

# Distributed Learning in Expert Referral Networks

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**Abstract.** Human experts or autonomous agents in a referral network must decide whether to accept a task or refer to a more appropriate expert, and if so to whom. In order for the referral network to improve over time, the experts must learn to estimate the topical expertise of other experts. This paper extends concepts from Reinforcement Learning and Active Learning to referral networks, to learn how to refer at the network level, based on the proposed distributed interval estimation learning (DIEL) algorithm. Diverse Monte Carlo simulations reveal that DIEL improves network performance significantly over both greedy and Q-learning baselines [3], approaching optimal given enough data.

## 1 INTRODUCTION

Consider a network of experts with differing expertise, where any expert may receive a problem (aka a task or a query) and must decide whether to work on it or to refer the problem, and if so to which other expert. How can a network, or its individual experts, learn how to refer tasks effectively?

This paper proposes a new Distributed Active Learning approach in referral networks. Our referral model assumes an initial sparse topology of a referral graph where each expert knows a handful of colleagues so that  $E \sim O(V)$  ( $E$  and  $V$  denote the number of edges and vertices in the network, respectively). Learning consists of each expert improving its estimates of the ability of colleagues to solve different classes of problems. We address learning to refer comparing overall network performance contrasting an exploitation-centered (greedy optimization) with a balanced exploration-exploitation trade-off (amortized optimization), showing that the former outperforms at first, and the latter overtakes as the network learns more effectively over time.

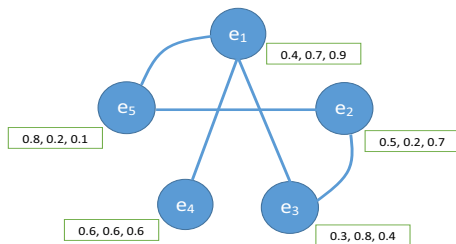


Figure 1. A referral network with five experts.

To illustrate the problem, consider the extremely simple graph, representing a five-expert network, shown in Figure 1. The nodes of the graph are the experts, and the edges indicate that the experts ‘know’ each other, that is, they can send or receive referrals and communicate results. We assume 3 different topics (subdomains) can be

distinguished – call them  $t_1$ ,  $t_2$ , and  $t_3$  – and the figures in brackets indicate an expert’s expertise in each of these.

In the example, with a query belonging to  $t_2$ , if there was no referral, the client may consult first  $e_2$  and then possibly  $e_5$ , leading to a probability of getting the correct answer of  $0.2 + (1 - 0.2) \times 0.2 = 0.36$ . With referrals, an expert handles a problem she knows how to answer, and otherwise if she had knowledge of all the other experts’ expertise she could ask  $e_2$  who would refer to  $e_3$  for the best skill in  $t_2$ , leading to a solution probability of  $0.2 + (1 - 0.2) \times 0.8 = 0.84$ .

Our referral mechanism consists of the following steps: 1) A user issues an *initial query* to an *initial expert*. 2) If the initial expert is able to solve it, she returns the solution; 3) if not, she selects a *referred expert* within her subnetwork, who solves or refers in turn. *Learning-to-refer* means improving the estimate of who is most likely to solve the problem.

Our primary contribution is the distributed learning-to-refer framework and a distributed learning algorithm. To learn referrals, we borrowed ideas from Reinforcement Learning [5] and Active Learning [1, 7], up till now rarely applied to referral networks, and compared performance under various conditions and learning algorithms. We extended Interval Estimation learning [4, 6] to DIEL, a distributed interval learning methods, and compare DIEL with well-known algorithms:  $\epsilon$ -greedy Q-learning and double Q-learning, and with an upper-bound where every expert has access to an oracle that knows the true topic-mean and thus can refer optimally.

## 2 REFERRAL NETWORK

We first present our notation and assumptions.

- A set of  $m$  instances  $(q_1, q_2, \dots, q_m)$  belonging to  $n$  topics ( $topic_1, topic_2, \dots, topic_n$ ) are to be addressed by  $k$  experts
- The experts are connected through a *referral network*, a graph  $(V, E)$  where each vertex  $v_i$  denotes an expert  $e_i$  and each edge  $\langle v_i, v_j \rangle$  indicates a *referral link*. The probability of an edge is:  $P(\text{ReferralLink}(v_i, v_j)) = \tau + c \text{Sim}(e_i, e_j)$ . We used cosine similarity of topic means for  $\text{Sim}$ . The *subnetwork* of each expert  $e_i$  is the set of all experts  $\{e_j\}$  that  $e_i$  can refer to.
- The *expertise* for an expert-instance pair,  $\langle e_i, q_j \rangle$ , is the probability that she can successfully solve the problem, i.e.,  $\text{Expertise}(e_i, q_j) = P(\text{solve}(e_i, q_j))$ .

**Topic-wise distributional assumption:** We take the expertise distribution for a given topic  $t$  to be a mixture of two truncated Gaussians (with parameters  $\lambda = \{w_i^t, \mu_i^t, \sigma_i^t\} i = 1, 2$ ). One of them  $(\mathcal{N}(\mu_2^t, \sigma_2^t))$  has higher mean ( $\mu_2^t > \mu_1^t$ ), smaller variance ( $\sigma_2^t < \sigma_1^t$ ) and lower mixture weight ( $w_2^t \ll w_1^t$ ).

**Instance-wise distributional assumption:** We model the expertise of a given expert on instances under a topic by a truncated Gaussian distribution with small variance. i.e.,  $\text{Expertise}(e_i, q_j) \sim \mathcal{N}(\mu_{topic_p, e_i}, \sigma_{topic_p, e_i})$ ,  $\forall q_j \in topic_p, \forall p, i : \sigma_{topic_p, e_i} \leq 0.2$ .

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### 3 DISTRIBUTED REFERRAL LEARNING

Action selection using Interval Estimation Learning [2] (IEL) estimates  $a$  the upper confidence interval for the mean reward by

$$UI(a) = m(a) + t_{\frac{\alpha}{2}}^{(n-1)} \frac{s(a)}{\sqrt{n}} \quad (1)$$

where  $m(a)$  is the mean observed reward for  $a$ ,  $s(a)$  is the sample standard deviation of the reward,  $n$  is the number of observed samples from  $a$ , and  $t_{\frac{\alpha}{2}}^{(n-1)}$  is the critical value for the Student's  $t$ -distribution ( $n - 1$  degrees of freedom,  $\frac{\alpha}{2}$  confidence level) (in our case, the action is the selection of a referred expert among possible choices in the subnetwork). Next, IEL selects the action with the highest upper confidence interval.

Algorithm 1 performs a single referral (per-task query budget  $Q = 2$ ). The function  $expR_h(e', topic)$  estimates  $e'$ 's topical expertise. DIEL (Distributed Interval Estimation Learning), and DMT (Distributed Mean-Tracking) differ in  $h$ , DIEL estimating reward by equation (1) and DMT by using the sample-mean.

**Input:** A set of  $k$  experts  $e_1, e_2, \dots, e_k$ . A set of  $n$  topics  $topic_1, topic_2, \dots, topic_n$ . A  $k \times k$  referral network.

Initialize rewards.

**for**  $iter \leftarrow 1$  **to**  $maxIter$  **do**

    Assign instance  $q$  to an initial expert  $e$  randomly

**if**  $e$  fails to solve  $q$  **then**

$topic \leftarrow getTopic(q)$

$expectedReward \leftarrow 0$

$bestExpert \leftarrow 0$

**for** each expert  $e'$  in the subnetwork of  $e$  **do**

**if**  $expR_h(e', topic) \geq expectedReward$  **then**

$bestExpert \leftarrow e'$

$expectedReward \leftarrow expR_h(e', topic)$

**end**

**end**

**end**

$referredExpert \leftarrow bestExpert$

**if**  $referredExpert$  solves  $q$  **then**

$update(reward(e, topic, referredExpert), 1)$

**else**

$update(reward(e, topic, referredExpert), 0)$

**end**

**end**

**Algorithm 1:** DISTRIBUTED REFERRAL LEARNING,  $Q = 2$

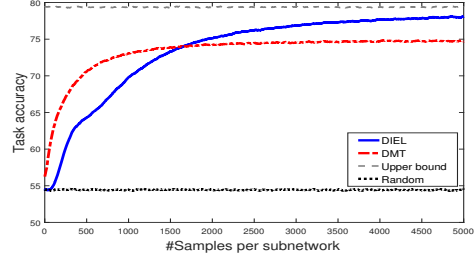
### 4 EXPERIMENTAL SETUP AND RESULTS

Parameter	Description	Distribution
$\tau$	$P(ReferralLink(v_i, v_j))$	Uniform(0.01, 0.1)
$c$	$= \tau + c Sim(e_i, e_j)$ .	Uniform(0.1, 0.2)
$\mu_1$	Truncated mixture of two Gaussians for topics	Uniform(0, $b$ )
$\mu_2$		Uniform( $b$ , 1)
$\sigma_1$		$b \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$
$\sigma_2$		Uniform(0.2, 0.4)
$w_2$		Uniform(0.05, 0.15)
		$\mathcal{N}(0.03, 0.01), w_2 \geq 0$

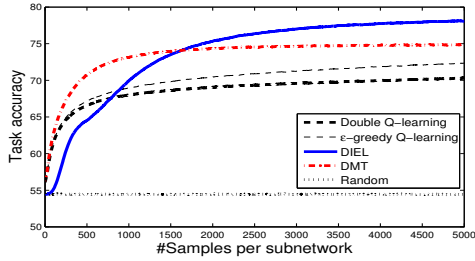
**Table 1.** Parameters for our synthetic data set.

We evaluated the performance of our learning algorithms on on 1000 scenarios, each with 100 experts, 10 topics and a referral network, whose parameters are in Table 1. Our measure of performance

is the overall task accuracy of the multi-expert network. As an upper bound we considered the performance achieved by a network where every expert has access to an oracle that knows the true topic-mean (i.e.,  $mean(Expertise(e_i, q) : q \in topic_p) \forall i, p$ ) of every expert-topic pair. Our baseline is a strategy where a task can be queried to maximum two randomly chosen experts.



(a) Comparison with baselines and upper bound.



(b) Comparison with Q-learning and DQ-learning.

**Figure 2.** Performance comparison of referral algorithms.

On every simulation, DIEL and DMT outperformed the baseline by a substantial margin. Over time DIEL clearly outperforms DMT and approaches the topical (oracle) upper bound. Figure 2(b) shows that both DIEL and DMT outperform Double Q-learning and  $\epsilon$ -greedy Q-learning (optimized using a rough parameter sweep) for the duration of the experiment, although unlike DMT, Double Q-learning and  $\epsilon$ -greedy Q-learning continue to improve.

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