with respect to the Laros spectrum. This was found to be the case for other methods of determining the spectrum and for other Crab scans.

SPECTRA was tested for different detector system parameters. For instance, the unfolding of a test spectrum gave more accurate results when 12 energy channels were used to span the 20-146 KeV energy range instead of the 7 channels in the June flight.
UNITs = CTS/(CM**2*SEC*KEV)

<table>
<thead>
<tr>
<th>ENERGY BIN</th>
<th>MEASURED SP</th>
<th>EFFECl CORR</th>
<th>EFFPSD CORR</th>
<th>NAI UNFOLDING</th>
<th>ESCAPE CORR</th>
<th>NAI ABSORPTION</th>
<th>TRINS</th>
<th>TRAI(FINAL)</th>
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<td>2.4776E-C3</td>
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<td>7.9913E-C4</td>
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<tr>
<td>115.00</td>
<td>146.00</td>
<td>1.4857E-C4</td>
<td>1.5605E-C4</td>
<td>1.8916E-C4</td>
<td>1.7350E-04</td>
<td>1.8340E-04</td>
<td>3.1862E-04</td>
<td>3.1886E-04</td>
</tr>
</tbody>
</table>

TOTAL INTEGRATED FLUX BETWEEN 20.00 AND 146.00 KEV IN UNITS OF ARAYS/(CM**2*SEC) IS 3.88579E-01

(Fig 4.4) Determination of Crab Nebula x-ray energy spectrum. There is close agreement between unfolded spectrum and power law with parameters $\alpha = -2.25$, $\beta = 22$. The Integrated flux between 20 and 146 KeV for the power law is $3.814 \times 10^{-1}$ x-rays/cm$^2$ sec.
Graphical representation of the determined x-ray energy spectrum for the Crab Nebula. The power law, given as a reference, is the Laros spectrum.
The discrete channel apodization technique yields the closest obtainable values to the true x-ray source energy spectrum at the top of the atmosphere in a single, efficient pass. These values may be matched to a theoretically predicted spectrum (e.g., under a $\chi^2$ minimization criterion) if one wishes to study the x-ray production mechanism. The calculated values for the source spectrum are independent of any fitting performed after the determination of the spectrum, unlike the repeated trial method, where nothing is known about the source spectrum until a reasonable fit is found. The determination of the source spectral parameters may be greatly facilitated by graphing the source spectrum and performing preliminary visual fitting, or, at least, setting severe constraints on the type of function and values for the free parameters chosen for the $\chi^2$ fit.

The discrete channel apodization algorithm may be applied to any x-ray detector with discrete channels and a Gaussian response function. It may be possible to generalize the method to include other types of response functions, but the necessity for discrete channels lies at the very heart of the algorithm.

There are some limitations to the apodization method. Only continuous spectra can be unfolded; line spectra
cannot be resolved by discrete channel apodization. In order to get accurate results for continuous spectra, the detector system needs to have at least five energy channels. On the whole, the discrete channel apodization algorithm is an efficient and fruitful process applied to the determination of the x-ray source spectrum at the top of the atmosphere. The author hopes that this method may be used by x-ray astronomers to facilitate and improve their spectrum determinations.
APPENDIX

FULL LISTING OF SPECTRA AND ITS SUBROUTINES

SPECTRA is a Fortran program which should be compatible with most Fortran IV implementations. The data is read in from a set of cards of specified format, fully explained in the first page of the listing. SPECTRA can process several detected spectra measured during a single balloon flight in one program run.

The discrete channel apodization algorithm is implemented in the subroutine GAUS, which may be used independently of SPECTRA for detector systems that are not balloon borne (e.g. satellite detectors), but which have a Gaussian response function and discrete channels.
FORTAN IV G LEVEL 19 PAIN

DATE = 75126 21/55/37

C-------MAIN PROGRAM TO LIFT BALLOON DETECTED X-RAY SPECTRA TO THE TOP CF
C THE ATMOSPHERE, ASSUMES DISCRETE CHANNELS AND GAUSSIAN RESPONSE
C
C-----WRITTEN AT MIT JANUARY THRU APRIL 1975 BY JAIME G. CARBONELL
C
C-----REQUIRES 4 SUBROUTINES:
C 1) GAUS = APODIZATION ALGORITHM FOR DISCRETE CHANNELS AND GAUSSIAN RESP.
C 2) RSIPC SOLVES SYSTEM OF N LINEAR EQUATIONS IN N UNKNOWNS
C 3) ESCAPE INVERTS ESCAPE EFFECT OF IODINE ABSORPTION K EDGE IN AI X-TAL.
C 4) EPSI CALCULATES EFFICIENCY OF PSC BY INTERPOLATION ON TIME AND ENERGY.
C
C-----REQUIRES 9 FUNCTIONS
C GAUSP XTRPOL ESLC TRAIR IRSN AI TRINS SIGF EFFEL FINTRP
C
C-----FIRST SET OF DATA CARDS IS THE EFFICIENCY TABLE PRECEDEED BY 1 CARD
C WITH THE (12) NUMBER OF ENTRIES (= NUMBER OF TIMES IN FLIGHT = NUMBER
C OF CARES IN EFFICIENCY TABLE.) EACH CARD CONSISTS OF A TIME IN CCT SEC.
C FOLLOWED BY 7 EFFICIENCY VALUES, ONE FOR EACH CHANNEL.
C
C-----SECOND SET OF DATA CARDS CONSISTS OF DETECTOR PARAMETERS
C 1) NUMBER OF ENERGY CHANNELS (12)
C 2) THICKNESS OF AI CRYSTAL IN GM/CM**2 (F7.3)
C 3) A,B,C VALUES FOR SIGF RESPONSE FUNCTION (3F7.3)
C 4) NEINS+1 ENERGY BOUNDARY VALUES (12F7.3)
C 5) NUMBER OF SPECTRA TO BE PROCESSED (12)
C-----THE FOLLOWING CARDS APPLY TO EACH SPECTRUM - MUST BE REPEATED
C 6) DETECTOR AREA IN C**2 (F10.1)
C 7) CCT TIME (F10.1)
C 8) AIR THICKNESS IN CM/CM**2 (F7.3)
C 9) COUNT RATES PER CHANNEL (DETECTED) IN CTS/SEC (10F7.3)
C
0001 REAL EKEV(51),EKVAL(50),TSPECT(5C),SPECT(5C,5)
0002 REAL SIGNAL(90),SICPAR(3)
0003 EFF=1.0
0004 CALL EFSD
0005 REAC(5,10) NBINS
0006 10 FORMAT(12)
0007 READ(5,11) TNAI
0008 11 FORMAT(F7.3)
0009 REAC(5,12) (SICPAR(I),I=1,3)
0010 12 FORMAT(3F7.3)
0011 KK=NEINS+1
0012 REAC(5,13) (EKEV(I),I=1,KK)
0013 13 FORMAT(10F7.2)
0014 ISPEC=0
0015 14 READ(5,14) NSPEC
0016 100 IF(NSPEC.LE.ISPEC) GO TO 101
0017 103 IF(NSPEC.EQ.ISPEC+1) GO TO 100
0018 ISPEC=ISPEC+1
0019 101 READ(5,18) AREA

39
C    --- PRINT CUT INPUT PARAMETERS READ
C
C020 18 FORMAT(F10.1)     READ(5,18) CETIME
C021     FORMAT(F10.11)   READ(5,11) TAIR
C022     READ(5,15) (SPECT(I,1),I=1,NBINS)
C023     FORMAT(10F7.3)
C024     C------ PRINT CUT INPUT PARAMETERS READ
C025     WRITE(6,16) ISPEC,TAIR,THNAI,(SIGPAR(I),I=1,3)
C026     16 FORMAT(1','' UNFOLDING CF SPECTRUM NUMBR ','12,’/)
C027     CC 29 J=1,NBINS
C028     29 EKEV(AI)=EKEV(J)+EKEV(J+1)/2
C029     CC 30 J=1,NBINS
C030     K=J+1
C031     SPECT(J,1)=SPECT(J,1)/(AREA*(EKEV(K)-EKEV(J)))
C032     SPECT(J,2)=SPECT(J,2)/EFFEL(EKEVA(I,J))
C033     CALL EFFPS(N,EFFL,EKEVA(I,J),EKEVA)
C034     SPECT(J,3)=SPECT(J,2)/EFF
C035     TSPCT(J)=SPECT(J,3)
C036     30 SIGVAL(J)=SIGF(EKEVA(I,J)),SIGPAR(1),SIGPAR(2),SIGPAR(3)/2.0
C037     CALL GAS(NBINS,EKEV,EKEV,SIGVAL,TSPCT,SIGPAR)
C038     WRITE(6,17) (SIGVAL(I),I=1,NBINS)
C039     17 FORMAT(05SIGVAL= ',10F7.2)
C    ------ NAI CRYSTAL UNFOLDING JUST COMPLETED -- NOW DO FIRST ORDER
C    ESCAPE CORRECTION AND BRING SPECTRUM TO TOP OF ATMOSPHERE
C040     TFLUX=0
C041     CC 41 J=1,NBINS
C042     41 SPECT(J,4)=TSPCT(J)
C043     CALL ESCAPE(EKEV,TSPCT,NBINS)
C044     CC 31 J=1,NBINS
C045     K=J+1
C046     SPECT(J,5)=TSPCT(J)
C047     SPECT(J,5)=SPECT(J,5)/(1.0-THNAI(EKEVA(J)))
C048     SPECT(J,7)=SPECT(J,6)/THNAI(EKEVA(J))
C049     SPECT(J,8)=SPECT(J,7)/TAIR(EKEVA(J))
C050     TFLUX=TFLUX+SPECT(J,8)*(EKEV(K)-EKEV(J))
C051     31 CC'TINUE
C    ------ PRINT SPECTRUM AT EACH STEP IN PROCESSING AND UNFOLDING
C052     WRITE(6,37)
C053     37 FORMAT(0 UNITS = CTS/(ICM**2*SEC*KEV)**)
C054     WRITE(6,32)
C055     32 FORMAT(00 ENERGY BIN MEASURED SP EFFEL CORR EFFPS(COR
C056     1R NAI UNFOLDING ESCAPE CORR NAI ABSORPTION TRINS TRAIR(F
C057     ZINAL)**)
C058     CC 34 J=1,NBINS
C059     K=J+1
C060     WRITE(6,38)
C061     38 FORMAT(0 I'' --- ''I'')
C062     2---''I'')
C063     WRITE(6,38)
C064     WRITE(6,39) EKEV(I),EKEV(I),TFLUX
C065     WRITE(6,35) ISPEC
C066     39 FORMAT(00,/' TOTAL INTEGRATED FLUX BETWEEN ''F7.2'' AND''F7.2''
1'KEV IN UNITS OF XRAYS/(CM**2*SEC) IS ,1PE14.5//)

C067 35 FORMAT('O',///' END OF PROCESSING FOR SPECTRUM ',12///)
C068 6C TO 100
C069 101 CONTINUE
C070 WRITE(6,36)
C071 36 FORMAT('O',///// COMPLETION OF UNFOLDING AND LIFTING OF SPECTRA

C072 1 '/////
C073 STOP
C073 FNC
C-----GAUSSIAN UNFOLDING CF XRAY SPECTRUM.

REAL XKEV(51),A1(50,50),LMDA(51),XSPEC1(50),XSPEC2(50)

C-----LIST EXTRAPOLATION CORRECTION
C-----MORE PRECISION FOR SMALLER E BINING)

XHIGH=XKEV(K)
XLIM=XKEV(J)

IF (I-J) 41,42,43

41 XPEA=XKEV(L)

42 XFEAN=XKEV(I)

15 XLIM=LMDA(I)

43 A1(I,J)=1.0

CONTINUE

C;AT ('OSZERCSALP-/, SCURR='3E14.4)

XSPEC1(NBINS)=XSPEC1(NBINS)-SCURR

C-----NOW SOLVE MATRIX OF CCNVCLUTION COEFF. FOR S ALPHA.

WRITE(6,30)

30 FORMAT('0','',PROBABILITY CCNVOLUTION MATRIX')/)

WRITE(6,31) (A1(I,J),J=1,NBINS)

WRITE(6,31) (A1(I,J),J=1,NBINS)

WRITE(6,31) (A1(I,J),J=1,NBINS)

WRITE(6,31) (A1(I,J),J=1,NBINS)
CALL RSIMQ(50,NBINS,A1,XSPEC2,0)
CONTINUE
DO 17 I=1,NBINS
K=I+1
XSPEC3(I)=XSPEC2(I)*(1.0+SQPI*LMCA(I)/(XKEV(K)-XKEV(I)))
CONTINUE
WRITE(6,18)
FORMAT('ENERGY BIN FOLDED SPECT ALPHA SPECT UNFOLDED SP')
UC 19 I=1,NBINS
K=I+1
WRITE(6,20)XKEV(I),XKEV(K),XSPEC1(I),XSPEC2(I),XSPEC3(I)
FORMAT(2F7.2,1P3E14.4/)
WRITE(6,21)
FORMAT('END OF GAUSSIAN UNFOLDING //\\')
DO 22 I=1,NBINS
XSPEC1(I)=XSPEC3(I)
RETURN
ENC

43
SUBROUTINE RSIMQ(NCIM, NORCER, COEFF, RHS, IERR)

REAL CCEFF, RHS, BIGC, SAVE, TOL, ABS
INTEGER NORDER, NCIM, I, J, K, IMAX, JP1, JJ, NMI

DIMENSION COEFF(NDIM, NCRCE), RHS(NCRCE)

C CHECK FOR ARGUMENT ERRORS.
IF (NDIM .GE. NORDER .AND. NORDER .GT. C) GC 10 10
IERR = 2
WRITE (6, 1001) CIM, NORDER
RETURN

10 TOL = 0.0E0
IERR = 0

CC FORWARD ELIMINATION, WITH PARTIAL PIVOTING.
CC 70 J = 1, NORDER

C CHOOSE LARGEST ELEMENT REMAINING IN THIS COLUMN.
BIGC = 0.0E0
CC 2) I = J, NORDER
IF (ABS(BIGC) GE. ABS(COEFF(I, J))) GO TO 20
BIGC = COEFF(I, J)
IMAX = I

20 CONTINUE

C IF ALL ELEMENTS HAVE MAGNITUDES LESS THAN OR EQUAL TO TOL, THEN
C MATRIX IS SINGULAR.
IF (ABS(BIGC) GT. TOL) GO TO 30
IERR = 1
WRITE (6, 1002)
RETURN

C INTERCHANGE ROWS IF NECESSARY, AND DIVIDE NEW CURRENT ROW BY
C PIVOT ELEMENT.
30 CC 4) K = J, NORDER
SAVE = COEFF(IMAX, K)
(COEFF(IMAX, K) = COEFF(J, K)
CCOEFF(J, K) = SAVE / BIGC

40 CONTINUE

C CC THE SAVE FOR THE RIGHT-HAND SIDE.
SAVE = RHS(IMAX)
RHS(IMAX) = RHS(J)
RHS(J) = SAVE / BIGC

C SUBTRACT MULTIPLES OF THIS ROW FROM ANY REMAINING ROWS TO MAKE
C LEADING COEFFICIENTS VANISH.
IF (J .GE. NORDER) GO TO 80

JP1 = J + 1
CC 60 I = JP1, NORDER
SAVE = COEFF(I, J)
CC 50 K = JP1, NORDER
CCOEFF(I, K) = COEFF(I, K) - SAVE * COEFF(J, K)
RHS(I) = RHS(I) - SAVE * RHS(J)

60 CONTINUE
DATE = 75126
21/55/37

C038    70 CONTINUE
C
C NEW FIND ELEMENTS OF SOLUTION VECTOR IN REVERSE ORDER BY DIRECT
C SUBSTITUTION.
C039    80 NP1 = NCORDER - 1
C040      NP1 = NCORDER + 1
C041    DC 100 JJ = 1, NM1
C042        J = NCORDER - JJ
C043       JP1 = J + 1
C044    DC 90 KK = 1, JJ
C045          K = NP1 - KK
C046    RHS(J) = RHS(J) - COEFF(J, K) * RHS(K)
C047   90 CONTINUE
C048   100 CONTINUE
C
C049    RETURN
C
C050   1001 FORMAT(23H RSIMQ ARGUMENT ERROR, 2111)
C051    1002 FORMAT(32H RSIMQ EQUATIONS ARE SINGULAR.)
C052    CND
SUBROUTINE ESCAPE(XKEV, TSPECT, NBINS)
C----- ESCAPE CORRECTION ON ENTIRE EXPANDED SPECTRUM
C RETURNS NEW SPECTRUM BY Clobbering TSPECT
C
REAL XKEV(N1), TSPECT(N0), FSPECT(NC)
INTEGER IKEV(N1)

DO 10 I = 1, KK
DO 20 I = 1, LD

IF (N.LT.IKEV(ICOUNT)) ICOUNT = ICOUNT - 1
I = I + IKEV(1)

IF (N.GE.IKEV(1)) FSPECT(N29) = FSPECT(N29) / (1 + ESC * FSPECT(N) / FSPECT(N29))

WRITE(6, 101) ICOUNT, IKEV(ICOUNT), TSPECT(ICOUNT), N, FSPECT(N)

CONTINUE

WRITE(6, 101) ICOUNT, IKEV(ICOUNT), TSPECT(ICOUNT), N, FSPECT(N)
CONTINUE

RETURN
DEBUX SUBCK

END
C-----EFFICIENCY OF PULSE HEICFT DISCRIPINATOR
C
C THIS ENTRY READS THE EFFICIENCY TABLE
C
C DIMENSION PSC(50,7),CDTX(50),CDTY(50),EX(7),EY(7)
C
C004 10 FORMAT(12)
C005 20 CC 20 1=1,NTIMES
C006 20 READ(5,30) CCTX(I),(PSC(I,J),J=1,7)
C007 30 FORMAT(F10.0,F10.3)
C008 RETURN
C009 -- ENTRY EFFPSD(EFF,T,E,EX)
C
C----INTERPOLATES EFFICIENCY TABLE FOR A GIVEN TIME AND ENERGY
C
C WRITE(6,81) EFF,T,E,(EX(I),I=1,7)
C010 81 FORMAT(10F10.2)
C011 40 CC 40 J=1,7
C012 40 CC 40 1=1,NTIMES
C013 40 CDTY(I)=PSC(I,J)
C014 50 EY(I)=FINTRP(T,NTIMES,CCTX,CDTY)
C015 C EFF=FINTRP(E,7,EX,EY)
C
C WRITE(6,82) EFF,(EY(I),I=1,7),(CDTX(I),CDTY(I),I=1,NTIMES)
C016 82 FORMAT(10F10.2/10F10.2/10F10.2,/
C        10F10.2,10F10.2)
C017 RETURN
C018 ENC
FUNCTION GAUSP(X1,X2,XMEAN,SIGMA)
C------COMPUTES THE INTEGRAL UNDER A NORMALIZED GAUSSIAN DISTRIBUTION
C APPEAR. ACCURATE TO SIX DIGITS. X1=LOWER AND X2=UPPER INTEGRATION
C LIMITS IN TERMS OF X COORDINATE, NOT SIGMA UNITS
 REAL BE(5)/.31938153, -.35656378, 1.7814779, -1.8212560, 1.3302744/
 REAL XX(2),RR(2),PP(2),EE(2)
 XX(1)=(X1-XMEAN)/SIGMA
 XX(2)=(X2-XMEAN)/SIGMA
 CC 30 I=1,2
 PP(I)=1./(1.+2316419*ABS(XX(I)))
 CC 30 I=1,2
 RR(1)=C.0
 RR(2)=C.0
 CC 30 J=1,5
 T=T*PP(I)
 RR(I)=RR(I)+T*BB(J)
 IF(XX(I).LT.0.0) RR(I)=1.0-RR(I)
 EE(I)=0.0
 IF(XX(I).LT.10.0.AND.XX(I).GT.-10.0) EE(I)=EXP(-XX(I)**2)/2.5665280
 CC WRITING 20 GAUSP,X1,X2,XMEAN,SIGMA
 CC 20 FORX1=10E12.3
 CC 21 RETURN
 CC 20 =NC

FUNCTION ESCR(E)
C------APPROXIMATE ESCAPE PROBABILITY FOR X-RAYS FROM IODINE K FLUORESCENCE
C VIA FRONT FACE OF 3 MM NAI CRYSTAL
 IF(E.LE.33.) GO TO 1
 IF(E.GT.33..AND.E.LE.50.) GO TO 2
 IF(E.GT.50..AND.E.LE.80.) GO TO 3
 IF(E.GT.80.) ESCR=0.55
 CC 1 CHILD ESCR=0.
 CC 2 ESCR=.27-((E-33.)/25.)*.21
 CC 3 ESCR=.14-((E-50.)/25.)*.C8
 CC 10 RETURN
 CC 2 RETURN
 CC 3 RETURN
 CC 10 =NC
FUNCTION XTRPOL(X1,X2,X3,X4,X5,X6)
C-----APPROXIMATE EXTRAPOLATION FOR DETECTED SPECTRUM FUNCTION.
SLOPE1=(S2-S1)/(X2-X1)
SLOPE2=(S3-S2)/(X3-X2)
SLOPE3=0
IF(SLOPE1.EQ.0) GO TO 5
IF (SLOPE1.GT.0 .AND. SLOPE2.GT.0) GO TO 1
IF (SLOPE1.LT.0 .AND. SLOPE2.LT.0) GO TO 1
SLPE3=SLOPE2-SLOPE1
GO TO 5
SLPE3=SLOPE2/SLOPE1
IF (ABS(SLOPE1).GT.4E14) SLOPE3=SLOPE2
XTRPOL=SLOPE3*(X4-X3)
WRITE(6,10) SLOPE1,SLOPE2,SLOPE3,XTRPOL
10 FORMAT('SLPE1,SLOPE2,SLOPE3,XTRPOL= ',4E14.6)
RETURN
ENC

FUNCTION FINTRP(X,NVALS,XVALS,YVALS)
C-----INTERPOLATES TABULATED FUNCTION, XVALS AND F(XVALS) = YVALS
C TO CALCULATE F(X). LINEAR EXTRAPOLATION IF X OUTSIDE TABULATED RANGE.
DIMENSION XVALS(50),YVALS(50)
IF(X.LE.XVALS(1)) GC TO 20
IF(X.GE.XVALS(NVALS)) GC TO 30
N=1
M=N+1
IF(N.GT.NVALS) GC TO 30
GO TO 10
FINTRP=YVALS(N1)*(X-XVALS(N1))*(YVALS(N)-YVALS(N1))/(XVALS(N)-XVALS(N1))
GO TO 40
C12 20 FINTRP=YVALS(1)-(XVALS(1)-X)*(YVALS(2)-YVALS(1))/(XVALS(2)-XVALS(1))
GO TO 40
C13 30 FINTRP=YVALS(NVALS)*(X-XVALS(NVALS))*(YVALS(NVALS)-YVALS(NVALS-1))/
; /XVALS(NVALS)-XVALS(NVALS-1))
GO TO 40
CONTINUE
WRITE(6,81) FINTRP,X,NVALS
81 FORMAT(*1,F10.2)
WRITE (6,82) XVALS(1),YVALS(1),I=1,NVALS)
82 FORMAT('XVALS,YVALS= *,1CF10.2)
RETURN
ENC
DEBUG SUBCPK
ENC
FUNCTION TRAIR(TAIR,E)
C-----PROBABILITY OF TRANSMISSION THROUGH ATMOSPHERE
TRAIR=EXP(-5.30*(1C./E)**2.50+.16)*THAIR
RETURN
END

FUNCTION TRINS(EE)
C-----PROBABILITY OF TRANSMISSION THRU STYROFOAM LAYER (JUNE, 1974)
TRINS=EXP(-1.0*(8.6/EE**2.65))
RETURN
END

FUNCTION TRNAI(TNAI,E)
C-----PROBABILITY OF TRANSMISSION THROUGH NAI CRYSTAL
IF(E<33.0) 1,1,2
1 C=5.8
GO TO 3
2 A=20.0
3 TRNAI=EXP(-TNAI*(A*(33.0/E)**2.65))
RETURN
END

FUNCTION SIGF(E,A,E,C)
C-----GIVES SIGMA FOR GAUSSIAN RESPONSE FUNCTION -- EMPIRICAL FORMULA
SIGF = (A+E + B*SQR(E) + C)
RETURN
END

FUNCTION EFFEL(E)
C-----EFFICIENCY OF ELECTRONICS ON SCALE 1 TO 0
EFFEL=C.94
RETURN
END
BIBLIOGRAPHY


5) Proposal to the National Science Foundation for x-ray balloon observations. Center for Space Research, M.I.T., 1974.

