Problem Set (Control)

1. **Muscle Modeling.** Simulate a quick-release experiment (Fig. 1A).

   a) Start with a contractile element CE. Use Matlab to implement the force-length relationship \( f_l(\ell_{CE}) \) and the force-velocity relationship \( f_v(v_{CE}) \) of a Hill-type CE. Plot \( M(\ell_{CE}, v_{CE}) = f_l f_v \) for \( \ell_{CE} \in [0, 2\ell_{opt}] \) and \( v_{CE} \in [-v_{max}, +v_{max}] \). Use this plot to discuss why we feel warm hiking up a hill but cool off on the way down. (2pts)

   b) In the second step, consider the muscle activation \( A \). Assume the muscle receives a neural stimulation \( S = 0.5 \sin(2\pi 5t) + 0.5 \). Model the excitation-contraction coupling \( A = f(S) \) as a first order systems with \( \tau_R = 30 \text{ ms} \) when \( A \) is rising and with \( \tau_F = 80 \text{ ms} \) when \( A \) is falling. Simulate 1 second, plot the time traces of \( S \) and \( A \), and discuss the behavior of \( A \). (2pts)

   c) Implement an activation-dependent contractile element in Simulink whose force output is \( F_{CE} = F_{iso} f_l(\ell_{CE}) f_v(v_{CE}) A \). Assume this element represents the muscle in Fig. 1 with \( l_m = \ell_{CE} \). Simulate the muscle behavior for the following experiment: At \( t = 0 \), the muscle stimulation is set to its maximum while the muscle is locked at the length \( l_m = l_{opt} \). At time \( t_q = 1 \text{ s} \), the lock is released instantaneously and the muscle pulls against the mass \( m = 400 \text{ kg} \). Determine the slope of the muscle length change right after \( t = t_q \)(use a \( v_{CE} \) plot as guide). Compare your time histories of muscle length and force to experimentally observed ones in quick-release experiments. Repeat the experiment for \( m = 1 \text{ kg}, 200 \text{ kg}, 600 \text{ kg}, 800 \text{ kg} \) and plot \( m \) over your recorded velocities \( v_{CE} \). Compare your results with the force-velocity relationship. (3pts)

   Muscle parameters: \( F_{iso}^{max} = 6000 \text{ N}, \ell_{opt} = 8 \text{ cm}, w = 0.56 \ell_{opt}, v_{max} = 12 \ell_{opt} s^{-1}, N_{ecc} = 1.5 \).

2. **Neural Control.** Develop a central pattern generator control for a biped (Fig 1B).

   a) Start with the left leg’s extensor neuron. Model the neuron output as \( y_{el} = \max(x_{el}, 0) \) where \( x_{el} \) is the neuron’s membrane potential, which changes according to \( \dot{x}_{el} = \frac{1}{\tau_x}(x_0 + \sum_i w_0 y_i - w' x' - x_{el}) \). The parameter \( x_0 \) describes a central input whereas \( \sum_i w_i y_i \) is the input from other neurons. Currently this sum is zero. The variable \( x' \) describes auto-adaptation of the neuron’s membrane potential with \( \dot{x'} = \frac{1}{\tau_a} (x - x') \). Simulate and explore this neuron model in Matlab or Simulink. How do \( x_0 \) and \( \tau_a \) influence the model behavior? (2pts)

   b) Continue with the left extensor-flexor pair, a basic neural oscillator. Implement the two models for the extensor and flexor neurons. Use the \( \sum_i w_i y_i \) in each neuron model to connect the
two models. Give these interconnections equal weights $w_{fe} = w_{ef} = -1.5$ and assign the extensor neuron an initial membrane potential of $x_{el} = 1$. Simulate and explore the neural oscillator. What happens if $\tau_a < 2\tau_x$? How does an imbalance $w_{fe} \neq w_{ef}$ influence the oscillator behavior? (2pts)

c) Now explore the two coupled neural oscillators for the left and right leg. Assume that whenever an extensor neuron is active, the biped’s corresponding leg pushes against the ground. The right side in figure 1B shows characteristic activity patterns for the leg extensor neurons in walking, running and two-legged hopping. Connect the four neurons according to the scheme in figure 1B. Explore different weight settings and initial membrane potentials, and see if you can identify the three different gait patterns. Use the paper ”Mechanisms of Frequency and Pattern Control in the Neural Rhythm Generators” of Matsuoka (1987, Biol Cybern) for inspiration. (3pts)

CPG parameters: $x_0 = 5$, $\tau_x = 1$, $\tau_a = 12$, $w' = 2.5$. 