

Linear Logical Voting Protocols

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Current Approach to Electronic Elections

Informal Specification

Legal Text



Human Translation

Java, C, etc.

Implementation

1. Translate legal text to imperative source code.

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 - ▶ How to trust this?

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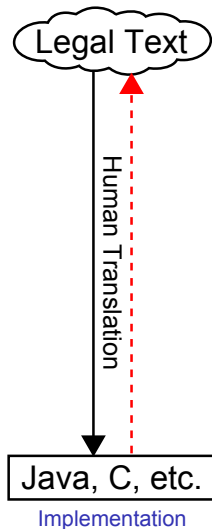
Java, C, etc.

Implementation

1. Translate legal text to imperative source code.
 - ▶ How to trust this?
2. Certify that code meets legal specification.
 - ▶ *Very hard!*

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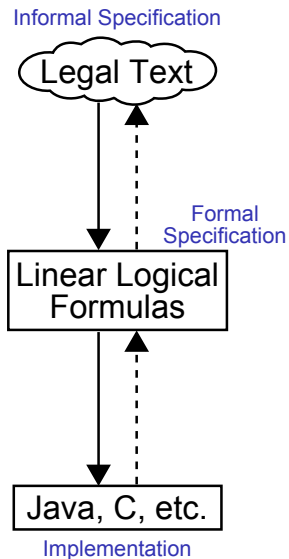
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Is this approach really trustworthy?

Key Idea

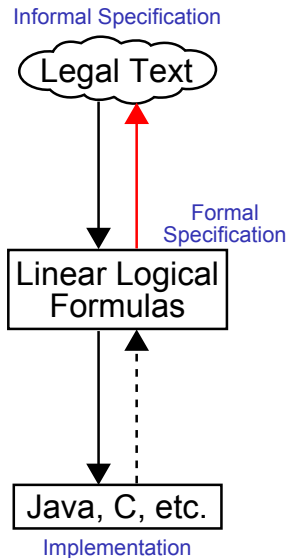
Formal logic, particularly linear logic, is well-suited to the trustworthy specification and implementation of voting protocols.

Our Approach to Electronic Elections



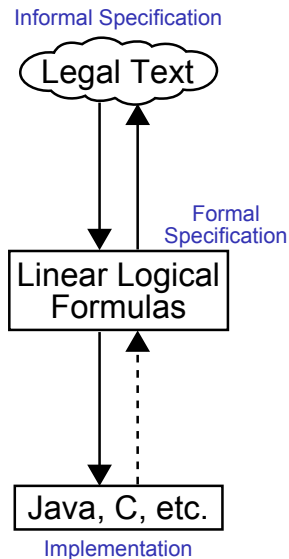
1. Translate legal text to logical formulas.
 - ▶ Algorithms at high level of abstraction

Our Approach to Electronic Elections



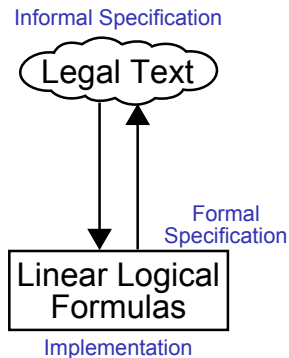
1. Translate legal text to logical formulas.
 - ▶ Algorithms at high level of abstraction
 - ▶ Much smaller gap from legal language!

Our Approach to Electronic Elections



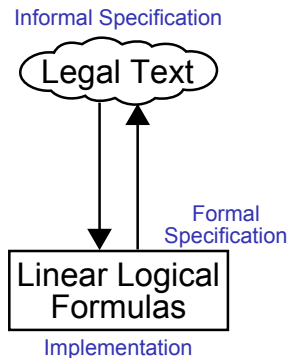
1. Translate legal text to logical formulas.
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2. Transliterate formulas to a logic program.
 - ▶ Formulas = source code

Our Approach to Electronic Elections



1. Translate legal text to logical formulas.
 - ▶ Algorithms at high level of abstraction
 - ▶ Much smaller gap from legal language!
2. Transliterate formulas to a logic program.
 - ▶ Formulas = source code
 - ▶ **No further translation necessary!**

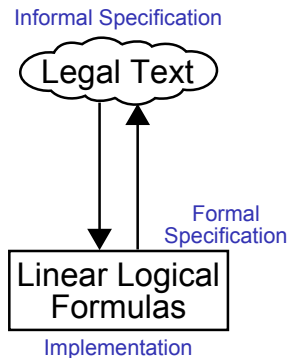
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What must still be trusted?

1. Translation to logical formulas
 - ▶ Much smaller gap from legal language
— more trustworthy!

Our Approach to Electronic Elections



What must still be trusted?

1. Translation to logical formulas
 - ▶ Much smaller gap from legal language — more trustworthy!
2. Correctness of logic programming engine
 - ▶ Equal to or easier than trusting compiler
 - ▶ Proof witnesses are audit trails for free.
 - ▶ Use a simpler proof checker to validate proof objects.

Contributions and Non-Contributions of Our Paper

Contributions:

- ▶ Full linear logical specifications of:
 - ▶ Single-winner first-past-the-post (SW-FPTP)
 - ▶ Single transferable vote (STV)
- ▶ Operational interpretation as Celf logic programs
- ▶ Proof sketch of partial correctness of STV specification

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Non-contributions:

- ▶ Focus is on *verified* elections, not *voter-verifiable* elections.
 - ▶ Complementary: E2E techniques detect errors at run time; verified software minimizes run-time errors.
- ▶ Only operational correctness, not security properties
 - ▶ Linear logical voting specs should be readily amenable to meta-reasoning [Reed '09] about security; left to future work.

In This Talk

- 1 A Brief Introduction to Linear Logic
- 2 A Linear Logical Specification of Single Transferable Vote
- 3 Conclusion

A Brief Introduction to Linear Logic

What is linear logic?

Traditional Logic

"Truth is free."

- ▶ Assumptions may be used any number of times.

Linear Logic

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"Truth is a consumable resource."

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What is linear logic?

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- ▶ The logic of **facts**.

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"Truth is a consumable resource."

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- ▶ The logic of **food**.

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Let's specify a voter check-in process:

- ▶ *Consume* an authorization card to prevent multiple check-ins.

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Let's specify a voter check-in process:

- ▶ *Consume* an authorization card to prevent multiple check-ins.
- ▶ Use linear implication, $A \multimap \{B\}$.
 - ▶ $A \multimap \{B\} \approx$ "consume resource A to produce B ."

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"If I *give* an authorization card, then I *get* a blank ballot."

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"If I *give* an authorization card, then I *get* a **blank ballot**."

“May I please see your identification?”

voting-auth-card “and photo ID” \multimap $\{blank-ballot\}$

“If I give an auth. card and a photo ID, then I get a ballot.”

Problem: How to express a pair of resources?

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Problem: How to express a pair of resources?

Solution: Use simultaneous conjunction, $A \otimes B$.

▶ $A \otimes B \approx$ “both resources A and B ”

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In linear logic, how do we only *show* the ID?

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Solution: Use the unrestricted modality, $!A$.

▶ $!A \approx$ “a version of A that is never consumed.”

“May I please see your identification?”

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“If I give an auth. card and **show** a photo ID, then I get a ballot.”

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Ensuring that the Card and ID Match

$\text{voting-auth-card} \otimes \neg \text{photo-ID} \multimap \{\text{blank-ballot}\}$

“If I give an auth. card and show a photo ID, then I get a ballot.”

Problem: Doesn't ensure that auth. card and photo ID match.

Ensuring that the Card and ID Match

$$\textit{voting-auth-card} \otimes !\textit{photo-ID} \multimap \{\textit{blank-ballot}\}$$

“If I give an auth. card and show a photo ID, then I get a ballot.”

Problem: Doesn't ensure that auth. card and photo ID match.

Solution: Use universal quantification, $\forall x.A$.

- ▶ Quantified variables are not resources.

Ensuring that the Card and ID Match

$\forall v. \text{voting-auth-card}(v) \otimes !\text{photo-ID}(v) \multimap \{\text{blank-ballot}\}$

“If I give an auth. card and show a **matching** ID, then I get a ballot.”

Problem: Doesn't ensure that auth. card and photo ID match.

Solution: Use universal quantification, $\forall x.A$.

- ▶ Quantified variables are not resources.

Ensuring that the Card and ID Match

$$\text{voting-auth-card}(V) \otimes !\text{photo-ID}(V) \multimap \{\text{blank-ballot}\}$$

“If I give an auth. card and show a matching ID, then I get a ballot.”

Problem: Doesn't ensure that auth. card and photo ID match.

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Problem: Doesn't ensure that the auth. card and ID are mine.

Ensuring that the Card and ID Match

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Problem: Doesn't ensure that auth. card and photo ID match.

Solution: Use universal quantification, $\forall x.A$.

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Problem: Doesn't ensure that the auth. card and ID are mine.

Future Work: Use possession modality [Garg⁺ '06], K has A .

- ▶ Location and secrecy of information?
Homomorphic encryption?

A Linear Logical Specification of Single Transferable Vote

Single Transferable Vote (STV)

Outline of STV Protocol:

0. Calculate the quota of votes.
1. Tally each ballot for its highest pref that is neither elected nor defeated.
 - ▶ Surplus votes go to next pref.
2. After all votes have been tallied:
 - ▶ If there are more cand. than seats, eliminate cand. with the fewest votes; transfer his votes and re-tally (go to 1).
 - ▶ If there are more seats than cand., then all remaining cand. are elected.

STV Ballot Form

Rank any number of candidates
in order of preference.

Alice

3

Bob

Charlie

1

Dave

2

Single Transferable Vote on a Single Slide

begin/1 :

begin(*S*, *H*, *U*) \otimes
 $!(Q = U/(S+1) + 1)$
 $\rightarrow \{!quota(Q) \otimes$
 tally-votes(*S*, *H*, *U*)}

tally/1 :

tally-votes(*S*, *H*, *U*) \otimes
uncounted-ballot(*C*, *L*) \otimes
hopeful(*C*, *N*) \otimes
 $!quota(Q) \otimes !(N+1 < Q)$
 $\rightarrow \{counted-ballot(C, L) \otimes$
 hopeful(*C*, *N*+1) \otimes
 tally-votes(*S*, *H*, *U*-1)}

tally/2 :

tally-votes(*S*, *H*, *U*) \otimes
uncounted-ballot(*C*, *L*) \otimes
hopeful(*C*, *N*) \otimes
 $!quota(Q) \otimes !(N+1 \geq Q) \otimes$
 $!(S \geq 1)$
 $\rightarrow \{counted-ballot(C, L) \otimes$
 !elected(*C*) \otimes
 tally-votes(*S*-1, *H*-1, *U*-1)}

tally/3 :

tally-votes(*S*, *H*, *U*) \otimes
uncounted-ballot(*C*, [*C'* | *L*]) \otimes
 $!(elected(C) \oplus !defeated(C))$
 $\rightarrow \{uncounted-ballot(C', L) \otimes$
 tally-votes(*S*, *H*, *U*)}

tally/4 :

tally-votes(*S*, *H*, *U*) \otimes
uncounted-ballot(*C*, []) \otimes
 $!(elected(C) \oplus !defeated(C))$
 $\rightarrow \{tally-votes(S, H, U-1)\}$

tally/5 :

tally-votes(*S*, *H*, 0) \otimes
 $!(S < H)$
 $\rightarrow \{defeat-min(S, H, 0)\}$

tally/6 :

tally-votes(*S*, *H*, 0) \otimes
 $!(S \geq H)$
 $\rightarrow \{!elect-all\}$

defeat-min/1 :

defeat-min(*S*, *H*, *M*) \otimes
hopeful(*C*, *N*)
 $\rightarrow \{minimum(C, N) \otimes$
 defeat-min(*S*, *H*-1, *M*+1)}

defeat-min/2 :

defeat-min(*S*, 0, *M*)
 $\rightarrow \{defeat-min'(S, 0, M)\}$

defeat-min'/1 :

defeat-min'(*S*, *H*, *M*) \otimes
minimum(*C*₁, *N*₁) \otimes
minimum(*C*₂, *N*₂) \otimes
 $!(N_1 \leq N_2)$
 $\rightarrow \{minimum(C_1, N_1) \otimes$
 hopeful(*C*₂, *N*₂) \otimes
 defeat-min'(*S*, *H*+1, *M*-1)}

defeat-min'/2 :

defeat-min'(*S*, *H*, 1) \otimes
minimum(*C*, *N*)
 $\rightarrow \{!defeated(C) \otimes$
 transfer(*C*, *N*, *S*, *H*, 0)}

transfer/1 :

transfer(*C*, *N*, *S*, *H*, *U*) \otimes
counted-ballot(*C*, *L*)
 $\rightarrow \{uncounted-ballot(C, L) \otimes$
 transfer(*C*, *N*-1, *S*, *H*, *U*+1)}

transfer/2 :

transfer(*C*, 0, *S*, *H*, *U*)
 $\rightarrow \{tally-votes(S, H, U)\}$

elect-all/1 :

!elect-all \otimes
hopeful(*C*, *N*)
 $\rightarrow \{!elected(C)\}$

Single Transferable Vote on a Single Slide

begin/1 :

$begin(S, H, U) \otimes$
 $!(Q = U/(S+1) + 1)$
 $\rightarrow \{!quota(Q) \otimes$
 $tally-votes(S, H, U)\}$

tally/1 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, L) \otimes$
 $hopeful(C, N) \otimes$
 $!quota(Q) \otimes !(N+1 < Q)$
 $\rightarrow \{counted-ballot(C, L) \otimes$
 $hopeful(C, N+1) \otimes$
 $tally-votes(S, H, U-1)\}$

tally/2 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, L) \otimes$
 $hopeful(C, N) \otimes$
 $!quota(Q) \otimes !(N+1 \geq Q) \otimes$
 $!(S \geq 1)$
 $\rightarrow \{counted-ballot(C, L) \otimes$
 $!elected(C) \otimes$
 $tally-votes(S-1, H-1, U-1)\}$

tally/3 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, [C' | L]) \otimes$
 $!(elected(C) \oplus !defeated(C))$
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tally/4 :

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 $!(S < H)$
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tally/6 :

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 $!(S \geq H)$
 $\rightarrow \{!elect-all\}$

defeat-min/1 :

$defeat-min(S, H, M) \otimes$
 $hopeful(C, N)$
 $\rightarrow \{minimum(C, N) \otimes$
 $defeat-min(S, H-1, M+1)\}$

defeat-min/2 :

$defeat-min(S, 0, M)$
 $\rightarrow \{defeat-min'(S, 0, M)\}$

defeat-min'/1 :

$defeat-min'(S, H, M) \otimes$
 $minimum(C_1, N_1) \otimes$
 $minimum(C_2, N_2) \otimes$
 $!(N_1 \leq N_2)$
 $\rightarrow \{minimum(C_1, N_1) \otimes$
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 $defeat-min'(S, H+1, M-1)\}$

defeat-min'/2 :

$defeat-min'(S, H, 1) \otimes$
 $minimum(C, N)$
 $\rightarrow \{!defeated(C) \otimes$
 $transfer(C, N, S, H, 0)\}$

transfer/1 :

$transfer(C, N, S, H, U) \otimes$
 $counted-ballot(C, L)$
 $\rightarrow \{uncounted-ballot(C, L) \otimes$
 $transfer(C, N-1, S, H, U+1)\}$

transfer/2 :

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 $tally-votes(S-1, H-1, U-1)\}$

tally/3 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, [C' | L]) \otimes$
 $!(elected(C) \oplus !defeated(C))$
 $\rightarrow \{uncounted-ballot(C', L) \otimes$
 $tally-votes(S, H, U)\}$

tally/4 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, []) \otimes$
 $!(elected(C) \oplus !defeated(C))$
 $\rightarrow \{tally-votes(S, H, U-1)\}$

tally/5 :

$tally-votes(S, H, 0) \otimes$
 $!(S < H)$
 $\rightarrow \{defeat-min(S, H, 0)\}$

tally/6 :

$tally-votes(S, H, 0) \otimes$
 $!(S \geq H)$
 $\rightarrow \{!elect-all\}$

defeat-min/1 :

$defeat-min(S, H, M) \otimes$
 $hopeful(C, N)$
 $\rightarrow \{minimum(C, N) \otimes$
 $defeat-min(S, H-1, M+1)\}$

defeat-min/2 :

$defeat-min(S, 0, M)$
 $\rightarrow \{defeat-min'(S, 0, M)\}$

defeat-min'/1 :

$defeat-min'(S, H, M) \otimes$
 $minimum(C_1, N_1) \otimes$
 $minimum(C_2, N_2) \otimes$
 $!(N_1 \leq N_2)$
 $\rightarrow \{minimum(C_1, N_1) \otimes$
 $hopeful(C_2, N_2) \otimes$
 $defeat-min'(S, H+1, M-1)\}$

defeat-min'/2 :

$defeat-min'(S, H, 1) \otimes$
 $minimum(C, N)$
 $\rightarrow \{!defeated(C) \otimes$
 $transfer(C, N, S, H, 0)\}$

transfer/1 :

$transfer(C, N, S, H, U) \otimes$
 $counted-ballot(C, L)$
 $\rightarrow \{uncounted-ballot(C, L) \otimes$
 $transfer(C, N-1, S, H, U+1)\}$

transfer/2 :

$transfer(C, 0, S, H, U)$
 $\rightarrow \{tally-votes(S, H, U)\}$

elect-all/1 :

$!elect-all \otimes$
 $hopeful(C, N)$
 $\rightarrow \{!elected(C)\}$

Aspect 1: From Legal Text to Formal Specification

Legal Text

“Tally the votes, assigning each ballot to its highest preference candidate who is neither elected nor defeated.”

Detailed Reading

If we are tallying votes and there is an uncounted vote for C and C is a hopeful with running tally N and the quota wouldn't be reached by this vote, then mark the ballot as counted and update C 's tally to $N+1$ votes and tally the remaining $U-1$ ballots.

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$tally-votes(S, H, U) \otimes$
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 $\quad \quad hopeful(C, N+1) \otimes$
 $\quad \quad tally\text{-votes}(S, H, U-1) \}$

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- ▶ Correspondence between legal text and logical formula is plain!
- ▶ Linearity: count ballots only once and update running tallies.

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Legal Text

"If a candidate reaches the quota, he is declared elected."

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Otherwise, if we are tallying votes and there is an uncounted vote for C and C is a hopeful with running tally N and this vote would meet the quota and there is at least one seat left, then mark the ballot as counted and declare candidate C to be elected and tally the remaining $U-1$ ballots among the $H-1$ hopefuls and $S-1$ seats left.

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 $!(S \geq 1)$
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- ▶ ! modality: once declared elected, always declared elected.

Aspect 1: From Legal Text to Formal Specification

Legal Text

“Any surplus votes go to the next preference listed on the ballot.”

Detailed Reading

If we are tallying votes and

there is an uncounted vote for C and C is either elected or defeated, then transfer it to the next pref. C' and tally the remaining U ballots.

tally/3 :

$tally\text{-votes}(S, H, U) \otimes$
 $uncounted\text{-ballot}(C, [C' \mid L]) \otimes$
 $(!elected(C) \oplus !defeated(C))$
 $\rightarrow \{uncounted\text{-ballot}(C', L) \otimes$
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tally/3 :

$$\begin{aligned} & \text{tally-votes}(S, H, U) \otimes \\ & \text{uncounted-ballot}(C, [C' \mid L]) \otimes \\ & (!\text{elected}(C) \oplus !\text{defeated}(C)) \\ & \rightarrow \{ \text{uncounted-ballot}(C', L) \otimes \\ & \quad \text{tally-votes}(S, H, U) \} \end{aligned}$$

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Aspect 2: From Formal Specification to Implementation

tally/1 :

tally-votes(*S*, *H*, *U*) \otimes
uncounted-ballot(*C*, *L*) \otimes
hopeful(*C*, *N*) \otimes
!quota(*Q*) \otimes *!(N+1 < Q)*
 \multimap { *counted-ballot*(*C*, *L*) \otimes
 hopeful(*C*, *N+1*) \otimes
 tally-votes(*S*, *H*, *U-1*) }

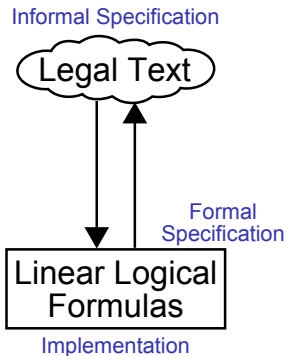
Celf Linear Logic Program

tally-votes *S* *H* (s *U'*) *
uncounted-ballot *C* *L* *
hopeful *C* *N* *
!quota *Q* * *!nat-less* (s *N*) *Q*
 \multimap { *counted-ballot* *C* *L* *
 hopeful *C* (s *N*) *
 tally-votes *S* *H* *U'* }

- ▶ Transliteration of formula to logic programming syntax
- ▶ Complete Celf source code of STV available online at <http://www.itu.dk/~carsten/files/voteid2011.tgz>

Conclusion

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Summary

Linear logic is well-suited to the trustworthy specification and implementation of voting protocols, including STV.

begin/1 :

$begin(S, H, U) \otimes$
 $!(Q = U/(S+1) + 1)$
 $\rightarrow \{!quota(Q) \otimes$
 $\quad tally-votes(S, H, U)\}$

tally/1 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, L) \otimes$
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 $!quota(Q) \otimes !(N+1 < Q)$
 $\rightarrow \{counted-ballot(C, L) \otimes$
 $\quad hopeful(C, N+1) \otimes$
 $\quad tally-votes(S, H, U-1)\}$

tally/2 :

$tally-votes(S, H, U) \otimes$
 $uncounted-ballot(C, L) \otimes$
 $hopeful(C, N) \otimes$
 $!quota(Q) \otimes !(N+1 \geq Q) \otimes$
 $!(S \geq 1)$
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tally/5 :

$tally\text{-votes}(S, H, 0) \otimes$
 $!(S < H)$
 $\rightarrow \{defeat\text{-min}(S, H, 0)\}$

tally/6 :

$tally\text{-votes}(S, H, 0) \otimes$
 $!(S \geq H)$
 $\rightarrow \{!elect\text{-all}\}$

defeat-min/1 :

$defeat-min(S, H, M) \otimes$

$hopeful(C, N)$

$\rightarrow \{minimum(C, N) \otimes$
 $defeat-min(S, H-1, M+1)\}$

defeat-min/2 :

$defeat-min(S, 0, M)$

$\rightarrow \{defeat-min'(S, 0, M)\}$

defeat-min'/1 :

$defeat-min'(S, H, M) \otimes$

$minimum(C_1, N_1) \otimes$

$minimum(C_2, N_2) \otimes$

$!(N_1 \leq N_2)$

$\rightarrow \{minimum(C_1, N_1) \otimes$

$hopeful(C_2, N_2) \otimes$

$defeat-min'(S, H+1, M-1)\}$

defeat-min'/2 :

$defeat-min'(S, H, 1) \otimes$

$minimum(C, N)$

$\rightarrow \{!defeated(C) \otimes$

$transfer(C, N, S, H, 0)\}$

transfer/1 :

transfer(C, N, S, H, U) \otimes

counted-ballot(C, L)

\multimap { *uncounted-ballot*(C, L) \otimes
transfer($C, N-1, S, H, U+1$) }

transfer/2 :

transfer($C, 0, S, H, U$)

\multimap { *tally-votes*(S, H, U) }

elect-all/1 :

!elect-all \otimes

hopeful(C, N)

\multimap { *!elected*(C) }

Why not use traditional first-order logic?

To motivate the use of linear logic and illustrate its connectives, let's develop a specification of the voter check-in process.

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Solution: Use linear logic!