Automatic Peak Number Detection in Image Symmetry Analysis*

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Abstract. In repeated pattern analysis, peak number detection in autocorrelation is of key importance, which subsequently determines the correctness of the constructed lattice. Previous work inevitably needs users to select peak number manually, which limits its generalization to applications in large image database. The main contribution of this paper is to propose an optimization-based approach for automatic peak number detection, i.e., we first formulate it as an optimization problem by a straightforward yet effective criterion function, and then resort to Simulated Annealing to optimize it. Based on this approach, we design a new feature to depict image symmetry property which can be automatically extracted for repeated pattern retrieval. Experimental results demonstrate the effectiveness of the optimization approach and the superiority of symmetry feature over wavelet feature in discriminating similar repeated patterns.

1 Introduction

Repeated pattern symmetry has been studied for decades and plays a nontrivial role in texture analysis. In a 2D repeated pattern, there exists a finite region bounded by two linearly independent vectors. Repeating along those two vectors, it produces simultaneously a covering (no gaps) and a packing (no overlaps) of the original image [5]. The finite region is the repeated unit and the two vectors are called translation vectors, which build up lattice structure. According to the theory of wallpaper groups, the infinite variety of repeated patterns can be well categorized into seventeen Crystallographic groups, which are characterized by four kinds of symmetry: translation symmetry, rotation symmetry, reflection symmetry and glide reflection symmetry [1]. Among them, rotation symmetry can only be 2-fold, 3-fold, 4-fold and 6-fold, where n-fold means $360/n$ degree rotation. Reflection and glide reflection symmetry can have four axes: two translation vectors and two diagonal vectors of the repeated unit. The fundamental

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problem in repeated pattern analysis is to determine whether or not it has a certain kind of symmetry, which relies on the construction of translation vectors, and to further classify it into one of the seventeen groups.

In lattice construction, Leung et al. [3] and Schaffalitzky et al. [6] both use parameterized affine transforms to establish correspondence between interesting regions. Starovoitov et al. [7] propose a more traditional approach which uses features extracted from co-occurrence matrix to detect translation vectors. It has been proved in [4] that if a texture image is of regular structure, autocorrelation is more appropriate than Fourier Transform in analyzing its structure. More recently, Liu et al. [5] first calculate the autocorrelation of the pattern, and then select a sparse set of points to designate the repeated units. Finally, they adapt a Hough transform approach [4] to find two translation vectors from the points. To select candidate points from autocorrelation, they propose an efficient approach based on the region of dominance instead of simple threshold method and the points are called peaks. For isolated patterns, this method proves to be most appropriate [5]. However, the problem of automatic peak number detection remains unsolved. The drawback limits its generalization to applications in large image database, as it tends to be a tedious task to manually set peak number for each image in the database. Although peak number is apparent to people, determining its value automatically is difficult due to the variability of autocorrelation of different patterns.

In this paper, we propose an optimization-based scheme to automatically select peak number. To be specific, firstly, we formulate peak number selection as an optimization problem by a straightforward yet effective criterion, which incorporates the highest score in the accumulator array, autocorrelation value, as well as the length of the translation vectors. Then, we resort to Simulated Annealing (SA) to optimize it in order to balance between optimization performance and processing time. Based on this scheme, we design a new symmetry feature using translation vectors for repeated pattern retrieval which can be applied in spin industry where users wish to find repeated patterns with similar symmetry property as the query from a large database. Experimental results demonstrate the effectiveness of our method.

The rest of this paper is organized as follows. In Section 2, we present our approach of automatic peak number detection; in Section 3, the symmetry feature extraction is proposed; experimental results are given in Section 4; finally, we conclude the paper in Section 5.

2 Automatic Peak Number Detection

To construct translation vectors correctly, peak number N should be an approximation of the number of repeated units in an image. Once peak number is determined, the Generalized Hough Transform (GHT) [4] can be utilized to find the two translation vectors [5]. The procedure can be summarized as follows. Initially, a 2D accumulator array is created, in which each entry is set to be zero. Secondly, each pair of non-collinear vectors are used as two translation vectors to span a parallelogram grid. For each peak, if it is located near any vertex of the parallelogram grid, the score that the peak belongs to the parallelogram grid will be high; otherwise the score will be low. Thirdly, the score is added to the entry corresponding to the pair of non-collinear vectors. Finally, two translation vectors are obtained by finding the entry with the highest score in the accumulator array.

In order to automatically detect peak number, we design an optimization-based approach. In this approach, the peak number is obtained by optimizing a criterion function. Details are presented below.

2.1 Criterion Function

Let the entry with the highest score in the accumulator array S_N locates at (i, j), where N denotes the peak number. When N is proper, almost all of the selected peaks are located at the lattice node, thus $S_N(i, j)$ will be fairly large. However, as N increases, more peaks will be selected, and more scores will be added to the entries of S_N. So the original maximum corresponding to proper peak number N will be overwhelmed. Thus we divide $S_N(i, j)$ by N to eliminate the accumulation effect and take this value as a criterion for selecting proper peak number N.

However, in GHT, the accumulating score is inverse proportional to the lengths of two translation vectors. In patterns with sub-units, translation vectors of sub-units are always shorter than those of real units. If peak number is well above proper, the criterion value will be larger than that of proper peak number and sub-units will be extracted. Inspired by the fact that the autocorrelation value of peaks corresponding to sub-units is not as conspicuous as that of real units, we add a height factor to the criterion to help distinguish real units from sub-units.

When peak number is small, one of the translation vectors may be zero, which is unreasonable. So we add another term $\min(|w_1|, |w_2|)$ to the criterion, which selects the shorter one of the two translation vectors to preclude unreasonable pair of vectors.

Based on the above discussion, the final criterion function can be written as follows:

$$C(N) = (\frac{S_N(i,j)}{N})^{\alpha} \cdot (\overline{height(N)})^{\beta} \cdot (\min(|w_1|, |w_2|))^{\gamma} \qquad (1)$$

where $\overline{height(N)}$ is the average autocorrelation value of the first N peaks, i.e. the height factor; α, β, γ are positive parameters controlling the contribution of the three terms to the overall criterion. In our current implementation, they are set to 1 for simplicity.

2.2 Optimization of the Criterion Function

As the criterion function contains several local maxima (Fig. 2(d)), traditional greedy algorithms may fail to find the global maximum. Although enumeration

1. Initialize peak number $N = N_0$, *temperature* $= t_0$, *time* $= 0$. Calculate $C(N_0)$, and set maximum criterion value $C_{max} = C(N_0)$, optimum peak number $N_{opt} = N_0$;
2. Repeat n_1 times:
   (a) Set $N1 = N + \Delta N$, where ΔN is an integer distributed uniformly distributed uniformly between $[-\Delta, +\Delta]$. Calculate $C(N_1)$. If $C(N_1)$ is bigger than $C(N)$, set $N = N_1$; otherwise, set $N = N_1$ with probability $\exp((C(N_1) - C(N))/temperature)$;
   (b) Update C_{max} and N_{opt};
3. Decrease *temperature* by Δt. If maximum criterion value C_{max} has not changed for n_2 times, output N_{opt} that produces C_{max} and stop; else go to 2.

Fig. 1. SA algorithm for peak number detection

methods can always find the optimal solution, it is very time-consuming due to the construction of accumulation array. To balance optimization performance and processing time, we use Simulation Annealing (SA) algorithm, which is listed in Fig. 1.

3 Symmetry Feature Extraction

To determine whether or not repeated patterns have a certain kind of symmetry, Liu et al. [5] apply the symmetry to be tested to the entire pattern, and check the similarity between the original and transformed images. However, repeated patterns are often corrupted by noise or distortion. Therefore, it is more reasonable to give a continuous value to measure the extent to which a pattern has a certain kind of symmetry than a yes or no conclusion.

3.1 Symmetry Measure

If a pattern has a certain kind of symmetry, the correlation between the original and transformed images will have peaks with similar underlying structure as autocorrelation. To make this structure similarity concrete, we extract translation vectors from both autocorrelation and correlation, and compare the two pairs of vectors.

Let w_1, w_2 denote the translation vectors calculated from autocorrelation. t_1, t_2 denote the translation vectors calculated from correlation between the original and transformed images. Using automatic peak number detection method discussed in Section 2, we can automatically determine how many peaks should be selected from autocorrelation and correlation, and construct translation vectors without the intervention of users. If a pattern has a certain kind of symmetry, the associated t_1 and t_2 will be approximately the same as w_1 and w_2, or sometimes a rotated version; otherwise they will be totally different. To correctly measure the similarity between the two pairs of translation vectors, we use three components to represent each pair: the lengths of w_1 and w_2 (t_1 and t_2) and the angle between them. Then the chessboard distance is calculated to measure the similarity:

$$D(w_1, w_2, t_1, t_2) = \frac{1}{|w_1|}||w_1| - |t_1|| + \frac{1}{|w_2|}||w_2| - |t_2|| + |\theta_w - \theta_t| \qquad (2)$$

where θ_w (θ_t) is the angle between w_1 and w_2 (t_1 and t_2) in radian. When calculating the distance automatically, we do not know if w_1 corresponds to t_1 or t_2, so we calculate both $D(w_1, w_2, t_1, t_2)$ and $D(w_1, w_2, t_2, t_1)$, and select the smaller the one as the final distance. We also normalize the distance to [0,1] by formalizing the following exponential form:

$$S = \exp(-\min(D(w_1, w_2, t_1, t_2), D(w_1, w_2, t_2, t_1))) \qquad (3)$$

If an image has a certain kind of symmetry, S will be near 1; otherwise, it will be small, sometimes near 0. For an image not strictly symmetrical due to noise or distortion or some other reason, S measures to what extent this pattern has a certain kind of symmetry.

3.2 Symmetry Feature

Every repeated pattern has translation symmetry. The translation vectors constructed from autocorrelation surface reflect this symmetry. To measure rotation symmetry likelihood, we perform the four kinds of rotation to the original image, construct translation vectors based on correlation, and get four measures S_2, S_3, S_4, S_6 using equation (3), where s_n denotes the symmetry measure for n-fold rotation symmetry. Reflection symmetry and glide reflection symmetry are essentially the same except for a translation factor. So we perform reflection transformations using each choice of the axes, construct translation vectors and get another four measures S_{T1}, S_{T2}, S_{D1}, S_{D2}, where footnote T denotes translation vector axes, and D denotes diagonal vector axes. The eight measures make up for the symmetry feature, which represents the symmetrical property of a repeated pattern, and can be written as follows:

$$f_S = [S_2, S_3, S_4, S_6, S_{T1}, S_{T2}, S_{D1}, S_{D2}]^t \qquad (4)$$

4 Experimental Results

4.1 Peak Number Detection

In our experiment, we have collected 487 repeated patterns from the web. These images include all seventeen wallpaper groups. The number of images belonging to each group is listed in Table 1. From the table, we can see that the number varies greatly from group to group, which reflects the non-uniformity of various kinds of symmetry in natural images. Most of the patterns are corrupted by noise or distortion, thus not strictly symmetrical. Figure 2 illustrates two examples of automatic peak number detection. Note that: 1) in both cases, the maximum

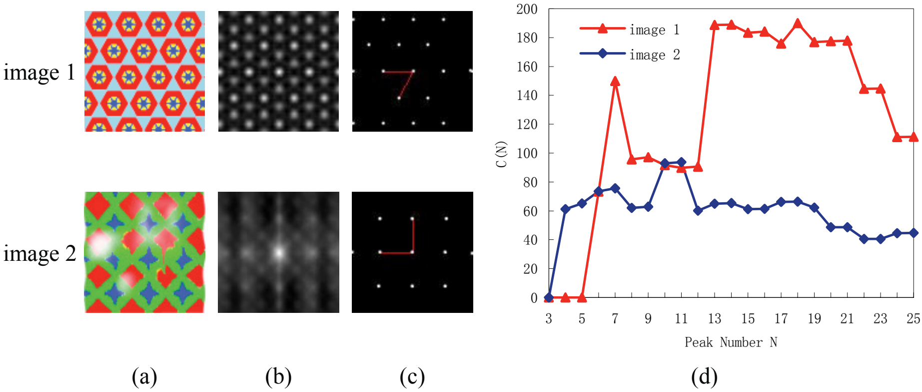

Fig. 2. Examples of automatic peak number detection via optimizing the criterion function. (a) repeated patterns [2]; (b) autocorrelation; (c) N_{opt} peaks and the translation vectors; (d) criterion value $C(N)$ (due to limited space, we only exhibit its value with peak number N from 3 to 25; when N is larger than 25, $C(N)$ is small and lacks variability)

value of the criterion is achieved at the proper peak number; 2) the response in the neighborhood of the proper peak number is somewhat flat, however it is still applicable for real applications (for explanation, see the next paragraph); 3) in both cases, there exists local maximum points, which indicates a greedy algorithm might fail to find the optimal number.

In order to test the performance of the criterion function, we first use an enumeration method to search for the optimal peak number that corresponds to maximum criterion value and extract the translation vectors. In 437 patterns, the constructed translation vectors are consistent with human perception, which means that the selected peak number is correct. Accordingly, the correct rate is 89.73%. For comparison, we also test the performance of the SA algorithm running several times. All the parameters are determined based on some prior knowledge of the database. In our experiment, we set $N_0 = 13$, $\Delta N = 5$, $t_0 = 0.01$, $\Delta t = 0.0001$, $n_1 = 3$, $n_2 = 5$. Currently we are doing research on optimal selection of these parameters. On average, 436 pairs of translation vectors are correctly extracted, i.e. the correct rate is 89.53%, which is very close to that of the enumeration approach. The number of patterns whose translation vectors are wrongly detected for each group by the two algorithms are also listed in Table 1. It is worth noticing that the image whose translation vectors are wrongly detected by enumeration method is NOT necessarily wrongly detected by SA method. This can be explained as follows: the robustness of GHT ensures that if peak number N lies in the neighborhood of repeated unit number N_0, i.e. $N \in [N_0 - \Delta N_1, N_0 + \Delta N_2]$, the constructed translation vectors will be correct. (This also explains the aforementioned problem) If peak number N_{opt} corresponding to maximum criterion value does not lie in this range, translation vectors will be unstable in case of peak number perturbation. Since SA is a kind of random search strategy, it may often return peak number in $[N_0 - \Delta N_1, N_0 + \Delta N_2]$ and

Table 1. Optimization method comparison. E: Enumeration, S: SA

Wallpaper groups		p1	p2	pm	pg	cm	pmm	pmg	pgg
Number		22	25	11	5	40	46	13	15
Error Num	E	1	10	1	2	3	3	2	0
	S	2	7	1	0	6	3	1	0

Wallpaper groups		cmm	p4	p4m	p4g	p3	p31m	p3m1	p6	p6m
Number		49	33	117	9	6	11	13	12	60
Error Num	E	5	2	8	0	0	2	1	1	9
	S	6	2	13	0	1	2	1	0	6

construct correct translation vectors. On the other hand, the overall correct rate of SA is less than that of enumeration. This may partially due to the prematurity property of SA.

As mentioned before, the enumeration scheme is very time consuming. For an image containing 72×128 pixels, the time needed with peak number varying from 3 to 100 is about 43 seconds (Intel(R) 500MHz, 256M RAM). But the optimization time of SA for the same image is less than a second on average.

4.2 Symmetry Feature Evaluation

To test the performance of symmetry feature in repeated pattern retrieval, we build this feature for the 487 repeated patterns, use each of the patterns as a query, and record average precision. The repeated patterns are classified into seventeen wallpaper groups as ground truth. We use wavelet feature for comparison, which is a widely accepted descriptor of texture. It consists of 18 coefficient moments, which is obtained after three-level Daubechies-8 wavelet transforms [8]. Both results are illustrated in Table 2. The precision values are not very high. That is because most of the repeated patterns are corrupted by noise or distortion. Therefore, although peak number is correctly identified, the underlying structure of correlation between the original and transformed images does not resemble that of autocorrelation, thus t_1 and t_2 will not be correctly calculated. For this reason, we only focus on the relative performance in subsequent discussion.

Comparing symmetry feature and wavelet feature, although the latter is a widely accepted descriptor of texture, and has more dimensions, its performance is not as good as symmetry feature. Moreover, the first retrieved images using symmetry feature are visually more similar with the query than those using wavelet feature. It demonstrates the effectiveness of our feature from another point of view.

Table 2. Feature performance comparison

Precision	P10	P20	P30	P40	P50
Symmetry Feature	0.1840	0.1738	0.1684	0.1647	0.1605
Wavelet Feature	0.1777	0.1567	0.1462	0.1416	0.1368

5 Conclusion

In this paper, we have proposed an optimization-based scheme for automatic peak number detection, which enables repeated patterns to be analyzed without human intervention. Moreover, we design a new symmetry feature which reflects the sym-metrical property of repeated patterns. Based on automatic peak number detection method, this feature can be extracted automatically for a large number of images. Currently we are doing research on generalizing this symmetry feature to natural images.

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