The Ground Reaction Force Pattern from the Hindlimb of the Horse Simulated by a Spring Model

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Abstract. A model consisting of a spring loaded by a time-dependent mass is presented simulating the vertical and longitudinal horizontal ground reaction force patterns obtained from the hindlimb of a walking horse.

Introduction

The horizontal and vertical ground reaction forces of the horse at normal walk show very characteristic patterns [Merkens et al., 1986]. The first peak in the vertical reaction force occurs following ground contact and loading of the limb. The second peak occurs during push-off at the end of the support phase. The transverse horizontal force shows also two peaks, but their amplitude is less than 10% of the vertical force. The positive and negative peaks in the longitudinal horizontal ground reaction force tracing correspond to the deceleration and propulsion phases, respectively. Similar force patterns can be observed in human walking [Jarrett et al., 1980].

Kinematic analysis revealed [Wentink, 1978] that the hindlimb, when rotating over the hoof during the stance phase, shows only small changes in knee, tarsal and proximal digital joint angles. Consequently, the total limb can be considered as a single rigid element that is slightly compressed along the hip-to-hoof line. Since most joints in the hindlimb of the horse are stabilized by several tendons, it may be expected that the mechanical behavior of the limb can be described by an elastic spring that rotates in a sagittal plane. The present study was undertaken to test this hypothesis and to simulate the ground reaction force pattern when this “rotating spring model” was loaded by a time-dependent mass. Comparison with experimental force plate tracings was used to optimize the model parameters.

Materials and Methods

Model Description

During normal walk of the horse both front and hind limbs are loaded sequentially with short overlaps at the start and end of the stance phase. When only the hindlimb has ground contact, the fraction of the animal’s body mass loading that limb is approximately 0.4 [Merkens et al., 1986]. Both hindlimbs contribute to bearing of the mass when they are simultaneously in contact with the ground. As an approximation, the mass loading on a hindlimb is assumed to increase linearly to the stance phase value \( m_s \) being 0.4 times the animal’s body mass, to remain at that value during single support and to decrease to zero at the end of the stance phase (Fig. 1) when multiple support occurs again. In mathematical terms:

\[
M(t) = \begin{cases} 
0 & \text{for } 0 \leq t < t_s \\
M_s & \text{for } t_s \leq t < t_2 \\
M(t) = m_a \left(1 - \frac{t - t_1}{t_2 - t_1}\right) & \text{for } t_1 \leq t < t_2 \\
0 & \text{for } t_2 \leq t < t_3 \\
\end{cases}
\]

In the model the limb is simplifies to a springless spring with constant \( k \) (Fig. 1), loaded by the time-dependent mass \( M(t) \) as defined above. The model is constrained to move in a sagittal plane. Let us assume initially...
Fig. 1. Model of the hindlimb of the horse consisting of a spring with constant k, loaded during the stance phase by a mass M(t) that varies as shown in the right hand panel. The hindlimb is loaded maximally with mH, 0.4 times the animal’s body mass. The angle $\theta(t)$, that the limb makes with the vertical increases linearly at rate $\omega$ (equation (5)).

that the limb does not rotate and is loaded vertically. The equation of motion is:

$$M(t)\ddot{x} - g + k\,x(t) = 0,$$

where $x(t)$ is the distance over which the spring is compressed and $g$ is the acceleration due to gravity. The magnitude of the vertical ground reaction force $F_y(t)$ equals the spring force $k\,x(t)$, or:

$$F_y(t) = k\,x(t) = M(t)\omega^2 = g - \omega^2 x(t).$$

However, if the rotation of the limb is taken into account, both the vertical and the longitudinal horizontal ground reaction forces can be calculated. If the angle the limb makes with the vertical is $\theta(t)$, the equation of motion becomes:

$$M(t)\ddot{\theta} + g\,l - k\,l\sin(\theta(t)) = l\,\omega(t)^2 \cos(\theta(t)) - k\,x(t),$$

where $l$ is the resting length of the spring and $\theta(t)$ the angular velocity of the hindlimb. The terms between $\sin(\theta(t))$ represent internal gravitational and centrifugal accelerations, respectively. The total ground reaction force force $F_y(t)$ can be calculated:

$$F_y(t) = k\,l\sin(\theta(t)) - l\,\omega(t)^2 \cos(\theta(t)) + x(t).$$

and the horizontal and vertical components become:

$$F_x(t) = F_y(t) \cos(\theta(t)),$$

$$F_y(t) = F_y(t) \sin(\theta(t)).$$

Movement Analysis

The overlap and stance phase durations of a group of 20 untrained, clinically sound horses were derived from high-speed film analysis (Merkenb et al., 1986). In three animals of this group the angle $\theta(t)$, the long axis of the limb (top to toe) makes with the vertical, the corresponding angular velocity and the "compression" of the limb along the hip-to-toe line were determined.

\[\theta(t) = \langle \theta(t) \rangle \pm \sigma\]

Results and Discussion

The ground reaction force curves of all 20 horses were similar in shape and amplitude (Merkenb et al., 1986). As a typical example we present the data of one horse of this group and the corresponding curves, calculated using the model described in this study (Fig. 2). The overlap and stance phase durations, equation (1), were 0.16 and 0.80 s, respectively. The angle that the hip-to-toe line makes with the vertical could be approximated by the relationship:

\[\theta(t) = \langle \theta(t) \rangle \pm \sigma\]
The numerical solutions of equations (6) and (7), giving both $F_z$ and $F_y$ of the rotating spring model, are plotted in figures 2a and b (curves c), using $m_0/k = 0.003 \text{ s}^2$. The loading-induced compression of the spring corresponding to this value was calculated to be 6 cm, a value that is close to the reduction in hip-to-hoof distance derived from film analysis. A better fit of the curves could be obtained using $m_0/k = 0.025$. This implies that the spring is slightly compressed prior to loading, or in other words, the limb has a certain pre-tension before ground contact occurs.

The present study illustrates that a simple model of the hindlimb of the horse, consisting of a rotating spring loaded by a time-constant mass, can be used to simulate longitudinal horizontal and vertical ground reaction forces that closely approximate those in the living animal.

References


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