

Towards Meta-Reasoning in the Concurrent Logical Framework CLF

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Objectives

- Concurrency and distribution are essential features in modern PL.
- Their formal semantics is not as well understood or studied as in the sequential case.
- Formal semantics will enable, e.g., development of formal verification frameworks, verifying program transformations, etc.

Logical frameworks

- Logical frameworks are formalisms used to specify PL and their metatheory
 - ▶ Coq, Agda, Twelf, Beluga, Delphin, ...
- Our goal is to develop logical frameworks for specifying concurrent and distributed PL.
- Two main approaches
 - ▶ Deep approach: specify a concurrency model in a general purpose LF (Coq, Agda)
 - ▶ Shallow approach: provide direct support in a special purpose LF (Twelf, Beluga, Delphin, LLF, HLF, CLF)
- We follow the shallow approach, using CLF as our LF

- CLF is an extension of the Edinburgh logical framework (LF) designed to specify distributed and concurrent systems
- Large number of examples: semantics of PL, Petri nets, voting protocols, etc.
- CLF extends LF with linear types and a monad to encapsulate concurrent effects:

$$\begin{aligned}
 A & ::= a \cdot S \mid \Pi !x : A. A \mid A \rightarrow B \mid A \multimap B \mid \{\Delta\} \\
 \Delta & ::= \cdot \mid \Delta, \downarrow x : A \mid \Delta, !x : A
 \end{aligned}$$

Example: $A \multimap B \multimap \{x : B, y : C\}$.

Substructural operational semantics

- Substructural operational semantics combines
 - ▶ Structural operational semantics
 - ▶ Substructural logics
- Extensible: we can add features without breaking previous developments
- Expressive: wide variety of concurrent and distributed mechanisms (Simmons12).

Higher-order abstract syntax

- Simply-typed λ -calculus

$$e ::= x \mid \lambda x.e \mid e e$$

- In (C)LF:

$\text{exp} : \text{type}.$

$\text{lam} : (\text{exp} \rightarrow \text{exp}) \rightarrow \text{exp}.$

$\text{app} : \text{exp} \rightarrow \text{exp} \rightarrow \text{exp}.$

- Linear-destination passing style (Pfenning04)
- Based on multiset rewriting; suitable for specifying in linear logic
- Multiset of facts:
 - $\text{eval } e \ d$ Evaluate expression e in destination d
 - $\text{ret } e \ d$ Value e in destination d
 - $\text{fapp } d_1 \ d_2 \ d$ Application: expects the function and argument to be evaluated in d_1 and d_2 , and the result is evaluated in d
- Evaluation rules transform multisets of facts

- Multiset of facts:

$\text{eval } e \ d, \quad \text{ret } e \ d, \quad \text{fapp } d_1 \ d_2 \ d$

- In CLF:

$\text{dest} : \text{type}.$

$\text{eval} : \text{exp} \rightarrow \text{dest} \rightarrow \text{type}.$

$\text{ret} : \text{exp} \rightarrow \text{dest} \rightarrow \text{type}.$

$\text{fapp} : \text{dest} \rightarrow \text{dest} \rightarrow \text{dest} \rightarrow \text{type}.$

Evaluation rules

- Multiset rewriting rules:

$\text{eval } e \ d \rightsquigarrow \text{ret } e \ d$ if e is a value

- In CLF:

$\text{step/eval} : \text{eval } e \ d \multimap \{\text{ret } e \ d\}.$

Evaluation rules

- Multiset rewriting rules:

$$\text{eval } (e_1 \ e_2) \ d \rightsquigarrow \text{eval } e_1 \ d_1, \text{eval } e_2 \ d_2, \text{fapp } d_1 \ d_2 \ d$$

where d_1, d_2 fresh

- In CLF:

$$\begin{aligned} \text{step/app: eval } (\text{app } e_1 \ e_2) \ d \\ \multimap \{!d_1 \ !d_2 : \text{dest}, \\ x_1 : \text{eval } e_1 \ d_1, x_2 : \text{eval } e_2 \ d_2, \\ y : \text{fapp } d_1 \ d_2 \ d\}. \end{aligned}$$

Evaluation rules

- Multiset rewriting rules:

$$\text{ret } (\lambda x. e_1) d_1, \text{ret } e_2 d_2, \text{fapp } d_1 d_2 d \rightsquigarrow \text{eval } (e_1[e_2/x]) d$$

- In CLF:

$$\begin{aligned} \text{step/beta : ret } (\text{lam } e_1) d_1 \\ \quad \multimap \text{ret } e_2 d_2 \\ \quad \multimap \text{fapp } d_1 d_2 d \\ \quad \multimap \{ \text{eval } (e_1 e_2) d \} \end{aligned}$$

Traces

- Evaluations (sequences of steps) are represented in CLF using traces.
- A trace is a sequence of computational steps, where independent steps can be permuted:

$$\varepsilon ::= \diamond \mid \{\Delta\} \leftarrow c \cdot S \mid \varepsilon_1; \varepsilon_2$$

- $\{\Delta\} \leftarrow c \cdot S$ means apply rule c to arguments S returning a new context Δ ; essentially a rewriting rule.
- Typing rules for traces:

$$\Delta \vdash \varepsilon : \Delta'$$

Traces

- Equality on traces: α -equivalence modulo permutation of independent steps
- Two steps are independent if they operate on different variables:

$$\{\Delta_1\} \leftarrow c_1 \cdot S_1; \{\Delta_2\} \leftarrow c_2 \cdot S_2 \equiv \{\Delta_2\} \leftarrow c_2 \cdot S_2; \{\Delta_1\} \leftarrow c_1 \cdot S_1$$

if $\text{dom}(\Delta_1) \cap \text{FV}(S_2) = \text{dom}(\Delta_2) \cap \text{FV}(S_1) = \emptyset$.

Example

$\text{eval } ((\lambda x.x)(\lambda y.y)) \ d$

In CLF:

$(!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) \ d)$

$\vdash \diamond$

$: (!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) \ d)$

Example

$$\text{eval } ((\lambda x.x)(\lambda y.y)) d \rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d$$

In CLF:

$$\begin{aligned} & (!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d) \\ & \quad \vdash \{!d_1, !d_2, x, y, z\} \leftarrow \text{step/app } x_0; \end{aligned}$$
$$\begin{aligned} & : (!d, !d_1, !d_2 : \text{dest})(x : \text{eval } (\text{lam } \lambda x.x) d_1)(y : \text{eval } (\text{lam } \lambda y.y) d_2) \\ & \quad (z : \text{fapp } d_1 d_2 d) \end{aligned}$$

Example

$$\begin{aligned} \text{eval } ((\lambda x.x)(\lambda y.y)) d &\rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \end{aligned}$$

In CLF:

$$\begin{aligned} (!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d) \\ \vdash \{!d_1, !d_2, x, y, z\} \leftarrow \text{step/app } x_0; \\ \quad \{x'\} \leftarrow \text{step/eval } x; \end{aligned}$$
$$\begin{aligned} : (!d, !d_1, !d_2 : \text{dest})(x' : \text{ret } (\text{lam } \lambda x.x) d_1)(y : \text{eval } (\text{lam } \lambda y.y) d_2) \\ (z : \text{fapp } d_1 d_2 d) \end{aligned}$$

Example

$$\begin{aligned} \text{eval } ((\lambda x.x)(\lambda y.y)) d &\rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{ret } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \end{aligned}$$

In CLF:

$$(!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d)$$
$$\begin{aligned} \vdash \{!d_1, !d_2, x, y, z\} &\leftarrow \text{step/app } x_0; \\ \{x'\} &\leftarrow \text{step/eval } x; \\ \{y'\} &\leftarrow \text{step/eval } y; \end{aligned}$$
$$\begin{aligned} &: (!d, !d_1, !d_2 : \text{dest})(x' : \text{ret } (\text{lam } \lambda x.x) d_1)(y' : \text{ret } (\text{lam } \lambda y.y) d_2) \\ &(z : \text{fapp } d_1 d_2 d) \end{aligned}$$

Example

$$\begin{aligned} \text{eval } ((\lambda x.x)(\lambda y.y)) d &\rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{ret } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{eval } (\lambda y.y) d \end{aligned}$$

In CLF:

$$(!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d)$$
$$\begin{aligned} \vdash \{!d_1, !d_2, x, y, z\} &\leftarrow \text{step/app } x_0; \\ \{x'\} &\leftarrow \text{step/eval } x; \\ \{y'\} &\leftarrow \text{step/eval } y; \\ \{w\} &\leftarrow \text{step/beta } x' y' z; \end{aligned}$$
$$: (!d, !d_1, !d_2 : \text{dest})(w : \text{eval } (\text{lam } \lambda y.y) d)$$

Example

$$\begin{aligned} \text{eval } ((\lambda x.x)(\lambda y.y)) d &\rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{ret } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{eval } (\lambda y.y) d \\ &\rightsquigarrow \text{ret } (\lambda y.y) d \end{aligned}$$

In CLF:

$$\begin{aligned} (!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d) \\ \vdash \{!d_1, !d_2, x, y, z\} \leftarrow \text{step/app } x_0; \\ \quad \{x'\} \leftarrow \text{step/eval } x; \\ \quad \{y'\} \leftarrow \text{step/eval } y; \\ \quad \{w\} \leftarrow \text{step/beta } x' y' z; \\ \quad \{w'\} \leftarrow \text{step/eval } w; \\ : (!d, !d_1, !d_2 : \text{dest})(w' : \text{ret } (\text{lam } \lambda y.y) d) \end{aligned}$$

Example

$$\begin{aligned} \text{eval } ((\lambda x.x)(\lambda y.y)) d &\rightsquigarrow \text{eval } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{eval } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{ret } (\lambda x.x) d_1, \text{ret } (\lambda y.y) d_2, \text{fapp } d_1 d_2 d \\ &\rightsquigarrow \text{eval } (\lambda y.y) d \\ &\rightsquigarrow \text{ret } (\lambda y.y) d \end{aligned}$$

In CLF:

$$\begin{aligned} (!d : \text{dest})(x_0 : \text{eval } (\text{app } (\text{lam } \lambda x.x) (\text{lam } \lambda y.y)) d) \\ \vdash \{!d_1, !d_2, x, y, z\} \leftarrow \text{step/app } x_0; \\ \quad \{y'\} \leftarrow \text{step/eval } y; \\ \quad \{x'\} \leftarrow \text{step/eval } x; \\ \quad \{w\} \leftarrow \text{step/beta } x' y' z; \\ \quad \{w'\} \leftarrow \text{step/eval } w; \\ : (!d, !d_1, !d_2 : \text{dest})(w' : \text{ret } (\text{lam } \lambda y.y) d) \end{aligned}$$

Safety

- Safety is the conjunction of the following properties:
 - ▶ Preservation: evaluation preserves well-typed states
 - ▶ Progress: a well-typed state is either final (result) or is possible to take a step
- Safety for SSOS can be proved by defining a suitable notion of well-typed multiset.
For example, $\text{eval } e_1 \ d, \text{eval } e_2 \ d$ is not well typed.
- Well-typed states can be defined by rewriting rules.
- Well-typed states are generated following the structure of the term.

Safety

- Well-typed states:

$\text{gen} \quad : \text{tp} \rightarrow \text{dest} \rightarrow \text{type}.$

$\text{gen/eval} : \text{gen } t \ d \multimap \text{of } e \ t \rightarrow \{\text{eval } e \ d\}.$

$\text{gen/ret} : \text{gen } t \ d \multimap \text{of } e \ t \rightarrow \{\text{ret } e \ d\}.$

$\text{gen/fapp} : \text{gen } t \ d \multimap \{!d_1 \ !d_2 : \text{dest},$
 $\quad \text{fapp } d_1 \ d_2 \ d,$
 $\quad \text{gen } (\text{arr } t_1 \ t) \ d_1,$
 $\quad \text{gen } t_1 \ d_2\}.$

- Generating well-typed states:

$$\text{gen } t \ d \rightsquigarrow^* \mathcal{A}$$

where \mathcal{A} contains no fact of the form $\text{gen } t_0 \ d_0$.

Safety

Lemma (Safety)

Preservation *If $\{\text{gen } t \ d\} \rightsquigarrow_{\text{gen}}^* \mathcal{A}$ and $\mathcal{A} \rightsquigarrow_{\text{step}} \mathcal{A}'$ then $\{\text{gen } t \ d\} \rightsquigarrow_{\text{gen}}^* \mathcal{A}'$.*

Progress *if $\{\text{gen } t \ d\} \rightsquigarrow_{\text{gen}}^* \mathcal{A}$, then either \mathcal{A} is of the form $\{\text{ret } e \ d\}$ or there exists \mathcal{A}' such that $\mathcal{A} \rightsquigarrow_{\text{step}} \mathcal{A}'$.*

Proof.

Preservation The proof proceeds by case analysis on the evaluation step.

Progress The proof proceeds by induction on the generating trace.



Limitations of CLF

- In CLF it is not possible to express preservation and progress.
- CLF lacks support for first-order traces, and quantification over contexts.
- We propose an extension of LF with trace types: Meta-CLF.
- Similar approaches are taken in Beluga, Delphin, Abella (in the sense of using a two-level approach).

Meta-CLF

- Meta-CLF is an extension of LF with trace types and quantification over contexts and names:

$$A ::= \dots \mid \{\Delta\} \Sigma^* \{\Delta\} \mid \{\Delta\} \Sigma^1 \{\Delta\} \mid \Pi \psi : \text{ctx}.A \mid \nabla x.A$$

- $\{\Delta\} \Sigma^* \{\Delta'\}$ is the type of all traces ε satisfying $\Delta \vdash \varepsilon : \Delta'$ that use only rules in the signature Σ .
- $\{\Delta\} \Sigma^1 \{\Delta'\}$ is the type of all 1-step traces ε satisfying $\Delta \vdash \varepsilon : \Delta'$ that use only rules in the signature Σ .

- In Meta-CLF we can express properties about traces:

preservation : $\nabla d. \nabla g. \Pi \psi_1 : \text{ctx}. \Pi \psi_2 : \text{ctx}.$

$$\{!d : \text{dest}, g : \text{gen } d \ t\} \Sigma_{\text{gen}}^* \{\psi_1\} \rightarrow \{\psi_1\} \Sigma_{\text{step}}^1 \{\psi_2\} \rightarrow \\ \{!d : \text{dest}, g : \text{gen } d \ t\} \Sigma_{\text{gen}}^* \{\psi_2\} \rightarrow \text{type}.$$

Meta-CLF

- The safety proof in Meta-CLF follows closely the paper proof.

$$\mathcal{A}, \text{eval } e \ d \rightsquigarrow_{\text{step}} \mathcal{A}, \text{ret } e \ d$$

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$\text{gen } t_0 \ d_0$

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$$\begin{array}{c} \text{gen } t_0 \ d_0 \\ \downarrow \text{gen}^* \\ \mathcal{A}, \text{gen } t \ d \\ \downarrow \text{gen} \\ \mathcal{A}, \text{eval } e \ d \rightsquigarrow_{\text{step}} \mathcal{A}, \text{ret } e \ d \end{array}$$

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- In Meta-CLF:

$$\begin{aligned} \text{tpres/ret} : & \text{tpres } (X_1; \{\downarrow x\} \leftarrow \text{gen/eval } e \ d_0 \ g_0 \ H) \\ & (\{\downarrow y\} \leftarrow \text{step/eval } e \ d_0 \ x \ H_v) \\ & (X_1; \{\downarrow y\} \leftarrow \text{gen/ret } e \ d_0 \ g_0 \ H \ H_v) \end{aligned}$$

Meta-CLF

- Both proofs of preservation and progress in Meta-CLF follow the pen-and-paper proofs.
- Preservation is performed by case analysis (no induction).
- Progress relies on induction, but termination is easy (size of the trace).
- However, we rely on coverage to ensure the proof is total.
- Coverage checking in the presence of traces is tricky, due to the possibility of permuting steps. (Left for future work.)

- We can extend this semantics with other features without invalidating the previous rules
- Example: store, futures, call/cc, communication,...

location : type.

loc : location \rightarrow exp .

get : exp \rightarrow exp .

ref : exp \rightarrow exp .

set : exp \rightarrow exp \rightarrow exp .

cell : location \rightarrow exp \rightarrow type.

step/ref : eval (ref e) $d \multimap \{!d_1 : \text{dest}, !l : \text{loc},$
 $\text{ref } d_1 \ l, \text{eval } e \ d_1, \text{ret } (\text{loc } l) \ d\}.$

step/fref : ret e $d \multimap \text{fref } d \ l \multimap \{\text{cell } l \ e\}.$

- We can extend this semantics with other features without invalidating the previous rules
- Example: store, futures, call/cc, communication,...

future : $\text{exp} \rightarrow \text{exp}$.

promise : $\text{dest} \rightarrow \text{exp}$.

deliver : $\text{exp} \rightarrow \text{dest} \rightarrow \text{type}$.

step/fut : $\text{eval} (\text{future } e) d \multimap \{!d_1 : \text{dest},$
 $\text{eval } e d_1, \text{fdel } d_1,$
 $\text{ret} (\text{promise } d_1) d\}$.

step/fdel : $\text{ret } e d \multimap \text{fdel } d_1 \multimap \{! \text{deliver } e d\}$.

step/promise : $\text{ret} (\text{promise } d_1) d \multimap \text{delivee } e d_1 \rightarrow \text{ret } e d$.

Conclusions and future work

- Our goal is to develop logical frameworks suitable for specifying concurrent and distributed systems.
- We introduced Meta-CLF, an extension of LF to reason about CLF specifications.
- We showed that it is expressive enough to write safety proofs of parallel/concurrent PL.
- Future work
 - ▶ Coverage checker
 - ▶ Termination checker
 - ▶ Implementation

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Thank you!