Simultaneous Localization and Map Building: A Global Topological Model with Local Metric Maps

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Abstract

In this paper an approach combining the metric and topological paradigm for simultaneous localization and map building is presented. The main idea is to connect local metric maps by means of a global topological map. This allows a compact environment model which does not require global metric consistency and permits both precision and robustness. The method uses a 360 degree laser scanner in order to extract corners and openings for the topological approach and lines for the metric localization. The approach has been tested in a 30 x 25 m portion of the institute building with the fully autonomous robot Donald Duck. An experiment consists of a complete exploration and a set of test missions. Three experiments have been performed for a total of 15 test missions, which have been randomly defined and completed with a success ratio of 87%.

1. Introduction

Research in simultaneous localization and automatic mapping has diverged into different approaches leading promising results. However solutions for precise and robust localization and mapping in unmodified, dynamic, real-world environments have not been found yet. The problem is highly complex due to the fact that it requires the robot to be localized with respect to the portion of the environment that has already been mapped in order to build a coherent map.

Metric, topological or hybrid navigation schemes have been proposed and studied. Metric approaches are defined here as methods, which permit the robot to estimate its (x, y, θ) position, while topological are those where the position is given by a location without metric information. Approaches using purely metric maps are vulnerable to inaccuracies in both map-making and dead-reckoning abilities of the robot. Even by taking into account all the relationships between features and the robot itself, in large environments the drift in the odometry makes the global consistency of the map difficult to maintain. Landmark-based approaches, which rely on the topology of the environment, can better handle this problem, because they only have to maintain topological global consistency, not metric. However these approaches are either less precise than fully metric approaches, due to the discretization of the localization space, or computationally intractable for fully autonomous robots, when fine grained grids are used. More recently, approaches combining the topological and the metric paradigm (mainly grid-based) have shown that positive characteristics of both can be integrated to compensate for the weakness of each single approach.

This paper proposes an integration of both the metric and topological paradigms, to gain the best characteristics of both universes. This includes that the precision for the metric approach has only to be bounded by the quality of the sensors and not by the approach itself. The model used here, embodies both a metric and a topological representation. The metric model consists of infinite lines that belong to the same place. These places are related to each other by means of a topological map that is composed of nodes and edges between nodes. Connections between a node and a place are a special case: Traveling along these edges causes a switch from the topological to the metric paradigm. The effectiveness of this method has already been shown in [18]. In this paper the approach is extended to include automatic mapping.

For the metric approach an *extended Kalman filter* (EKF) is used. This approach has already proven its strength for localization [2]. Map building can therefore be done with the *stochastic map* [15]. Topological navigation uses a *partially observable Markov decision process* (POMDP) [3] for state estimation. This permits efficient planning in the large, has an advantageous symbolic representation for man-machine interaction and is robust due to its multi hypothesis tracking.

2. Environment Modeling

The environment is described by a global topological map, which permits moving in the whole environment. However the environment model is characterized by two different levels of abstraction (fig. 1):

- Places are defined as a local metric map which allows navigation within the neighbourhood.
- To go from one place to another, the system moves metrically in the start place, then topologically after leaving that place and switches back to metric when reaching the goal place.

In order to switch from topological to metric, a detectable metric feature is needed to determine the transition point and to initialize the metric localization (i.e. relocation). This is the only specific requirement for this approach. Given this transition feature, a metric place can be defined everywhere in the environment.

Leaving a metric map and switching to topological reduces to a metric navigation to the initialization position for the current local place where the robot restarts its topological navigation.

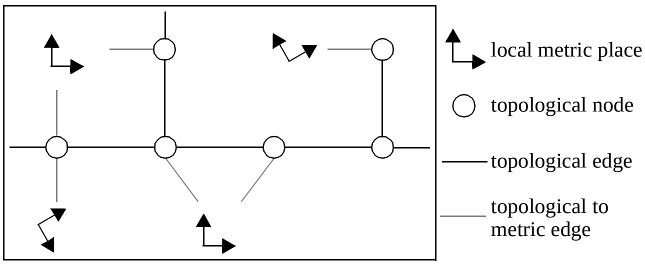

Figure 1: *The environment is represented by places given by their metric maps and nodes representing topological locations. When travelling from a node to a place, the system switches from topological to metric and vice-versa.*

2.1 Global Topological Map

For the topological model the landmarks are:

- Corners, which are characterized by their orientation.
- Openings, which are also used for model transition.

The topological map can be viewed as a graph. Topological locations are represented by nodes containing the information about the way to reach the connected topological location/metric place. Furthermore the list of the landmarks lying between two locations is represented as a list between the two nodes. In fig. 2 the graph representing the topological model is viewed for a portion of the environment.

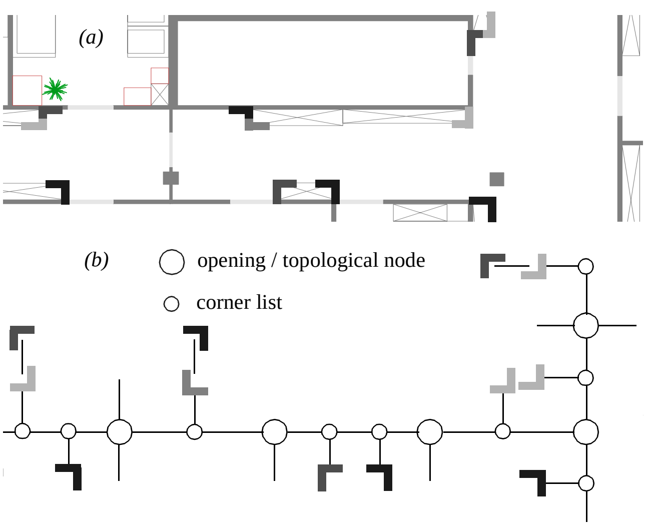

Figure 2: *(a) A portion of an hallway with the extracted corner and opening features. (b) The topological map is represented by a graph. It contains nodes connected to each other with the list of corner features lying between them. Openings (topological nodes) can either be a transition to a room or be a connection to another hallway. The colour of the corners helps distinguishing between corners with different orientation.*

The corner extractor returns a set of (x, y, θ) parameters representing the position and orientation of the corners and an extraction confidence parameter p_c for each corner.

Openings are either large steps perpendicular to the direction of motion in hallways or transitions from rooms to hallways. They can either be a transition between an hallway and a room or between two perpendicular hallways. This characteristic will be used for the map building strategy (3.1).

Note that, because the sensor used is a 360 degree laser scanner, an observation contains many landmarks which are transformed in a graph compatible to the environment model, as shown in fig. 3.

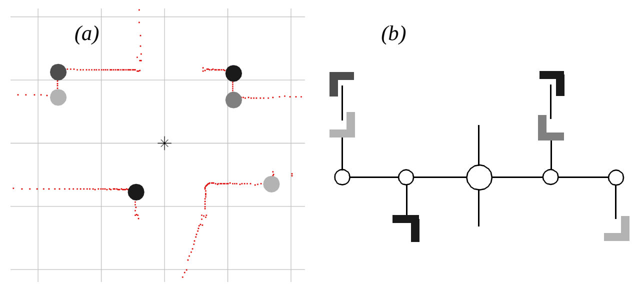

Figure 3: *(a) Laser data and the extracted features. (b) The resulting observation graph. Different colours represent different corner orientation.*

2.2 Local Metric Maps

The features used for metric environmental representation are infinite lines. They are less informative than line segments, but have a better probabilistic model with analytical solution and permit a very compact representation of structured geometric environments (i.e. long hallway represented by only two infinite lines) requiring only about 10 bytes per m^2. In fig. 4 a typical office is shown with the lines used for its local metric map. The line model is

$$\rho\cos(\varphi - \alpha) - r = 0 \qquad (1)$$

where (ρ, φ) is the raw measurement and (α, r) the model parameters. α is the angle of the perpendicular to the line, r its length. The used extraction algorithm has been described in [1]. Its result is a set of (α, r) parameters with their 2 x 2 covariance matrix, which results by propagating the uncertainty from the laser measures.

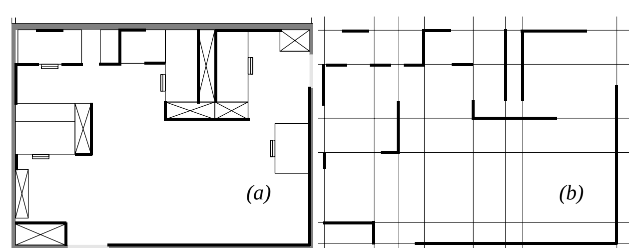

Figure 4: *An office of the institute (a) and the lines representing it in the local metric map(b). The black segments permit to see the correspondence between the two figures.*

3. Localization and Map Building

Both environment models require a different navigation method with different characteristics. The metric approach permits a very precise positioning at the goal point [2] whereas the topological method [3] guarantees robustness against getting lost due to the multimodal representation of the robot's location.

3.1 Map Building Strategy

As explained in section 2, the environment model is composed of a global topological map and a set of local metric maps. Given a metric transition feature, local metric maps can be everywhere in the environment. Therefore a suitable strategy has to be adopted.

For many possible application scenarios it can be expected that the robot will have to be very precise in the rooms, where most of its tasks have to be executed (e.g. docking for power recharging; manipulation tasks with objects on a table; human-robot interaction). While navigating in the large (i.e. hallways), precision with respect to the features is less important, but robustness and global consistency take an important role. Because of this, the two different levels of abstraction are used in combination of the different type of environmental structures:

- While navigating in hallways the robot firstly creates and then extends the global topological map
- When it enters a room, it creates a new local metric map

These two environmental structures are differentiated by means of the laser sensor: Thin and long open spaces are assumed to be hallways, while other open spaces will be defined as rooms.

3.2 Exploration Strategy

The proposed exploration strategy is simple: The robot first explores all the hallways in a depth-first way. It then explores each room it encounters in the same way. Note that, in general, for each hallway the room exploration reduces to a linear list traversal. Rooms with multiple openings cause two special cases, which are treated in the following sections.

Rooms with opening to another room: In this case the robot continues building the current metric map. However this can lead to the next case if the neighbor room has an opening to a hallway.

Rooms with opening to a hallway: Due to the metric navigation mode during room exploration, the robot knows the direction of the opening and can therefore deduce if it opens to the same hallway, a known one or a new one. In the case of known hallways, the robot simply goes back to the hallway it was coming from and continues its exploration. This could cause having two metric maps for the same metric place, one for each opening. In the case of a new hallway, the exploration continues in a hallway depth-first way.

3.3 Topological Localization and Map Building

The current experimental test bed is a part of the institute building. This environment is rectilinear and mainly composed of offices, meeting rooms and hallways. Therefore only four directions of travel are employed: N, E, S, W. However this limitation is not an inherent loss of generality because it is not a general requirement of the algorithm.

Position Estimator: Given a finite set of environment states *S*, a finite set of actions *A* and a state transition model *T*, the model can be defined by introducing partial observability. This includes a finite set *O* of possible observations and an observation function *OS*, mapping *S* into a discrete probability distribution over *O*. $T(s, a, s')$ represents the probability that the environment makes a transition from state *s* to state s' when action *a* is taken. $OS(o, s)$ is the probability of making an observation *o* in state *s*. The probability of being in state s' (*belief state* of s') after having made observation *o* while performing action *a* is then given by the equation:

$$SE_{s'}(k+1) = \frac{OS(o, s') \sum_{s \in S} T(s, a, s') SE_s(k)}{P(o|a, SE(k))} \tag{2}$$

where $SE_s(k)$ is the belief state of *s* at the last step, $SE(k)$ is the belief state vector of last step and $P(o|a, SE(k))$ is a normalizing factor. The observation function *OS* is made robust by the fact that an observation is composed of many landmarks (fig. 3), rising its distinctiveness.

Heading Estimator: Because the position estimator does not take into account the heading of the robot, this is done separately like in [9]. The orientation is estimated by a weighted mean of each observed line that is either horizontal or vertical with respect to the environment. The success of this method is guaranteed by the fact that, in general, lines given by the environmental structures are either parallel or perpendicular to the direction of travel. Infinite lines are matched by means of the validation test $(z^{[i]} - \hat{z}^{[j]}) S_{ij}^{-1} (z^{[i]} - \hat{z}^{[j]})^T \le \chi^2_{\alpha,n}$, where prediction $\hat{z}^{[j]}$ is directly the odometry state vector variable θ. In this case, $\chi^2_{\alpha,n}$ is a number taken from a χ^2 distribution with $n = 1$ degrees of freedom. This can be interpreted as a Kalman filter for heading only.

Control Strategy: Since it is computationally intractable to compute the optimal POMDP control strategy for a large environment [3], simple suboptimal heuristics are introduced. For the system presented here the *most likely state* policy has been adopted: The world state with the highest probability is found and the action that would be optimal for that state is executed. However it can happen that the robot is not sure about its current state. This is calculated by mean of the unconfident function $U(SE(k))$, which is the entropy of the probability distribution over the states of the map. The POMDP is confident when

$$U(SE(k)) = -\sum SE_s(k)\log SE_s(k) < U_{max} \quad (3)$$

where U_{max} is determined by experience by performing tests allowing to see at which level of unconfidence state estimates are effectively false. When the robot is unconfident, it follows the hallway in the direction where it expects to found more information (landmarks). What has to be avoided at any cost is to switch from the multimodal topological navigation to the unimodal metric navigation when the robot is unconfident about its location, otherwise it could enter a false local metric place and therefore be lost.

Map Building: Instead of using a complex scheme for model learning like in [10] and [17], where an extension of the *Baum-Welch algorithm* is adopted, here the characteristics of the observation graph are used. When the robot feels confident about its position, it can decide if an extracted landmark is new by comparing the observation graph to the node in the map corresponding to the most likely state. This can happen either in an unexplored portion of the environment or in a know portion, where new landmarks appear due to the environment dynamic. As explained in section 2.1, landmarks come with their extraction confidence p_l. This characteristic is firstly used to decide if the new landmark can be integrated in the map. Furthermore, for each integrated landmark, the confidence is used to model the probability of seeing that landmark the next time p_{lmap}. When it is re-observed, the probability in the map is averaged with the confidence of the extracted one. If the robot does not see an expected landmark the probability $1 - p_{lmap}$ is used instead.

$$p_{lmap}(t_i) = \sum^{n} \frac{p_l(t_i)}{i} \quad (4)$$

$$\text{where, } p_l(t_i) = \begin{cases} p_l(t_i) & observed \\ 1 - p_{lmap}(t_{i-1}) & \neg observed \end{cases} \quad (5)$$

When the confidence p_{lmap} decreases and is below a minimum the corresponding landmark is deleted from the map. This allows for dynamics in the environment, where landmarks that disappear in the real world, will be deleted from the map too.

When an opening landmark is integrated in the map, a new state node is created (fig. 2).

3.4 Metric Localization and Map Building

This section describes briefly the main characteristics of the *stochastic map* approach [15], which permits using an *extended Kalman filter* [6], [13] for localization.

With this approach both the robot position $x_r = (x, y, \theta)'$ and the features $x_i = (\alpha, r)'$ are represented in the system state vector:

$$x = \begin{bmatrix} x_r \\ x_1 \\ \\ x_n \end{bmatrix} \quad C(x) = \begin{bmatrix} C_{rr} & C_{r1} & & C_{rn} \\ C_{1r} & C_{11} & & C_{1n} \\ & & & \\ C_{nr} & C_{n1} & & C_{nn} \end{bmatrix} \quad (6)$$

This represents the uncertain spatial relationship between objects in the map, which is changed by three actions:

- Robot displacement
- Observation of a new object
- Re-observation of an object already existing in the map

Robot Displacement: When the robot moves with an uncertain displacement $\boldsymbol{u}$ given by its two first moments (u, C_u), which are measured by the odometry, the robot state is updated to $g(x_r, \boldsymbol{u})$. The updated position and uncertainty of the robot pose are obtained by error propagation on g:

$$x_r(k+1) = g(x_r(k), u) = x_r(k) \oplus u \quad (7)$$

$$C_{rr}(k+1) = G\begin{bmatrix} C_{rr}(k) & C_{ru}(k) \\ C_{ur}(k) & C_u \end{bmatrix} G^T \quad (8)$$

where $\oplus$ is compounding operator and G is the Jacobian of g with respect to x_r and $\boldsymbol{u}$.

New Object: When a new object is found, a new entry must be made in the system state vector. A new row and column are also added to the system covariance matrix to describe the uncertainty in the object's location and the inter-dependencies with the other objects. The new object $(\hat{x}_{new}, C_{new})$ can be integrated in the map by computing the following equations of uncertainty propagation:

$$x_{N+1}(k) = g(x_r(k), x_{new}) = x_r(k) \oplus x_{new} \quad (9)$$

$$C_{N+1N+1}(k) = G_{x_r} C_{rr}(k) G_{x_r}^T + G_{x_{new}} C_{new} G_{x_{new}}^T \quad (10)$$

$$C_{N+1i}(k) = G_{x_r} C_{ri}(k) \quad (11)$$

Re-Observation: Let x_{new} be the new observation in the robot frame. The measurement equation is defined as:

$$z = h(x_r, x_{new}, x_i) = g(x_r, x_{new}) - x_i \quad (12)$$

x_{new} is temporarily included in the state to apply the extended Kalman filter. However if prediction x_i satisfies the validation test

$$(x_{new} - x_i)\, S_{newi}^{-1}\, (x_{new} - x_i)^T \le \chi^2_{\alpha,n} \quad (13)$$

where $S_{newi} = C_{newnew} + C_{ii} - C_{newi} - C_{inew}$, $\chi^2_{\alpha,n}$ is a number taken from a χ^2 distribution with $n = 2$ degrees of freedom and α is the level on which the hypothesis of pairing correctness is rejected, then x_{new} is a re-observation of x_i.

Extended Kalman Filter: When a spatial relationship is re-observed, the updated estimate is a weighted average of the two estimates calculated by means of an Extended Kalman filter. It permits to update a subset of the state vector while maintaining the consistency by means of the covariance matrices. A measurement equation $z = h(x_1, x. x_m)$ is considered as a function of *m* relationships included in *x*. All of the *n* estimates x_i of the state vector *x* are updated by a value that is proportional to the difference $\delta = z - \hat{z}$ between the ideal measurement *z* and the actual measurement $\hat{z}$:

$$x_i(k+1) = x_i(k) + \Gamma_{iz}\Gamma_{zz}^{-1}\delta \qquad (14)$$

$$\Gamma_{iz} = E[x_i\delta^T] = \sum_{j=1}^{M} C_{ij}H_{xj}^T \qquad (15)$$

$$\Gamma_{zz} = E[\delta\delta^T] = \sum_{j=1}^{M}\sum_{k=1}^{M} H_{x_j}C_{jk}H_{x_k}^T \qquad (16)$$

where H_{x_j} is the Jacobian matrix of *h* with respect to x_j . The variance and covariance C_{ij} are also updated:

$$C_{ij}(k+1) = C_{ij}(k) - \Gamma_{iz}\Gamma_{zz}^{-1}\Gamma_{jz}^T \qquad (17)$$

4. Experimental Results

For the experiments, Donald Duck (fig. 5) has been used. It is a fully autonomous mobile vehicle running XO/2, a deadline driven hard real-time operating system. Donald navigates locally by means of a motion control algorithm, which plays the role of both position controller and obstacle avoidance: It reaches the given (x, y, θ) or (x, y) goal by planning a collision free path (with respect to the current local data), and reacting to the dynamic environment either by merely replanning the path or by changing heading direction and replanning when an object appears in front of the robot.

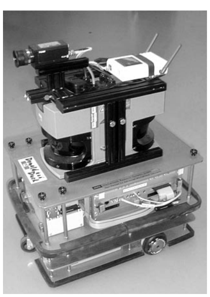

Figure 5: *The fully autonomous robot Donald Duck. Its controller consists of a VME standard backplane with a Motorola PowerPC 604 microprocessor clocked at 300 Mhz running XO/2. Among its peripheral devices, the most important are the wheel encoders, a 360° laser range finder and a grey-level CCD camera (not used here).*

4.1 Experiments

The approach has been tested in the portion of the institute building shown in figure 6.

An experiment is structured as follows:

- Explore the whole environment once and construct a hybrid map
- Evaluate the usability of the map by defining random generated navigation missions

number of missions	15
success rate	87%
number of state estimates	588
unconfident state rate	7%
total travel distance	0.4 km

Table 1: *Summary of the experiments. The results demonstrate the feasibility of this hybrid approach for office environments.*

The environment is closed, so that the exploration procedure is finite. Test missions are defined from a local metric place to another one. The robot is localized with respect to the local metric map at the start position. By leaving the room it switches to topological localization and map building. When it reaches the goal place, it initializes the Kalman filter and navigates metrically to the goal point.

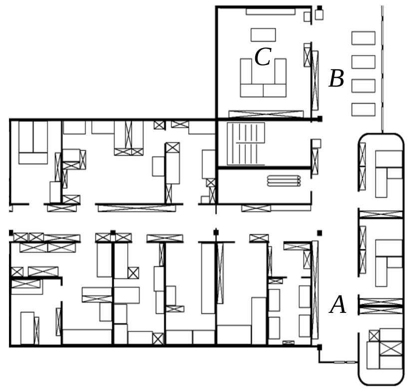

Figure 6: *The test environment. It is closed so that the exploration is finite. In A the robot had problems distinguishing between the two locations (doors). B and C are detected as rooms and represented by a single local metric map.*

4.2 Results

A set of three experiments has been performed. Each map resulting from a complete exploration has been tested with 5 randomly generated test missions. Donald performed the 15 mission with a success rate of 87%. In two cases the robot encountered some troubles distinguishing between two locations where only few landmarks are visible (fig. 6: A): It entered the false office. The precision at the goal is comparable to the results in [18] with mean error of less than 1 cm.

5. Related Work

Successful navigation of embedded systems for real applications relies on the precision that the vehicle can achieve, the capacity of not getting lost and the practicability of their algorithms on the limited resources of the autonomous system. Furthermore the fact that *a priori* maps are rarely available and, even when given, not in the format required by the robot, and that they are mainly unsatisfactory due to imprecision, incorrectness and incompleteness, makes automatic mapping a real need for application like scenarios.

Simultaneous localization and map building research can be divided into two main categories: Metric and topological.

With the first precise mathematical definition of the *stochastic map* [15] and early experiments [6], [13], fully metric simultaneous localization and map building have shown their quality: Highly precise localization, which is only bounded by the quality of the sensors [2]. However these approaches also suffer of some limitations. Firstly they rely strongly on dead-reckoning. For automatic mapping this makes the global consistency of the map difficult to maintain in large environments, where the drift in the odometry becomes too important. Furthermore they represent the robot pose with a single probability distribution. This means that an unmodeled event (i.e. collision) could cause a divergence between the ground truth and the estimated pose (lost situation) from which the system is unable to recover. In [4] it has been shown that the correlations, are very important for the global consistency, but not sufficient, as confirmed by a recent work [5], where a solution is proposed by extending the absolute localization to include local reference frames.

On the other hand topological approaches [11] can handle multi hypothesis tracking and have a topological global consistency, which is easier to maintain. The robustness of such approaches has firstly been proven by the application of the *state set progression* [14], which has then been generalized to the POMDP approach [3], [9]. In [10] the *Baum-Welch algorithm* is used for model learning. [12] proposes a topological approach, which heavily rely on odometry in order better to handle dynamic environments. While being robust, the drawback of these approaches is the loss of precision: The robot pose is represented by a location without precise metric information. To face this, the *Markov localization* [8] has been proposed: A fine grained grid guarantees both precision and multimodality. But this approach remains computationally intractable for current embedded systems. A more efficient alternative has recently been proposed, but the *Monte Carlo localization* [7] has not been extended yet for automatic mapping.

Metric and topological approaches are converging, like [5], [7] and [8], to hybrid solutions. Moving in this direction, in [16] the approach consists in extracting a topological map from a grid map by means of a Voronoi based method, while [17] proposes to use the *Baum-Welch algorithm* as in [10], but to build a topologically consistent global metric map.

In contrast to the above mentioned approaches, for this system a natural integration of the metric and topological paradigm is proposed. The approaches are completely separated into two levels of abstraction. Metric maps are used only locally for structures (rooms) that are naturally defined by the environment. There, a fully metric method is adopted. As it has been shown in [4], for such small environments, where the drift in the odometry remains uncritical, *stochastic map* allows for precise and consistent automatic mapping. The topological approach is used to connect the local metric maps that can be far away from each other. With this the robot can take advantage of the precision of a fully metric, Kalman based navigation, added to the robustness in the large of the POMDP approach while maintaining a compactness of the environment representation that allows the implementation of the method on a fully autonomous system.

6. Conclusions and Outlook

This paper presents a new hybrid approach for simultaneous localization and map building. The metric and topological parts are completely separated into two levels of abstraction. Together they permit a very compact and computationally efficient representation of the environment. Furthermore this combination permits both precision with the non-discrete metric estimator and robustness by means of the multimodal topological approach. The success rate over the 0.5 km of the 15 tests missions is 87%. Only 7% of the states are unconfident. They are uncritical in the experiments, nevertheless they cause a loss of time when travelling for gaining further information.

Future work will extend experiments to the whole floor of the institute building. Moreover the main future goal is to face the problem of closing loops in large environments [17].

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