Commuting Conversions and the π-Calculus

Henry DeYoung
hdeyoung@cs.cmu.edu

Frank Pfenning
fp@cs.cmu.edu

January 8, 2012

1 Introduction

We would like to exploit latent parallelism in π-calculus processes. For example, in the process $x(w).z(y).P_{wxyz}$, the two inputs are independent (assuming $z \neq x, w$), and this process is observationally equivalent to $z(y).x(w).P_{wxyz}$.

We propose that such observational equivalences are justified by commuting conversions in the proof theory. To support this claim, this document provides a catalog of commuting conversions for linear logic (currently excluding &, $\oplus$, $\top$, and $\mathbf{0}$).

A smaller question is whether the commuting conversions involving $\mathbf{1}\mathsf{L}$ suffice to show that the process assignment

$$\frac{}{\Gamma; \cdot \vdash \mathbf{0} :: x{:}\mathbf{1}}\ \mathbf{1}R \qquad\qquad \frac{\Gamma; \Delta \vdash P :: z{:}C}{\Gamma; \Delta, x{:}\mathbf{1} \vdash P :: z{:}C}\ \mathbf{1}L$$

is equivalent to the alternate process assignment

$$\frac{}{\Gamma; \cdot \vdash x\langle\rangle.\mathbf{0} :: x{:}\mathbf{1}}\ \mathbf{1}R' \qquad\qquad \frac{\Gamma; \Delta \vdash P :: z{:}C}{\Gamma; \Delta, x{:}\mathbf{1} \vdash x().P :: z{:}C}\ \mathbf{1}L'.$$

A similar question exists for the process assignment for $!\mathsf{L}$.

2 Catalog of Commuting Conversions

The commuting conversions are organized into subsections according to their top-level proof term constructor. Within each subsection, each proof term constructor is considered as a possible secondary constructor in each of the possible subterm positions.

2.1 $\multimap\mathsf{R}$

In the following cases, there are no commuting conversions in the proof theory. In the process assignment, this is reflected by a dependence of names, as in $z(y).z(x).P_{yxz}$.

$\multimap\mathsf{R}$-$\multimap\mathsf{R}$ No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \multimap\mathsf{R}\ (\hat{\lambda}x.\ D\ y\ x))$
$z(y).z(x).P_{yxz}$

$\multimap\mathsf{R}$-$\otimes\mathsf{R}$-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \otimes\mathsf{R}\ (D_1\ y)\ D_2)$
$z(y).(\nu x)z\langle x\rangle.(P_{yx} \mid Q_z)$

$\multimap\mathsf{R}$-$\otimes\mathsf{R}$-**2** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \otimes\mathsf{R}\ D_1\ (D_2\ y))$
$z(y).(\nu x)z\langle x\rangle.(P_x \mid Q_{yz})$

$\multimap$R-**1**R $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\mathbf{1}\mathsf{R})$ is not a well-formed proof term.

$\multimap$R-!R $\multimap\mathsf{R}\ (\hat{\lambda}y.\,!\mathsf{R}\ D)$ is not a well-formed proof term.

$\multimap$R-$\forall$R No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\forall\mathsf{R}\ (\lambda a.\,D\,y\,a))$

$$z(y).z(a).P_{yaz}$$

$\multimap$R-$\exists$R No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\exists\mathsf{R}\ M\ (D\,y))$

$$z(y).z\langle M\rangle.P_{yz}$$

The next three cases exemplify three other common patterns. In the first, $\multimap$R-$\multimap$L-1, the run-time value of y is known only after the input along z occurs, and so the output depends on the input. In the second, $\multimap$R-$\multimap$L-2, the input $z(y)$ cannot occur after the output along x since the input contributes to both the output process, P_{yw}, and the continuation process, Q_{xz}. In the third, a conversion exists because the input and output occur along independent names and the input contributes only to the continuation process, Q_{yxz}.

$\multimap$R-$\multimap$L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\multimap\mathsf{L}\ y\ D_1\ (\hat{\lambda}y.\,D_2\,y))$

$$z(y).(\nu x)y\langle x\rangle.(P_x \mid Q_{yz})$$

$\multimap$R-$\multimap$L-**2** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\multimap\mathsf{L}\ x\ (D_1\,y)\ (\hat{\lambda}x.\,D_2\,x))$

$$z(y).(\nu w)x\langle w\rangle.(P_{yw} \mid Q_{xz})$$

$\multimap$R-$\multimap$L-**3** $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\,D_2\,y\,x)) \Leftrightarrow \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\,\multimap\mathsf{R}\ (\hat{\lambda}y.\,D_2\,y\,x))$

$$z(y).(\nu w)x\langle w\rangle.(P_w \mid Q_{yxz}) \Leftrightarrow (\nu w)x\langle w\rangle.(P_w \mid z(y).Q_{yxz})$$

The following cases are similar to those described above.

$\multimap$R-$\otimes$L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\otimes\mathsf{L}\ y\ (\hat{\lambda}x.\hat{\lambda}y.\,D\,x\,y))$

$$z(y).y(x).P_{xyz}$$

$\multimap$R-$\otimes$L-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,y\,w\,x)) \Leftrightarrow \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\multimap\mathsf{R}\ (\hat{\lambda}y.\,D\,y\,w\,x))$

$$z(y).x(w).P_{ywxz} \Leftrightarrow x(w).z(y).P_{ywxz}$$

$\multimap$R-**1**L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\mathbf{1}\mathsf{L}\ y\ D)$

$$z(y).y().P_z$$

$\multimap$R-**1**L-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\mathbf{1}\mathsf{L}\ x\ (D\,y)) \Leftrightarrow \mathbf{1}\mathsf{L}\ x\ (\multimap\mathsf{R}\ (\hat{\lambda}y.\,D\,y))$

$$z(y).x().P_{yz} \Leftrightarrow x().z(y).P_{yz}$$

$\multimap$R-!L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,!\mathsf{L}\ y\ (\lambda u.\,D\,u))$

$$z(y).y/u.P_{uz}$$

$\multimap$R-!L-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\,!\mathsf{L}\ x\ (\lambda u.\,D\,y\,u)) \Leftrightarrow !\mathsf{L}\ x\ (\lambda u.\,\multimap\mathsf{R}\ (\hat{\lambda}y.\,D\,y\,u))$

$$z(y).x/u.P_{yuz} \Leftrightarrow x/u.z(y).P_{yuz}$$

$\multimap$R-copy $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D\,y\,u\,x)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}x.\,(\multimap\mathsf{R}\ (\hat{\lambda}y.\,D\,y\,u\,x)))$

$$z(y).(\nu x)u\langle x\rangle.P_{yuxz} \Leftrightarrow (\nu x)u\langle x\rangle.z(y).P_{yuxz}$$

$\multimap$R-$\forall$L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,D\,y))$

$$z(y).y\langle M\rangle.P_{yz}$$

$\multimap$R-$\forall$L-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\,\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,D\,y\,x)) \Leftrightarrow \forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\multimap\mathsf{R}\ (\hat{\lambda}y.\,D\,y\,x))$

$$z(y).x\langle M\rangle.P_{yxz} \Leftrightarrow x\langle M\rangle.z(y).P_{yxz}$$

$\multimap$R-$\exists$L-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\ D\,a\,y))$
$$z(y).y(a).P_{ayz}$$

$\multimap$R-$\exists$L-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D\,y\,a\,x)) \Leftrightarrow \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \multimap\mathsf{R}\ (\hat{\lambda}y.\ D\,y\,a\,x))$
$$z(y).x(a).P_{yaxz} \Leftrightarrow x(a).z(y).P_{yaxz}$$

$\multimap$R-cut-**1** No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \mathsf{cut}\ (D_1\,y)\ (\hat{\lambda}x.\ D_2\,x))$
$$z(y).(\nu x)(P_{yx} \mid Q_{xz})$$

$\multimap$R-cut-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \mathsf{cut}\ D_1\ (\hat{\lambda}x.\ D_2\,y\,x)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}x.\ \multimap\mathsf{R}\ (\hat{\lambda}y.\ D_2\,y\,x))$
$$z(y).(\nu x)(P_x \mid Q_{yxz}) \Leftrightarrow (\nu x)(P_x \mid z(y).Q_{yxz})$$

$\multimap$R-init No conversion for $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \mathsf{init}\ y)$
$$z(y).[y \leftrightarrow z]$$

$\multimap$R-cut$^!$-**1** $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \mathsf{cut}^!\ (D_1\,y)\ (\lambda u.\ D_2\,u))$ is not a well-formed proof term.

$\multimap$R-cut$^!$-**2** $\multimap\mathsf{R}\ (\hat{\lambda}y.\ \mathsf{cut}^!\ D_1\ (\lambda u.\ D_2\,y\,u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\ \multimap\mathsf{R}\ (\hat{\lambda}y.\ D_2\,y\,u))$
$$z(y).(\nu u)(!u(x).P_x \mid Q_{yuz}) \Leftrightarrow (\nu u)(!u(x).P_x \mid z(y).Q_{yuz})$$

2.2 $\otimes$R

$\otimes$R-$\multimap$R-**1** No conversion for $\otimes\mathsf{R}\ (\multimap\mathsf{R}\ (\hat{\lambda}x.\ D_1\,x))\ D_2$
$$(\nu y)z\langle y\rangle.(y(x).P_{xy} \mid Q_z)$$

$\otimes$R-$\multimap$R-**2** No conversion for $\otimes\mathsf{R}\ D_1\ (\multimap\mathsf{R}\ (\hat{\lambda}x.\ D_2\,x))$
$$(\nu y)z\langle y\rangle.(P_y \mid z(x).Q_{xz})$$

$\otimes$R-$\otimes$R-**1** No conversion for $\otimes\mathsf{R}\ (\otimes\mathsf{R}\ D_1\ D_2)\ D_3$
$$(\nu y)z\langle y\rangle.((\nu x)y\langle x\rangle.(P_x \mid Q_y) \mid R_z)$$

$\otimes$R-$\otimes$R-**2** No conversion for $\otimes\mathsf{R}\ D_1\ (\otimes\mathsf{R}\ D_2\ D_3)$
$$(\nu y)z\langle y\rangle.(P_y \mid (\nu x)z\langle x\rangle.(Q_x \mid R_z))$$

$\otimes$R-**1**R-**1** No conversion for $\otimes\mathsf{R}\ \mathbf{1}\mathsf{R}\ D$
$$(\nu y)z\langle y\rangle.(y\langle\rangle.\mathbf{0} \mid P_z)$$

$\otimes$R-**1**R-**2** No conversion for $\otimes\mathsf{R}\ D\ \mathbf{1}\mathsf{R}$
$$(\nu y)z\langle y\rangle.(P_y \mid z\langle\rangle.\mathbf{0})$$

$\otimes$R-!R-**1** No conversion for $\otimes\mathsf{R}\ (!\mathsf{R}\ D_1)\ D_2$
$$(\nu y)z\langle y\rangle.(!y(x).P_x \mid Q_z)$$

$\otimes$R-!R-**2** No conversion for $\otimes\mathsf{R}\ D_1\ (!\mathsf{R}\ D_2)$
$$(\nu y)z\langle y\rangle.(P_y \mid !z(x).Q_z)$$

$\otimes$R-$\forall$R-**1** No conversion for $\otimes\mathsf{R}\ (\forall\mathsf{R}\ (\lambda a.\ D_1\,a))\ D_2$
$$(\nu y)z\langle y\rangle.(y(a).P_{ay} \mid Q_z)$$

$\otimes$R-$\forall$R-**2** No conversion for $\otimes\mathsf{R}\ D_1\ (\forall\mathsf{R}\ (\lambda a.\ D_2\,a))$
$$(\nu y)z\langle y\rangle.(P_y \mid z(a).Q_{az})$$

$\otimes$R-$\exists$R-**1** No conversion for $\otimes\mathsf{R}\ (\exists\mathsf{R}\ M\ D_1)\ D_2$
$(\nu y)z\langle y\rangle.(y\langle M\rangle.P_y \mid Q_z)$

$\otimes$R-$\exists$R-**2** No conversion for $\otimes\mathsf{R}\ D_1\ (\exists\mathsf{R}\ M\ D_2)$
$(\nu y)z\langle y\rangle.(P_y \mid z\langle M\rangle.Q_z)$

$\otimes$R-$\multimap$L-**1** $\otimes\mathsf{R}\ (\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x))\ D_3 \Leftrightarrow \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_2\, x)\ D_3)$
$(\nu y)z\langle y\rangle.((\nu w)x\langle w\rangle.(P_w \mid Q_{xy}) \mid R_z) \Leftrightarrow (\nu w)x\langle w\rangle.(P_w \mid (\nu y)z\langle y\rangle.(Q_{xy} \mid R_z))$

$\otimes$R-$\multimap$L-**2** $\otimes\mathsf{R}\ D_1\ (\multimap\mathsf{L}\ x\ D_2\ (\hat{\lambda}x.\, D_3\, x)) \Leftrightarrow \multimap\mathsf{L}\ x\ D_2\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_3\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid (\nu w)x\langle w\rangle.(Q_w \mid R_{xz})) \Leftrightarrow (\nu w)x\langle w\rangle.(Q_w \mid (\nu y)z\langle y\rangle.(P_y \mid R_{xz}))$

$\otimes$R-$\otimes$L-**1** $\otimes\mathsf{R}\ (\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, D_1\, w\, x))\ D_2 \Leftrightarrow \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_1\, w\, x)\ D_2)$
$(\nu y)z\langle y\rangle.(x(w).P_{wxy} \mid Q_z) \Leftrightarrow x(w).(\nu y)z\langle y\rangle.(P_{wxy} \mid Q_z)$

$\otimes$R-$\otimes$L-**2** $\otimes\mathsf{R}\ D_1\ (\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, D_2\, w\, x)) \Leftrightarrow \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_2\, w\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid x(w).Q_{wxz}) \Leftrightarrow x(w).(\nu y)z\langle y\rangle.(P_y \mid Q_{wxz})$

$\otimes$R-**1**L-**1** $\otimes\mathsf{R}\ (\mathbf{1}\mathsf{L}\ x\ D_1)\ D_2 \Leftrightarrow \mathbf{1}\mathsf{L}\ x\ (\otimes\mathsf{R}\ D_1\ D_2)$
$(\nu y)z\langle y\rangle.(x().P_y \mid Q_z) \Leftrightarrow x().(\nu y)z\langle y\rangle.(P_y \mid Q_z)$

$\otimes$R-**1**L-**2** $\otimes\mathsf{R}\ D_1\ (\mathbf{1}\mathsf{L}\ x\ D_2) \Leftrightarrow \mathbf{1}\mathsf{L}\ x\ (\otimes\mathsf{R}\ D_1\ D_2)$
$(\nu y)z\langle y\rangle.(P_y \mid x().Q_z) \Leftrightarrow x().(\nu y)z\langle y\rangle.(P_y \mid Q_z)$

$\otimes$R-!L-**1** $\otimes\mathsf{R}\ (!\mathsf{L}\ x\ (\lambda u.\, D_1\, u))\ D_2 \Leftrightarrow !\mathsf{L}\ x\ (\lambda u.\, \otimes\mathsf{R}\ (D_1\, u)\ D_2)$
$(\nu y)z\langle y\rangle.(x/u.P_{uy} \mid Q_z) \Leftrightarrow x/u.(\nu y)z\langle y\rangle.(P_{uy} \mid Q_z)$

$\otimes$R-!L-**2** $\otimes\mathsf{R}\ D_1\ (!\mathsf{L}\ x\ (\lambda u.\, D_2\, u)) \Leftrightarrow !\mathsf{L}\ x\ (\lambda u.\, \otimes\mathsf{R}\ D_1\ (D_2\, u))$
$(\nu y)z\langle y\rangle.(P_y \mid x/u.Q_{uz}) \Leftrightarrow x/u.(\nu y)z\langle y\rangle.(P_y \mid Q_{uz})$

$\otimes$R-copy-**1** $\otimes\mathsf{R}\ (\mathsf{copy}\ u\ (\hat{\lambda}x.\, D_1\, u\, x))\ D_2 \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_1\, u\, x)\ D_2)$
$(\nu y)z\langle y\rangle.((\nu x)u\langle x\rangle.P_{uxy} \mid Q_z) \Leftrightarrow (\nu x)u\langle x\rangle.(\nu y)z\langle y\rangle.(P_{uxy} \mid Q_z)$

$\otimes$R-copy-**2** $\otimes\mathsf{R}\ D_1\ (\mathsf{copy}\ u\ (\hat{\lambda}x.\, D_2\, u\, x)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_2\, u\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid (\nu x)u\langle x\rangle.Q_{uxz}) \Leftrightarrow (\nu x)u\langle x\rangle.(\nu y)z\langle y\rangle.(P_y \mid Q_{uxz})$

$\otimes$R-$\forall$L-**1** $\otimes\mathsf{R}\ (\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, D_1\, x))\ D_2 \Leftrightarrow \forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_1\, x)\ D_2)$
$(\nu y)z\langle y\rangle.(x\langle M\rangle.P_{xy} \mid Q_z) \Leftrightarrow x\langle M\rangle.(\nu y)z\langle y\rangle.(P_{xy} \mid Q_z)$

$\otimes$R-$\forall$L-**2** $\otimes\mathsf{R}\ D_1\ (\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, D_2\, x)) \Leftrightarrow \forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_2\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid x\langle M\rangle.Q_{xz}) \Leftrightarrow x\langle M\rangle.(\nu y)z\langle y\rangle.(P_y \mid Q_{xz})$

$\otimes$R-$\exists$L-**1** $\otimes\mathsf{R}\ (\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, D_1\, a\, x))\ D_2 \Leftrightarrow \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_1\, a\, x)\ D_2)$
$(\nu y)z\langle y\rangle.(x(a).P_{axy} \mid Q_z) \Leftrightarrow x(a).(\nu y)z\langle y\rangle.(P_{axy} \mid Q_z)$

$\otimes$R-$\exists$L-**2** $\otimes\mathsf{R}\ D_1\ (\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, D_2\, a\, x)) \Leftrightarrow \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_2\, a\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid x(a).Q_{axz}) \Leftrightarrow x(a).(\nu y)z\langle y\rangle.(P_y \mid Q_{axz})$

$\otimes$R-cut-**1** $\otimes\mathsf{R}\ (\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, D_2\, x))\ D_3 \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_2\, x)\ D_3)$
$(\nu y)z\langle y\rangle.((\nu x)(P_x \mid Q_{xy}) \mid R_z) \Leftrightarrow (\nu x)(P_x \mid (\nu y)z\langle y\rangle.(Q_{xy} \mid R_z))$

⊗R-cut-**2** $\otimes\mathsf{R}\ D_1\ (\mathsf{cut}\ D_2\ (\hat{\lambda}x.\, D_3\, x)) \Leftrightarrow \mathsf{cut}\ D_2\ (\hat{\lambda}x.\, \otimes\mathsf{R}\ D_1\ (D_3\, x))$
$(\nu y)z\langle y\rangle.(P_y \mid (\nu x)(Q_x \mid R_{xz})) \Leftrightarrow (\nu x)(Q_x \mid (\nu y)z\langle y\rangle.(P_y \mid R_{xz}))$

⊗R-init-**1** No conversion for $\otimes\mathsf{R}\ (\mathsf{init}\ x)\ D$
$(\nu y)z\langle y\rangle.([x \leftrightarrow y] \mid P_z)$

⊗R-init-**2** No conversion for $\otimes\mathsf{R}\ D\ (\mathsf{init}\ x)$
$(\nu y)z\langle y\rangle.(P_y \mid [x \leftrightarrow z])$

⊗R-cut$^!$-**1** $\otimes\mathsf{R}\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ D_3 \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \otimes\mathsf{R}\ (D_2\, u)\ D_3)$
$(\nu y)z\langle y\rangle.((\nu u)(!u(x).P_x \mid Q_{uy}) \mid R_z) \Leftrightarrow (\nu u)(!u(x).P_x \mid (\nu y)z\langle y\rangle.(Q_{uy} \mid R_z))$

⊗R-cut$^!$-**2** $\otimes\mathsf{R}\ D_1\ (\mathsf{cut}^!\ D_2\ (\lambda u.\, D_3\, u)) \Leftrightarrow \mathsf{cut}^!\ D_2\ (\lambda u.\, \otimes\mathsf{R}\ D_1\ (D_3\, u))$
$(\nu y)z\langle y\rangle.(P_y \mid (\nu u)(!u(x).Q_x \mid R_{uz})) \Leftrightarrow (\nu u)(!u(x).Q_x \mid (\nu y)z\langle y\rangle.(P_y \mid R_{uz}))$

2.3 **1**R

There are no cases here because **1**R has no subterms. (However, there will be cases involving **1**R as a secondary constructor—⊸R-**1**R and cut$^!$-**1**R-2, for example.)

2.4 !R

!R-⊸R No conversion for $!\mathsf{R}\ (\multimap\mathsf{R}\ (\hat{\lambda}x.\, D\, x))$
$!z(y).y(x).P_{xy}$

!R-⊗R No conversion for $!\mathsf{R}\ (\otimes\mathsf{R}\ D_1\ D_2)$
$!z(y).(\nu x)y\langle x\rangle.(P_x \mid Q_y)$

!R-**1**R No conversion for $!\mathsf{R}\ \mathbf{1}\mathsf{R}$
$!z(y).y\langle\rangle.\mathbf{0}$

!R-!R No conversion for $!\mathsf{R}\ (!\mathsf{R}\ D)$
$!z(y).!y(x).P_x$

!R-∀R No conversion for $!\mathsf{R}\ (\forall\mathsf{R}\ (\lambda a.\, D\, a))$
$!z(y).y(a).P_{ay}$

!R-∃R No conversion for $!\mathsf{R}\ (\exists\mathsf{R}\ M\ D)$
$!z(y).y\langle M\rangle.P_y$

!R-⊸L !R $(\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x))$ is not a well-formed proof term.

!R-⊗L !R $(\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, D\, w\, x))$ is not a well-formed proof term.

!R-**1**L !R $(\mathbf{1}\mathsf{L}\ x\ D)$ is not a well-formed proof term.

!R-!L !R $(!\mathsf{L}\ x\ (\lambda u.\, D\, u))$ is not a well-formed proof term.

!R-copy No conversion for $!\mathsf{R}\ (\mathsf{copy}\ u\ (\hat{\lambda}x.\, D\, u\, x))$
$!z(y).(\nu x)u\langle x\rangle.P_{uxy}$

!R-∀L !R $(\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, D\, x))$ is not a well-formed proof term.

!R-$\exists$L $\mathsf{!R}\ (\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, D\, a\, x))$ is not a well-formed proof term.

!R-cut No conversion for $\mathsf{!R}\ (\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, D_2\, x))$
$$!z(y).(\nu x)(P_x \mid Q_{xy})$$

!R-init $\mathsf{!R}\ (\mathsf{init}\ x)$ is not a well-formed proof term.

The following case is noteworthy because it allows us to lift the definition of a server out of another server's body.

!R-cut$^!$ $\mathsf{!R}\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \mathsf{!R}\ (D_2\, u))$
$$!z(y).(\nu u)(!u(x).P_x \mid Q_{uy}) \Leftrightarrow (\nu u)(!u(x).P_x \mid !z(y).Q_{uy})$$

2.5 $\forall$R

$\forall$R-$\multimap$R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \multimap\mathsf{R}\ (\hat{\lambda}x.\, D\, a\, x))$
$$z(a).z(x).P_{axz}$$

$\forall$R-$\otimes$R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \otimes\mathsf{R}\ (D_1\, a)\ (D_2\, a))$
$$z(a).(\nu x)z\langle x\rangle.(P_{ax} \mid Q_{az})$$

$\forall$R-**1**R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \mathbf{1}\mathsf{R})$
$$z(a).z\langle\rangle.\mathbf{0}$$

$\forall$R-!R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \mathsf{!R}\ (D\, a))$
$$z(a).!z(x).P_{ax}$$

$\forall$R-$\forall$R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \forall\mathsf{R}\ (\lambda b.\, D\, a\, b))$
$$z(a).z(b).P_{abz}$$

In the following case, the process has type $\forall x.\exists y.\, A(x, y)$. This is a case where we could apply Skolemization to get $\exists f.\forall x.\, A(x, f\, x)$. However, this would not be a commuting conversion because it changes the type of the process.

$\forall$R-$\exists$R No conversion for $\forall\mathsf{R}\ (\lambda a.\, \exists\mathsf{R}\ (M\, a)\ (D\, a))$
$$z(a).z\langle M\, a\rangle.P_{az}$$

The other cases are similar to ones described above.

$\forall$R-$\multimap$L-**1** No conversion for $\forall\mathsf{R}\ (\lambda a.\, \multimap\mathsf{L}\ x\ (D_1\, a)\ (\hat{\lambda}x.\, D_2\, a\, x))$ when the first occurrence of a is strict.
$$z(a).(\nu w)x\langle w\rangle.(P_{aw} \mid Q_{axz})$$

$\forall$R-$\multimap$L-**2** $\forall\mathsf{R}\ (\lambda a.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, a\, x)) \Leftrightarrow \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \forall\mathsf{R}\ (\lambda a.\, D_2\, a\, x))$
$$z(a).(\nu w)x\langle w\rangle.(P_w \mid Q_{axz}) \Leftrightarrow (\nu w)x\langle w\rangle.(P_w \mid z(a).Q_{axz})$$

$\forall$R-$\otimes$L $\forall\mathsf{R}\ (\lambda a.\, \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, D\, a\, w\, x)) \Leftrightarrow \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, \forall\mathsf{R}\ (\lambda a.\, D\, a\, w\, x))$
$$z(a).x(w).P_{awxz} \Leftrightarrow x(w).z(a).P_{awxz}$$

$\forall$R-**1**L $\forall\mathsf{R}\ (\lambda a.\, \mathbf{1}\mathsf{L}\ x\ (D\, a)) \Leftrightarrow \mathbf{1}\mathsf{L}\ x\ (\forall\mathsf{R}\ (\lambda a.\, D\, a))$
$$z(a).x().P_{az} \Leftrightarrow x().z(a).P_{az}$$

$\forall$R-!L $\forall\mathsf{R}\ (\lambda a.\, \mathsf{!L}\ x\ (\lambda u.\, D\, a\, u)) \Leftrightarrow \mathsf{!L}\ x\ (\lambda u.\, \forall\mathsf{R}\ (\lambda a.\, D\, a\, u))$
$$z(a).x/u.P_{auz} \Leftrightarrow x/u.z(a).P_{auz}$$

$\forall$R-copy $\forall\mathsf{R}\ (\lambda a.\, \mathsf{copy}\ u\ (\hat{\lambda} x.\, D\, a\, x)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda} x.\, \forall\mathsf{R}\ (\lambda a.\, D\, a\, x))$

$$z(a).(\nu x)u\langle x\rangle.P_{uaxz} \Leftrightarrow (\nu x)u\langle x\rangle.z(a).P_{uaxz}$$

$\forall$R-$\forall$L-**1** No conversion for $\forall\mathsf{R}\ (\lambda a.\, \forall\mathsf{L}\ x\ (M\, a)\ (\hat{\lambda} x.\, D\, a\, x))$ when the first occurrence of a is strict.

$$z(a).x\langle M\, a\rangle.P_{axz}$$

$\forall$R-$\forall$L-**2** $\forall\mathsf{R}\ (\lambda a.\, \forall\mathsf{L}\ x\ M\ (\hat{\lambda} x.\, D\, a\, x)) \Leftrightarrow \forall\mathsf{L}\ x\ M\ (\hat{\lambda} x.\, \forall\mathsf{R}\ (\lambda a.\, D\, a\, x))$

$$z(a).x\langle M\rangle.P_{axz} \Leftrightarrow x\langle M\rangle.z(a).P_{axz}$$

$\forall$R-$\exists$L $\forall\mathsf{R}\ (\lambda a.\, \exists\mathsf{L}\ x\ (\lambda b.\hat{\lambda} x.\, D\, a\, b\, x)) \Leftrightarrow \exists\mathsf{L}\ x\ (\lambda b.\hat{\lambda} x.\, \forall\mathsf{R}\ (\lambda a.\, D\, a\, b\, x))$

$$z(a).x(b).P_{abxz} \Leftrightarrow x(b).z(a).P_{abxz}$$

$\forall$R-cut-**1** No conversion for $\forall\mathsf{R}\ (\lambda a.\, \mathsf{cut}\ (D_1\, a)\ (\hat{\lambda} x.\, D_2\, a\, x))$ when the first occurrence of a is strict.

$$z(a).(\nu x)(P_{ax} \mid Q_{axz})$$

$\forall$R-cut-**2** $\forall\mathsf{R}\ (\lambda a.\, \mathsf{cut}\ D_1\ (\hat{\lambda} x.\, D_2\, a\, x)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda} x.\, \forall\mathsf{R}\ (\lambda a.\, D_2\, a\, x))$

$$z(a).(\nu x)(P_x \mid Q_{axz}) \Leftrightarrow (\nu x)(P_x \mid z(a).Q_{axz})$$

$\forall$R-init No conversion for $\forall\mathsf{R}\ (\lambda a.\, \mathsf{init}\ x)$

$$z(a).[x \leftrightarrow z]$$

$\forall$R-cut$^!$-**1** No conversion for $\forall\mathsf{R}\ (\lambda a.\, \mathsf{cut}^!\ (D_1\, a)\ (\lambda u.\, D_2\, a\, u))$ when the first occurrence of a is strict.

$$z(a).(\nu u)(!u(x).P_{ax} \mid Q_{auz})$$

$\forall$R-cut$^!$-**2** $\forall\mathsf{R}\ (\lambda a.\, \mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, a\, u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \forall\mathsf{R}\ (\lambda a.\, D_2\, a\, u))$

$$z(a).(\nu u)(!u(x).P_x \mid Q_{auz}) \Leftrightarrow (\nu u)(!u(x).P_x \mid z(a).Q_{auz})$$

2.6 $\exists$R

$\exists$R-$\multimap$R No conversion for $\exists\mathsf{R}\ M\ (\multimap\mathsf{R}\ (\hat{\lambda} x.\, D\, x))$

$$z\langle M\rangle.z(x).P_{xz}$$

$\exists$R-$\otimes$R No conversion for $\exists\mathsf{R}\ M\ (\otimes\mathsf{R}\ D_1\ D_2)$

$$z\langle M\rangle.(\nu x)z\langle x\rangle.(P_x \mid Q_z)$$

$\exists$R-**1**R No conversion for $\exists\mathsf{R}\ M\ \mathbf{1}\mathsf{R}$

$$z\langle M\rangle.z\langle\rangle.\mathbf{0}$$

$\exists$R-!R No conversion for $\exists\mathsf{R}\ M\ (!\mathsf{R}\ D)$

$$z\langle M\rangle.!z(x).P_x$$

$\exists$R-$\forall$R No conversion for $\exists\mathsf{R}\ M\ (\forall\mathsf{R}\ (\lambda a.\, D\, a))$

$$z\langle M\rangle.z(a).P_{az}$$

$\exists$R-$\exists$R No conversion for $\exists\mathsf{R}\ M_1\ (\exists\mathsf{R}\ M_2\ D)$

$$z\langle M_1\rangle.z\langle M_2\rangle.P_z$$

$\exists$R-$\multimap$L $\exists\mathsf{R}\ M\ (\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda} x.\, D_2\, x)) \Leftrightarrow \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda} x.\, \exists\mathsf{R}\ M\ (D_2\, x))$

$$z\langle M\rangle.(\nu w)x\langle w\rangle.(P_w \mid Q_{xz}) \Leftrightarrow (\nu w)x\langle w\rangle.(P_w \mid z\langle M\rangle.Q_{xz})$$

$\exists$R-$\otimes$L $\exists\mathsf{R}\ M\ (\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ D\,w\,x)) \Leftrightarrow \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ \exists\mathsf{R}\ M\ (D\,w\,x))$
$z\langle M\rangle.x(w).P_{wxz} \Leftrightarrow x(w).z\langle M\rangle.P_{wxz}$

$\exists$R-**1**L $\exists\mathsf{R}\ M\ (\mathbf{1}\mathsf{L}\ x\ D) \Leftrightarrow \mathbf{1}\mathsf{L}\ x\ (\exists\mathsf{R}\ M\ D)$
$z\langle M\rangle.x().P_z \Leftrightarrow x().z\langle M\rangle.P_z$

$\exists$R-!L $\exists\mathsf{R}\ M\ (!\mathsf{L}\ x\ (\lambda u.\ D\,u)) \Leftrightarrow !\mathsf{L}\ x\ (\lambda u.\ \exists\mathsf{R}\ M\ (D\,u))$
$z\langle M\rangle.x/u.P_{uz} \Leftrightarrow x/u.z\langle M\rangle.P_{uz}$

$\exists$R-copy $\exists\mathsf{R}\ M\ (\mathsf{copy}\ u\ (\hat{\lambda}x.\ D\,x)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}x.\ \exists\mathsf{R}\ M\ (D\,x))$
$z\langle M\rangle.(\nu x)u\langle x\rangle.P_{uxz} \Leftrightarrow (\nu x)u\langle x\rangle.z\langle M\rangle.P_{uxz}$

$\exists$R-$\forall$L $\exists\mathsf{R}\ M_1\ (\forall\mathsf{L}\ x\ M_2\ (\hat{\lambda}x.\ D\,x)) \Leftrightarrow \forall\mathsf{L}\ x\ M_2\ (\hat{\lambda}x.\ \exists\mathsf{R}\ M_1\ (D\,x))$
$z\langle M_1\rangle.x\langle M_2\rangle.P_{xz} \Leftrightarrow x\langle M_2\rangle.z\langle M_1\rangle.P_{xz}$

$\exists$R-$\exists$L $\exists\mathsf{R}\ M\ (\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D\,a\,x)) \Leftrightarrow \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \exists\mathsf{R}\ M\ (D\,a\,x))$
$z\langle M\rangle.x(a).P_{axz} \Leftrightarrow x(a).z\langle M\rangle.P_{axz}$

$\exists$R-cut $\exists\mathsf{R}\ M\ (\mathsf{cut}\ D_1\ (\hat{\lambda}x.\ D_2\,x)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}x.\ \exists\mathsf{R}\ M\ (D_2\,x))$
$z\langle M\rangle.(\nu x)(P_x \mid Q_{xz}) \Leftrightarrow (\nu x)(P_x \mid z\langle M\rangle.Q_{xz})$

$\exists$R-init No conversion for $\exists\mathsf{R}\ M\ (\mathsf{init}\ x)$
$z\langle M\rangle.[x \leftrightarrow z]$

$\exists$R-cut$^!$ $\exists\mathsf{R}\ M\ (\mathsf{cut}^!\ D_1\ (\lambda u.\ D_2\,u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\ \exists\mathsf{R}\ M\ (D_2\,u))$
$z\langle M\rangle.(\nu u)(!u(x).P_x \mid Q_{uz}) \Leftrightarrow (\nu u)(!u(x).P_x \mid z\langle M\rangle.Q_{uz})$

2.7 $\multimap$L

$\multimap$L-$\multimap$R-**1** No conversion for $\multimap\mathsf{L}\ x\ (\multimap\mathsf{R}\ (\hat{\lambda}y.\ D_1\,y))\ (\hat{\lambda}x.\ D_2\,x)$
$(\nu w)x\langle w\rangle.(w(y).P_{wy} \mid Q_{xz})$

$\multimap$L-$\multimap$R-**2** Refer to $\multimap$R-$\multimap$L-3.

$\multimap$L-$\otimes$R-**1** No conversion for $\multimap\mathsf{L}\ x\ (\otimes\mathsf{R}\ D_1\ D_2)\ (\hat{\lambda}x.\ D_3\,x)$
$(\nu w)x\langle w\rangle.((\nu y)w\langle y\rangle.(P_y \mid Q_w) \mid R_{xz})$

$\multimap$L-$\otimes$R-**2** Refer to $\otimes$R-$\multimap$L-1.

$\multimap$L-$\otimes$R-**3** Refer to $\otimes$R-$\multimap$L-2.

$\multimap$L-**1**R-**1** No conversion for $\multimap\mathsf{L}\ x\ \mathbf{1}\mathsf{R}\ (\hat{\lambda}x.\ D\,x)$
$(\nu w)x\langle w\rangle.(w\langle\rangle.\mathbf{0} \mid P_{xz})$

$\multimap$L-**1**R-**2** $\multimap\mathsf{L}\ x\ D\ (\hat{\lambda}x.\ \mathbf{1}\mathsf{R})$ is not a well-formed proof term.

$\multimap$L-!R-**1** No conversion for $\multimap\mathsf{L}\ x\ (!\mathsf{R}\ D_1)\ (\hat{\lambda}x.\ D_2\,x)$
$(\nu w)x\langle w\rangle.(!w(y).P_y \mid Q_{xz})$

$\multimap$L-!R-**2** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\ !\mathsf{R}\ D_2)$ is not a well-formed proof term.

$\multimap$L-$\forall$R-**1** No conversion for $\multimap\mathsf{L}\ x\ (\forall\mathsf{R}\ (\lambda a.\ D_1\,a))\ (\hat{\lambda}x.\ D_2\,x)$
$(\nu w)x\langle w\rangle.(w(a).P_{aw} \mid Q_{xz})$

$\multimap$L-$\forall$R-**2** Refer to $\forall$R-$\multimap$L-2.

$\multimap$L-$\exists$R-**1** No conversion for $\multimap\mathsf{L}\ x\ (\exists\mathsf{R}\ M\ D_1)\ (\hat{\lambda}x.\, D_2\, x)$
$(\nu w)x\langle w\rangle.(w\langle M\rangle.P_w \mid Q_{xz})$

$\multimap$L-$\exists$R-**2** Refer to $\exists$R-$\multimap$L.

$\multimap$L-$\multimap$L-**1** $\multimap\mathsf{L}\ x\ (\multimap\mathsf{L}\ y\ D_1\ (\hat{\lambda}y.\, D_2\, y))\ (\hat{\lambda}x.\, D_3\, x) \Leftrightarrow \multimap\mathsf{L}\ y\ D_1\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_2\, y)\ (\hat{\lambda}x.\, D_3\, x))$
$(\nu w)x\langle w\rangle.((\nu v)y\langle v\rangle.(P_v \mid Q_{yw}) \mid R_{xz}) \Leftrightarrow (\nu v)y\langle v\rangle.(P_v \mid (\nu w)x\langle w\rangle.(Q_{yw} \mid R_{xz}))$

$\multimap$L-$\multimap$L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \multimap\mathsf{L}\ x\ D_2\ (\hat{\lambda}x.\, D_3\, x))$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu y)x\langle y\rangle.(Q_y \mid R_{xz}))$

$\multimap$L-$\multimap$L-**3** Refer to $\multimap$L-$\multimap$L-1.

$\multimap$L-$\multimap$L-**4** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \multimap\mathsf{L}\ y\ D_2\ (\hat{\lambda}y.\, D_3\, x\, y)) \Leftrightarrow \multimap\mathsf{L}\ y\ D_2\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_3\, x\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu v)y\langle v\rangle.(Q_v \mid R_{xyz})) \Leftrightarrow (\nu v)y\langle v\rangle.(Q_v \mid (\nu w)x\langle w\rangle.(P_w \mid R_{xyz}))$

$\multimap$L-$\otimes$L-**1** $\multimap\mathsf{L}\ x\ (\otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\, D_1\, v\, y))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_1\, v\, y)\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(y(v).P_{vyw} \mid Q_{xz}) \Leftrightarrow y(v).(\nu w)x\langle w\rangle.(P_{vyw} \mid Q_{xz})$

$\multimap$L-$\otimes$L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \otimes\mathsf{L}\ x\ (\hat{\lambda}y.\hat{\lambda}x.\, D_2\, y\, x))$
$(\nu w)x\langle w\rangle.(P_w \mid x(y).Q_{yxz})$

$\multimap$L-$\otimes$L-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\, D_2\, x\, v\, y)) \Leftrightarrow \otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x\, v\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid y(v).Q_{xvyz}) \Leftrightarrow y(v).(\nu w)x\langle w\rangle.(P_w \mid Q_{xvyz})$

$\multimap$L-**1**L-**1** $\multimap\mathsf{L}\ x\ (\mathbf{1}\mathsf{L}\ y\ D_1)\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \mathbf{1}\mathsf{L}\ y\ (\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(y().P_w \mid Q_{xz}) \Leftrightarrow y().(\nu w)x\langle w\rangle.(P_w \mid Q_{xz})$

$\multimap$L-**1**L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathbf{1}\mathsf{L}\ x\ D_2)$
$(\nu w)x\langle w\rangle.(P_w \mid x().Q_z)$

$\multimap$L-**1**L-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathbf{1}\mathsf{L}\ y\ (D_2\, x)) \Leftrightarrow \mathbf{1}\mathsf{L}\ y\ (\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(P_w \mid y().Q_{xz}) \Leftrightarrow y().(\nu w)x\langle w\rangle.(P_w \mid Q_{xz})$

$\multimap$L-!L-**1** $\multimap\mathsf{L}\ x\ (!\mathsf{L}\ y\ (\lambda u.\, D_1\, u))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow !\mathsf{L}\ y\ (\lambda u.\, \multimap\mathsf{L}\ x\ (D_1\, u)\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(y/u.P_{uw} \mid Q_{xz}) \Leftrightarrow y/u.(\nu w)x\langle w\rangle.(P_{uw} \mid Q_{xz})$

$\multimap$L-!L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, !\mathsf{L}\ x\ (\lambda u.\, D_2\, u))$
$(\nu w)x\langle w\rangle.(P_w \mid x/u.Q_{uz})$

$\multimap$L-!L-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, !\mathsf{L}\ y\ (\lambda u.\, D_2\, x\, u)) \Leftrightarrow !\mathsf{L}\ y\ (\lambda u.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x\, u))$
$(\nu w)x\langle w\rangle.(P_w \mid y/u.Q_{xuz}) \Leftrightarrow y/u.(\nu w)x\langle w\rangle.(P_w \mid Q_{xuz})$

$\multimap$L-copy-**1** $\multimap\mathsf{L}\ x\ (\mathsf{copy}\ u\ (\hat{\lambda}y.\, D_1\, u\, y))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_1\, u\, y)\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.((\nu y)u\langle y\rangle.P_{uyw} \mid Q_{xz}) \Leftrightarrow (\nu y)u\langle y\rangle.(\nu w)x\langle w\rangle.(P_{uyw} \mid Q_{xz})$

$\multimap$L-copy-**2** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathsf{copy}\ u\ (\hat{\lambda}y.\, D_2\, x\, u\, y)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x\, u\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu y)u\langle y\rangle.Q_{xuyz}) \Leftrightarrow (\nu w)x\langle w\rangle.(P_w \mid (\nu y)u\langle y\rangle.Q_{xuyz})$

$\multimap$L-$\forall$L-**1** $\multimap\mathsf{L}\ x\ (\forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, D_1\, y))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_1\, y)\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(y\langle M\rangle.P_{yw} \mid Q_{xz}) \Leftrightarrow y\langle M\rangle.(\nu w)y\langle w\rangle.(P_{yw} \mid Q_{xz})$

$\multimap$L-$\forall$L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(P_w \mid x\langle M\rangle.Q_{xz})$

$\multimap$L-$\forall$L-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, D_2\, x\, y)) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid y\langle M\rangle.Q_{xyz}) \Leftrightarrow y\langle M\rangle.(\nu w)x\langle w\rangle.(P_w \mid Q_{xyz})$

$\multimap$L-$\exists$L-**1** $\multimap\mathsf{L}\ x\ (\exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\, D_1\, a\, y))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_1\, a\, y)\ (\hat{\lambda}x.\, D_2\, x))$
$(\nu w)x\langle w\rangle.(y(a).P_{ayw} \mid Q_{xz}) \Leftrightarrow y(a).(\nu w)y\langle w\rangle.(P_{ayw} \mid Q_{xz})$

$\multimap$L-$\exists$L-**2** No conversion for $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\, D_2\, a\, x))$
$(\nu w)x\langle w\rangle.(P_w \mid x(a).Q_{axz})$

$\multimap$L-$\exists$L-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\, D_2\, x\, a\, y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x\, a\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid y(a).Q_{xayz}) \Leftrightarrow y(a).(\nu w)x\langle w\rangle.(P_w \mid Q_{xayz})$

$\multimap$L-cut-**1** $\multimap\mathsf{L}\ x\ (\mathsf{cut}\ D_1\ (\hat{\lambda}y.\, D_2\, y))\ (\hat{\lambda}x.\, D_3\, x) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ (D_2\, y)\ (\hat{\lambda}x.\, D_3\, x))$
$(\nu w)x\langle w\rangle.((\nu y)(P_y \mid Q_{yw}) \mid R_{xz}) \Leftrightarrow (\nu y)(P_y \mid (\nu w)x\langle w\rangle.(Q_{yw} \mid R_{xz}))$

$\multimap$L-cut-**2** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}\ (D_2\, x)\ (\hat{\lambda}y.\, D_3\, y)) \Leftrightarrow \mathsf{cut}\ (\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_2\, x))\ (\hat{\lambda}y.\, D_3\, y)$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu y)(Q_{xy} \mid R_{yz})) \Leftrightarrow (\nu y)((\nu w)x\langle w\rangle.(P_w \mid Q_{xy}) \mid R_{yz})$

$\multimap$L-cut-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}\ D_2\ (\hat{\lambda}y.\, D_3\, x\, y)) \Leftrightarrow \mathsf{cut}\ D_2\ (\hat{\lambda}y.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_3\, x\, y))$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu y)(Q_y \mid R_{xyz})) \Leftrightarrow (\nu y)(Q_y \mid (\nu w)x\langle w\rangle.(P_w \mid R_{xyz}))$

$\multimap$L-init-**1** No conversion for $\multimap\mathsf{L}\ x\ (\mathsf{init}\ y)\ (\hat{\lambda}x.\, D\, x)$
$(\nu w)x\langle w\rangle.([y \leftrightarrow w] \mid P_{xz})$

$\multimap$L-init-**2** No conversion for $\multimap\mathsf{L}\ x\ D\ (\hat{\lambda}x.\, \mathsf{init}\ x)$
$(\nu w)x\langle w\rangle.(P_w \mid [x \leftrightarrow z])$

$\multimap$L-init-**3** $\multimap\mathsf{L}\ x\ D\ (\hat{\lambda}x.\, \mathsf{init}\ y)$ is not a well-formed proof term.

$\multimap$L-cut$^!$-**1** $\multimap\mathsf{L}\ x\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\hat{\lambda}x.\, D_3\, x) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \multimap\mathsf{L}\ x\ (D_2\, u)\ (\hat{\lambda}x.\, D_3\, x))$
$(\nu w)x\langle w\rangle.((\nu u)(!u(y).P_y \mid Q_{uw}) \mid R_{xz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid (\nu w)x\langle w\rangle.(Q_{uw} \mid R_{xz}))$

$\multimap$L-cut$^!$-**2** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}^!\ (D_2\, x)\ (\lambda u.\, D_3\, u))$ is not a well-formed proof term.

$\multimap$L-cut$^!$-**3** $\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}^!\ D_2\ (\lambda u.\, D_3\, x\, u)) \Leftrightarrow \mathsf{cut}^!\ D_2\ (\lambda u.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\, D_3\, x\, u))$
$(\nu w)x\langle w\rangle.(P_w \mid (\nu u)(!u(y).Q_y \mid R_{xuz})) \Leftrightarrow (\nu u)(!u(y).Q_y \mid (\nu w)x\langle w\rangle.(P_w \mid R_{xuz}))$

2.8 $\otimes$L

$\otimes$L-$\multimap$R Refer to $\multimap$R-$\otimes$L-2.

$\otimes$L-$\otimes$R-**1** Refer to $\otimes$R-$\otimes$L-1.

$\otimes$L-$\otimes$R-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, \otimes\mathsf{R}\ (D_1\, w)\ (D_2\, x))$
$x(w).(\nu y)z\langle y\rangle.(P_{wy} \mid Q_{xz})$

$\otimes$L-$\otimes$R-**3** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\otimes\mathsf{R}\ (D_1\,x)\ (D_2\,w))$
$x(w).(\nu y)z\langle y\rangle.(P_{xy} \mid Q_{wz})$

$\otimes$L-$\otimes$R-**4** Refer to $\otimes$R-$\otimes$L-2.

$\otimes$L-**1**R $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ \mathbf{1}\mathsf{R})$ is not a well-formed proof term.

$\otimes$L-!R $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ !\mathsf{R}\ (D\,w\,x))$ is not a well-formed proof term.

$\otimes$L-$\forall$R Refer to $\forall$R-$\otimes$L.

$\otimes$L-$\exists$R Refer to $\exists$R-$\otimes$L.

$\otimes$L-$\multimap$L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ w\ (D_1\,x)\ (\hat{\lambda}w.\,D_2\,w))$
$x(w).(\nu v)w\langle v\rangle.(P_{xv} \mid Q_{wz})$

$\otimes$L-$\multimap$L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ w\ D_1\ (\hat{\lambda}w.\,D_2\,x\,w))$
$x(w).(\nu v)w\langle v\rangle.(P_v \mid Q_{xwz})$

$\otimes$L-$\multimap$L-**3** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ x\ (D_1\,w)\ (\hat{\lambda}x.\,D_2\,x))$
$x(w).(\nu v)x\langle v\rangle.(P_{wv} \mid Q_{xz})$

$\otimes$L-$\multimap$L-**4** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\,D_2\,w\,x))$
$x(w).(\nu v)x\langle v\rangle.(P_v \mid Q_{wxz})$

$\otimes$L-$\multimap$L-**5** Refer to $\multimap$L-$\otimes$L-1.

$\otimes$L-$\multimap$L-**6** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ y\ (D_1\,w)\ (\hat{\lambda}y.\,D_2\,x\,y))$
$x(w).(\nu v)y\langle v\rangle.(P_{wv} \mid Q_{xyz})$

$\otimes$L-$\multimap$L-**7** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\multimap\mathsf{L}\ y\ (D_1\,x)\ (\hat{\lambda}y.\,D_2\,w\,y))$
$x(w).(\nu v)y\langle v\rangle.(P_{xv} \mid Q_{wyz})$

$\otimes$L-$\multimap$L-**8** Refer to $\multimap$L-$\otimes$L-3.

$\otimes$L-$\otimes$L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\otimes\mathsf{L}\ w\ (\hat{\lambda}y.\hat{\lambda}w.\,D\,x\,y\,w))$
$x(w).w(y).P_{xywz}$

$\otimes$L-$\otimes$L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\otimes\mathsf{L}\ x\ (\hat{\lambda}y.\hat{\lambda}x.\,D\,w\,y\,x))$
$x(w).x(y).P_{wyxz}$

$\otimes$L-$\otimes$L-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\,D\,w\,x\,v\,y)) \Leftrightarrow \otimes\mathsf{L}\ y\ (\hat{\lambda}v.\hat{\lambda}y.\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x\,v\,y))$
$x(w).y(v).P_{wxvyz} \Leftrightarrow y(v).x(w).P_{wxvyz}$

$\otimes$L-**1**L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ \mathbf{1}\mathsf{L}\ w\ (D\,x))$
$x(w).w().P_{xz}$

$\otimes$L-**1**L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ \mathbf{1}\mathsf{L}\ x\ (D\,w))$
$x(w).x().P_{wz}$

$\otimes$L-**1**L-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\ \mathbf{1}\mathsf{L}\ y\ (D\,w\,x)) \Leftrightarrow \mathbf{1}\mathsf{L}\ y\ (\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x))$
$x(w).y().P_{wxz} \Leftrightarrow y().x(w).P_{wxz}$

⊗L-!L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,!\mathsf{L}\ w\ (\lambda u.\,D\,x\,u))$
$$x(w).w/u.P_{xuz}$$

⊗L-!L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,!\mathsf{L}\ x\ (\lambda u.\,D\,w\,u))$
$$x(w).x/u.P_{wuz}$$

⊗L-!L-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,!\mathsf{L}\ y\ (\lambda u.\,(D\,w\,x\,u))) \Leftrightarrow\ !\mathsf{L}\ y\ (\lambda u.\,(\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x\,u)))$
$$x(w).y/u.P_{wxuz} \Leftrightarrow y/u.x(w).P_{wxuz}$$

⊗L-copy $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{copy}\ u\ (\hat{\lambda}y.\,D\,w\,x\,u\,y)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\,(\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x\,u\,y)))$
$$x(w).(\nu y)u\langle y\rangle.P_{wxuyz} \Leftrightarrow (\nu y)u\langle y\rangle.x(w).P_{wxuyz}$$

⊗L-∀L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\forall\mathsf{L}\ w\ M\ (\hat{\lambda}w.\,D\,x\,w))$
$$x(w).w\langle M\rangle.P_{xwz}$$

⊗L-∀L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,D\,w\,x))$
$$x(w).x\langle M\rangle.P_{wxz}$$

⊗L-∀L-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,D\,w\,x\,y)) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x\,y))$
$$x(w).y\langle M\rangle.P_{wxyz} \Leftrightarrow y\langle M\rangle.x(w).P_{wxyz}$$

⊗L-∃L-**1** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\exists\mathsf{L}\ w\ (\lambda a.\hat{\lambda}w.\,D\,x\,a\,w))$
$$x(w).w(a).P_{xawz}$$

⊗L-∃L-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\,D\,w\,a\,x))$
$$x(w).x(a).P_{waxz}$$

⊗L-∃L-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,D\,w\,x\,a\,y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D\,w\,x\,a\,y))$
$$x(w).y(a).P_{wxayz} \Leftrightarrow y(a).x(w).P_{wxayz}$$

⊗L-cut-**1** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}\ (D_1\,w\,x)\ (\hat{\lambda}y.\,D_2\,y)) \Leftrightarrow \mathsf{cut}\ (\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D_1\,w\,x))\ (\hat{\lambda}y.\,D_2\,y)$
$$x(w).(\nu y)(P_{wxy} \mid Q_{yz}) \Leftrightarrow (\nu y)(x(w).P_{wxy} \mid Q_{yz})$$

⊗L-cut-**2** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}\ (D_1\,w)\ (\hat{\lambda}y.\,D_2\,x\,y))$
$$x(w).(\nu y)(P_{wy} \mid Q_{xyz})$$

⊗L-cut-**3** No conversion for $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}\ (D_1\,x)\ (\hat{\lambda}y.\,D_2\,w\,y))$
$$x(w).(\nu y)(P_{xy} \mid Q_{wyz})$$

⊗L-cut-**4** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}\ D_1\ (\hat{\lambda}y.\,D_2\,w\,x\,y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\,\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D_2\,w\,x\,y))$
$$x(w).(\nu y)(P_y \mid Q_{wxyz}) \Leftrightarrow (\nu y)(P_y \mid x(w).Q_{wxyz})$$

⊗L-init-**1** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{init}\ w)$ is not a well-formed proof term.

⊗L-init-**2** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{init}\ x)$ is not a well-formed proof term.

⊗L-init-**3** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{init}\ y)$ is not a well-formed proof term.

⊗L-cut$^!$-**1** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}^!\ (D_1\,w\,x)\ (\lambda u.\,D_2\,u))$ is not a well-formed proof term.

⊗L-cut$^!$-**2** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}^!\ (D_1\,w)\ (\lambda u.\,D_2\,x\,u))$ is not a well-formed proof term.

$\otimes$L-cut$^!$-**2** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}^!\ (D_1\ x)\ (\lambda u.\,D_2\,w\,u))$ is not a well-formed proof term.

$\otimes$L-cut$^!$-**4** $\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,\mathsf{cut}^!\ D_1\ (\lambda u.\,D_2\,w\,x\,u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\,\otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\,D_2\,w\,x\,u))$

$$x(w).(\nu u)(!u(y).P_y \mid Q_{wxuz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid x(w).Q_{wxuz})$$

2.9 1L

1L-$\multimap$R Refer to $\multimap$R-**1L**-2.

1L-$\otimes$R**-1** Refer to $\otimes$R-**1L**-1.

1L-$\otimes$R**-2** Refer to $\otimes$R-**1L**-2.

The next two cases distinguish the process assignment that uses a silent **1L** from the process assignment that uses an explicit $x()$. In the former, $!z(y).P_y$ could immediately offer a replicated input action to the external environment, but, in the latter, the replicated input action can only be offered to the external environment once x terminates. We cannot postpone waiting on x to terminate because there is no commuting conversion here; !R (**1L** x D) is not a well-formed proof term. More comments on this issue appear at the end of this document.

1L-1R No conversion for $\mathbf{1}\mathsf{L}\ x\ \mathbf{1}\mathsf{R}$

$$x().z\langle\rangle.\mathbf{0}$$

1L-!R No conversion for $\mathbf{1}\mathsf{L}\ x\ (!\mathsf{R}\ D)$

$$x().!z(y).P_y$$

The following cases are similar to those described above.

1L-$\forall$R Refer to $\forall$R-**1L**.

1L-$\exists$R Refer to $\exists$R-**1L**.

1L-$\multimap$L**-1** Refer to $\multimap$L-**1L**-1.

1L-$\multimap$L**-2** Refer to $\multimap$L-**1L**-3.

1L-$\otimes$L Refer to $\otimes$L-**1L**-3.

1L-1L $\mathbf{1}\mathsf{L}\ x\ (\mathbf{1}\mathsf{L}\ y\ D) \Leftrightarrow \mathbf{1}\mathsf{L}\ y\ (\mathbf{1}\mathsf{L}\ x\ D)$

$$x().y().P \Leftrightarrow y().x().P$$

1L-!L $\mathbf{1}\mathsf{L}\ x\ (!\mathsf{L}\ y\ (\lambda u.\,D\,u)) \Leftrightarrow !\mathsf{L}\ y\ (\lambda u.\,\mathbf{1}\mathsf{L}\ x\ (D\,u))$

$$x().y/u.P_u \Leftrightarrow y/u.x().P_u$$

1L-copy $\mathbf{1}\mathsf{L}\ x\ (\mathsf{copy}\ u\ (\hat{\lambda}y.\,D\,u\,y)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\,\mathbf{1}\mathsf{L}\ x\ (D\,u\,y))$

$$x().(\nu y)u\langle y\rangle.P_{uyz} \Leftrightarrow (\nu y)u\langle y\rangle.x().P_{uyz}$$

1L-$\forall$L $\mathbf{1}\mathsf{L}\ x\ (\forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,D\,y)) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,\mathbf{1}\mathsf{L}\ x\ (D\,y))$

$$x().y\langle M\rangle.P_{yz} \Leftrightarrow y\langle M\rangle.x().P_{yz}$$

1L-$\exists$L $\mathbf{1}\mathsf{L}\ x\ (\exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,D\,a\,y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,\mathbf{1}\mathsf{L}\ x\ (D\,a\,y))$

$$x().y(a).P_{ayz} \Leftrightarrow y(a).x().P_{ayz}$$

1L-cut-1 $\mathbf{1}\mathsf{L}\ x\ (\mathsf{cut}\ D_1\ (\hat{\lambda}y.\,D_2\,y)) \Leftrightarrow \mathsf{cut}\ (\mathbf{1}\mathsf{L}\ x\ D_1)\ (\hat{\lambda}y.\,D_2\,y)$

$$x().(\nu y)(P_y \mid Q_{yz}) \Leftrightarrow (\nu y)(x().P_y \mid Q_{yz})$$

1L-cut-2 $\mathbf{1}\mathsf{L}\ x\ (\mathsf{cut}\ D_1\ (\hat{\lambda}y.\, D_2\, y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\, \mathbf{1}\mathsf{L}\ x\ (D_2\, y))$
$x().(\nu y)(P_y \mid Q_{yz}) \Leftrightarrow (\nu y)(P_y \mid x().Q_{yz})$

Here is another instance where we cannot postpone waiting on x to terminate.

1L-init No conversion for $\mathbf{1}\mathsf{L}\ x\ (\mathsf{init}\ y)$
$x().[y \leftrightarrow z]$

And then one last case:

1L-cut$^!$ $\mathbf{1}\mathsf{L}\ x\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \mathbf{1}\mathsf{L}\ x\ (D_2\, u))$
$x().(\nu u)(!u(y).P_y \mid Q_{uz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid x().Q_{uz})$

2.10 !L

!L-⊸R Refer to ⊸R-!L-2.

!L-⊗R-1 Refer to ⊗R-!L-1.

!L-⊗R-2 Refer to ⊗R-!L-2.

!L-⊗R-3 No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \otimes\mathsf{R}\ (D_1\, u)\ (D_2\, u))$ when both occurrences of u are strict.
$x/u.(\nu y)z\langle y\rangle.(P_{uy} \mid Q_{uz})$

!L-1R No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \mathbf{1}\mathsf{R})$
$x/u.z\langle\rangle.\mathbf{0}$

!L-!R No conversion for $!\mathsf{L}\ x\ (\lambda u.\, !\mathsf{R}\ (D\, u))$
$x/u.!z(y).P_{uy}$

!L-∀R Refer to ∀R-!L.

!L-∃R Refer to ∃R-!L.

!L-⊸L-1 Refer to ⊸L-!L-1.

!L-⊸L-2 Refer to ⊸L-!L-3.

!L-⊸L-3 No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \multimap\mathsf{L}\ y\ (D_1\, u)\ (\hat{\lambda}y.\, D_2\, u\, y))$ when both occurrences of u are strict.
$x/u.(\nu w)y\langle w\rangle.(P_{uw} \mid Q_{uyz})$

!L-⊗L Refer to ⊗L-!L-3.

!L-1L Refer to **1**L-!L.

!L-!L $!\mathsf{L}\ x\ (\lambda u.\, !\mathsf{L}\ y\ (\lambda v.\, D\, u\, v)) \Leftrightarrow !\mathsf{L}\ y\ (\lambda v.\, !\mathsf{L}\ x\ (\lambda u.\, D\, u\, v))$
$x/u.y/v.P_{uvz} \Leftrightarrow y/v.x/u.P_{uvz}$

!L-copy-1 No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \mathsf{copy}\ u\ (\hat{\lambda}y.\, D\, u\, y))$
$x/u.(\nu y)u\langle y\rangle.P_{uyz}$

!L-copy-2 $!\mathsf{L}\ x\ (\lambda u.\, \mathsf{copy}\ v\ (\hat{\lambda}y.\, D\, u\, v\, y)) \Leftrightarrow \mathsf{copy}\ v\ (\hat{\lambda}y.\, !\mathsf{L}\ x\ (\lambda u.\, D\, u\, v\, y))$
$x/u.(\nu y)v\langle y\rangle.P_{uvyz} \Leftrightarrow (\nu y)v\langle y\rangle.x/u.P_{uvyz}$

!L-∀L $!\mathsf{L}\ x\ (\lambda u.\, \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, D\, u\, y)) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\, !\mathsf{L}\ x\ (\lambda u.\, D\, u\, y))$
$x/u.y\langle M\rangle.P_{uyz} \Leftrightarrow y\langle M\rangle.x/u.P_{uyz}$

!L-∃L $\quad !\mathsf{L}\ x\ (\lambda u.\, \exists \mathsf{L}\ y\ (\lambda a.\hat{\lambda} y.\, D\, u\, a\, y)) \Leftrightarrow \exists \mathsf{L}\ y\ (\lambda a.\hat{\lambda} y.\, !\mathsf{L}\ x\ (\lambda u.\, D\, u\, a\, y))$

$$x/u.y(a).P_{uayz} \Leftrightarrow y(a).x/u.P_{uayz}$$

!L-cut-1 $\quad !\mathsf{L}\ x\ (\lambda u.\, \mathsf{cut}\ (D_1\, u)\ (\hat{\lambda} y.\, D_2\, y)) \Leftrightarrow \mathsf{cut}\ (!\mathsf{L}\ x\ (\lambda u.\, D_1\, u))\ (\hat{\lambda} y.\, D_2\, y)$

$$x/u.(\nu y)(P_{uy} \mid Q_{yz}) \Leftrightarrow (\nu y)(x/u.P_{uy} \mid Q_{yz})$$

!L-cut-2 $\quad !\mathsf{L}\ x\ (\lambda u.\, \mathsf{cut}\ D_1\ (\hat{\lambda} y.\, D_2\, u\, y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda} y.\, !\mathsf{L}\ x\ (\lambda u.\, D_2\, u\, y))$

$$x/u.(\nu y)(P_y \mid Q_{uyz}) \Leftrightarrow (\nu y)(P_y \mid x/u.Q_{uyz})$$

!L-cut-3 No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \mathsf{cut}\ (D_1\, u)\ (\hat{\lambda} y.\, D_2\, u\, y))$ when both occurrences of u are strict.

$$x/u.(\nu y)(P_{uy} \mid Q_{uyz})$$

!L-init No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \mathsf{init}\ y)$

$$x/u.[y \leftrightarrow z]$$

!L-cut!-1 No conversion for $!\mathsf{L}\ x\ (\lambda u.\, \mathsf{cut}^!\ (D_1\, u)\ (\lambda v.\, D_2\, u\, v))$ when the first occurrence of u is strict.

$$x/u.(\nu v)(!v(y).P_{uy} \mid Q_{uvz})$$

!L-cut!-2 $\quad !\mathsf{L}\ x\ (\lambda u.\, \mathsf{cut}^!\ D_1\ (\lambda v.\, D_2\, u\, v)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda v.\, !\mathsf{L}\ x\ (\lambda u.\, D_2\, u\, v))$

$$x/u.(\nu v)(!v(y).P_y \mid Q_{uvz}) \Leftrightarrow (\nu v)(!v(y).P_y \mid x/u.Q_{uvz})$$

2.11 copy

copy-⊸R Refer to ⊸R-copy.

copy-⊗R-1 Refer to ⊗R-copy-1.

copy-⊗R-2 Refer to ⊗R-copy-2.

copy-1R $\quad \mathsf{copy}\ u\ (\hat{\lambda} x.\, \mathbf{1}\mathsf{R})$ is not a well-formed proof term.

copy-!R $\quad \mathsf{copy}\ u\ (\hat{\lambda} x.\, !\mathsf{R}\ D)$ is not a well-formed proof term.

copy-∀R Refer to ∀R-copy.

copy-∃R Refer to ∃R-copy.

copy-⊸L-1 No conversion for $\mathsf{copy}\ u\ (\hat{\lambda} x.\, \multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda} x.\, D_2\, x))$

$$(\nu x)u\langle x\rangle.(\nu y)x\langle y\rangle.(P_{uy} \mid Q_{uxz})$$

copy-⊸L-2 Refer to ⊸L-copy-1.

copy-⊸L-3 Refer to ⊸L-copy-2.

copy-⊗L-1 No conversion for $\mathsf{copy}\ u\ (\hat{\lambda} x.\, \otimes\mathsf{L}\ x\ (\hat{\lambda} y.\hat{\lambda} x.\, D\, y\, x))$

$$(\nu x)u\langle x\rangle.x(y).P_{uyxz}$$

copy-⊗L-2 Refer to ⊗L-copy.

copy-1L-1 No conversion for $\mathsf{copy}\ u\ (\hat{\lambda} x.\, \mathbf{1}\mathsf{L}\ x\ D)$

$$(\nu x)u\langle x\rangle.x().P_{uz}$$

copy-1L-2 Refer to 1L-copy.

copy-!L-1 No conversion for $\mathsf{copy}\ u\ (\hat{\lambda} x.\, !\mathsf{L}\ x\ (\lambda v.\, D\, v))$

$$(\nu x)u\langle x\rangle.x/v.P_{uvz}$$

copy-!L-**2** Refer to !L-copy-2.

copy-copy $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{copy}\ v\ (\hat{\lambda}y.\,D\,x\,y)) \Leftrightarrow \mathsf{copy}\ v\ (\hat{\lambda}y.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D\,x\,y))$
$(\nu x)u\langle x\rangle.(\nu y)v\langle y\rangle.P_{uxvyz} \Leftrightarrow (\nu y)v\langle y\rangle.(\nu x)u\langle x\rangle.P_{uxvyz}$

copy-∀L-**1** No conversion for $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,D\,x))$
$(\nu x)u\langle x\rangle.x\langle M\rangle.P_{uxz}$

copy-∀L-**2** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,D\,x\,y)) \Leftrightarrow \forall\mathsf{L}\ y\ M\ (\hat{\lambda}y.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D\,x\,y))$
$(\nu x)u\langle x\rangle.y\langle M\rangle.P_{uxyz} \Leftrightarrow y\langle M\rangle.(\nu x)u\langle x\rangle.P_{uxyz}$

copy-∃L-**1** No conversion for $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\,D\,a\,x))$
$(\nu x)u\langle x\rangle.x(a).P_{uaxz}$

copy-∃L-**2** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,D\,x\,a\,y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D\,x\,a\,y))$
$(\nu x)u\langle x\rangle.y(a).P_{uxayz} \Leftrightarrow y(a).(\nu x)u\langle x\rangle.P_{uxayz}$

copy-cut-**1** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{cut}\ (D_1\,x)\ (\hat{\lambda}y.\,D_2\,y)) \Leftrightarrow \mathsf{cut}\ (\mathsf{copy}\ u\ (\hat{\lambda}x.\,D_1\,x))\ (\hat{\lambda}y.\,D_2\,y)$
$(\nu x)u\langle x\rangle.(\nu y)(P_{uxy} \mid Q_{yz}) \Leftrightarrow (\nu y)((\nu x)u\langle x\rangle.P_{uxy} \mid Q_{yz})$

copy-cut-**2** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{cut}\ D_1\ (\hat{\lambda}y.\,D_2\,x\,y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D_2\,x\,y))$
$(\nu x)u\langle x\rangle.(\nu y)(P_y \mid Q_{uxyz}) \Leftrightarrow (\nu y)(P_y \mid (\nu x)u\langle x\rangle.Q_{uxyz})$

copy-init-**1** No conversion for $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{init}\ x)$
$(\nu x)u\langle x\rangle.[x \leftrightarrow z]$

copy-init-**2** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{init}\ y)$ is not a well-formed proof term.

copy-cut$^!$-**1** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{cut}^!\ (D_1\,x)\ (\lambda v.\,D_2\,v))$ is not a well-formed proof term.

copy-cut$^!$-**2** $\mathsf{copy}\ u\ (\hat{\lambda}x.\,\mathsf{cut}^!\ D_1\ (\lambda v.\,D_2\,x\,v)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda v.\,\mathsf{copy}\ u\ (\hat{\lambda}x.\,D_2\,x\,v))$
$(\nu x)u\langle x\rangle.(\nu v)(!v(y).P_{uy} \mid Q_{uxvz}) \Leftrightarrow (\nu v)(!v(y).P_{uy} \mid (\nu x)u\langle x\rangle.Q_{uxvz})$

2.12 ∀L

∀L-⊸R Refer to ⊸R-∀L-2.

∀L-⊗R-**1** Refer to ⊗R-∀L-1.

∀L-⊗R-**2** Refer to ⊗R-∀L-2.

∀L-**1**R $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathbf{1}\mathsf{R})$ is not a well-formed proof term.

∀L-!R $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,!\mathsf{R}\ D)$ is not a well-formed proof term.

∀L-∀R Refer to ∀R-∀L-2.

∀L-∃R Refer to ∃R-∀L.

∀L-⊸L-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\multimap\mathsf{L}\ x\ D_1\ (\hat{\lambda}x.\,D_2\,x))$
$x\langle M\rangle.(\nu w)x\langle w\rangle.(P_w \mid Q_{xz})$

∀L-⊸L-**2** Refer to ⊸L-∀L-1.

∀L-⊸L-**3** Refer to ⊸L-∀L-3.

$\forall$L-$\otimes$L-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\otimes\mathsf{L}\ x\ (\hat{\lambda}y.\hat{\lambda}x.\ D\,y\,x))$

$$x\langle M\rangle.x(y).Q_{yxz}$$

$\forall$L-$\otimes$L-**2** Refer to $\otimes$L-$\forall$L-3.

$\forall$L-**1**L-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ \mathbf{1}\mathsf{L}\ x\ D)$

$$x\langle M\rangle.x().P_z$$

$\forall$L-**1**L-**2** Refer to **1**L-$\forall$L.

$\forall$L-!L-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ !\mathsf{L}\ x\ (\lambda u.\ D\,u))$

$$x\langle M\rangle.x/u.P_{uz})$$

$\forall$L-!L-**2** Refer to !L-$\forall$L.

$\forall$L-copy Refer to copy-$\forall$L-2.

$\forall$L-$\forall$L-**1** No conversion for $\forall\mathsf{L}\ x\ M_1\ (\hat{\lambda}x.\ \exists\mathsf{L}\ x\ (\lambda M_2.\hat{\lambda}x.\ D\,x))$

$$x\langle M_1\rangle.x\langle M_2\rangle.P_{xz}$$

$\forall$L-$\forall$L-**2** $\forall\mathsf{L}\ x\ M_1\ (\hat{\lambda}x.\forall\mathsf{L}\ y\ M_2\ (\hat{\lambda}y.\ D\,x\,y)) \Leftrightarrow \forall\mathsf{L}\ y\ M_2\ (\hat{\lambda}y.\forall\mathsf{L}\ x\ M_1\ (\hat{\lambda}x.\ D\,x\,y))$

$$x\langle M_1\rangle.y\langle M_2\rangle.P_{xyz} \Leftrightarrow y\langle M_2\rangle.x\langle M_1\rangle.P_{xyz}$$

$\forall$L-$\exists$L-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D\,a\,x))$

$$x\langle M\rangle.x(a).P_{axz}$$

$\forall$L-$\exists$L-**2** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\ D\,x\,a\,y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda}y.\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ D\,x\,a\,y))$

$$x\langle M\rangle.y(a).P_{xayz} \Leftrightarrow y(a).x\langle M\rangle.P_{xayz}$$

$\forall$L-cut-**1** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{cut}\ (D_1\,x)\ (\hat{\lambda}y.\ D_2\,y)) \Leftrightarrow \mathsf{cut}\ (\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ D_1\,x))\ (\hat{\lambda}y.\ D_2\,y)$

$$x\langle M\rangle.(\nu y)(P_{xy} \mid Q_{yz}) \Leftrightarrow (\nu y)(x\langle M\rangle.P_{xy} \mid Q_{yz})$$

$\forall$L-cut-**2** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{cut}\ D_1\ (\hat{\lambda}y.\ D_2\,x\,y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ D_2\,x\,y))$

$$x\langle M\rangle.(\nu y)(P_y \mid Q_{xyz}) \Leftrightarrow (\nu y)(P_y \mid x\langle M\rangle.Q_{xyz})$$

$\forall$L-init-**1** No conversion for $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{init}\ x)$

$$x\langle M\rangle.[x \leftrightarrow z]$$

$\forall$L-init-**2** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{init}\ y)$ is not a well-formed proof term.

$\forall$L-cut$^!$-**1** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{cut}^!\ (D_1\,x)\ (\lambda u.\ D_2\,u))$ is not a well-formed proof term.

$\forall$L-cut$^!$-**2** $\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\,\mathsf{cut}^!\ D_1\ (\lambda u.\ D_2\,x\,u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\forall\mathsf{L}\ x\ M\ (\hat{\lambda}x.\ D_2\,x\,u))$

$$x\langle M\rangle.(\nu u)(!u(y).P_y \mid Q_{xuz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid x\langle M\rangle.Q_{xuz})$$

2.13 $\exists$L

$\exists$L-$\multimap$R Refer to $\multimap$R-$\exists$L-2.

$\exists$L-$\otimes$R-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\otimes\mathsf{R}\ (D_1\,a\,x)\ (D_2\,a))$ when the second occurrence of a is strict.

$$x(a).(\nu y)z\langle y\rangle.(P_{axy} \mid Q_{az})$$

$\exists$L-$\otimes$R-**2** Refer to $\otimes$R-$\exists$L-1.

$\exists$L-$\otimes$R-**3** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\otimes\mathsf{R}\ (D_1\,a)\ (D_2\,a\,x))$ when the first occurrence of a is strict.

$$x(a).(\nu y)z\langle y\rangle.(P_{ay} \mid Q_{axz})$$

$\exists$L-$\otimes$R-**4** Refer to $\otimes$R-$\exists$L-2.

$\exists$L-**1**R $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\mathbf{1}\mathsf{R})$ is not a well-formed proof term.

$\exists$L-!R $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.!\mathsf{R}\ (D\,a))$ is not a well-formed proof term.

$\exists$L-$\forall$R Refer to $\forall$R-$\exists$L.

$\exists$L-$\exists$R-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\exists\mathsf{R}\ (M\,a)\ (D\,a\,x))$ when the first occurrence of a is strict.

$$x(a).z\langle M\,a\rangle.P_{axz}$$

$\exists$L-$\exists$R-**2** Refer to $\exists$R-$\exists$L.

$\exists$L-$\multimap$L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\multimap\mathsf{L}\ x\ (D_1\,a)\ (\hat{\lambda}x.\,D_2\,a\,x))$

$$x(a).(\nu w)x\langle w\rangle.(P_{aw} \mid Q_{axz})$$

$\exists$L-$\multimap$L-**2** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\multimap\mathsf{L}\ y\ (D_1\,a\,x)\ (\hat{\lambda}y.\,D_2\,a\,y))$ when the second occurrence of a is

$$x(a).(\nu w)y\langle w\rangle.(P_{axw} \mid Q_{ayz})$$

strict.

$\exists$L-$\multimap$L-**3** Refer to $\multimap$L-$\exists$L-1.

$\exists$L-$\multimap$L-**4** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\multimap\mathsf{L}\ y\ (D_1\,a)\ (\hat{\lambda}y.\,D_2\,a\,x\,y))$ when the first occurrence of a is strict.

$$x(a).(\nu w)y\langle w\rangle.(P_{aw} \mid Q_{axyz})$$

$\exists$L-$\multimap$L-**5** Refer to $\multimap$L-$\exists$L-3.

$\exists$L-$\otimes$L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\otimes\mathsf{L}\ x\ (\hat{\lambda}y.\hat{\lambda}x.\,D\,a\,y\,x))$

$$x(a).x(y).Q_{ayxz}$$

$\exists$L-$\otimes$L-**2** Refer to $\otimes$L-$\exists$L-3.

$\exists$L-**1**L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\mathbf{1}\mathsf{L}\ x\ (D\,a))$

$$x(a).x().P_{az}$$

$\exists$L-**1**L-**2** Refer to **1**L-$\exists$L.

$\exists$L-!L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.!\mathsf{L}\ x\ (\lambda u.\,D\,a\,u))$

$$x(a).x/u.P_{auz})$$

$\exists$L-!L-**2** Refer to !L-$\exists$L.

$\exists$L-copy Refer to copy-$\exists$L-2.

$\exists$L-$\forall$L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\forall\mathsf{L}\ x\ (M\,a)\ (\hat{\lambda}x.\,D\,a\,x))$

$$x(a).x\langle M\,a\rangle.P_{axz}$$

$\exists$L-$\forall$L-**2** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\forall\mathsf{L}\ y\ (M\,a)\ (\hat{\lambda}y.\,D\,a\,x\,y))$ when the first occurrence of a is strict.

$$x(a).y\langle M\,a\rangle.P_{axyz}$$

$\exists$L-$\forall$L-**3** Refer to $\forall$L-$\exists$L-2.

$\exists$L-$\exists$L-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\exists\mathsf{L}\ x\ (\lambda b.\hat{\lambda}x.\,D\,a\,b\,x))$

$$x(a).x(b).P_{abxz}$$

$\exists$L-$\exists$L-**2** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \exists\mathsf{L}\ y\ (\lambda b.\hat{\lambda}y.\ D\,a\,x\,b\,y)) \Leftrightarrow \exists\mathsf{L}\ y\ (\lambda b.\hat{\lambda}y.\ \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D\,a\,x\,b\,y))$

$$x(a).y(b).P_{axbyz} \Leftrightarrow y(b).x(a).P_{axbyz}$$

$\exists$L-cut-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}\ (D_1\,a\,x)\ (\hat{\lambda}y.\ D_2\,a\,y))$ when the second occurrence of a is strict.

$$x(a).(\nu y)(P_{axy} \mid Q_{ayz})$$

$\exists$L-cut-**2** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}\ (D_1\,a\,x)\ (\hat{\lambda}y.\ D_2\,y)) \Leftrightarrow \mathsf{cut}\ (\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D_1\,a\,x))\ (\hat{\lambda}y.\ D_2\,y)$

$$x(a).(\nu y)(P_{axy} \mid Q_{yz}) \Leftrightarrow (\nu y)(x(a).P_{axy} \mid Q_{yz})$$

$\exists$L-cut-**3** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}\ (D_1\,a)\ (\hat{\lambda}y.\ D_2\,a\,x\,y))$ when the first occurrence of a is strict.

$$x(a).(\nu y)(P_{ay} \mid Q_{axyz})$$

$\exists$L-cut-**4** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}\ D_1\ (\hat{\lambda}y.\ D_2\,a\,x\,y)) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\ \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D_2\,a\,x\,y))$

$$x(a).(\nu y)(P_y \mid Q_{axyz}) \Leftrightarrow (\nu y)(P_y \mid x(a).Q_{axyz})$$

$\exists$L-init-**1** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{init}\ x)$

$$x(a).[x \leftrightarrow z]$$

$\exists$L-init-**2** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{init}\ y)$ is not a well-formed proof term.

$\exists$L-cut$^!$-**1** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}^!\ (D_1\,a\,x)\ (\lambda u.\ D_2\,a\,u))$ is not a well-formed proof term.

$\exists$L-cut$^!$-**2** No conversion for $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}^!\ (D_1\,a)\ (\lambda u.\ D_2\,a\,x\,u))$ when the first occurrence of a is strict.

$$x(a).(\nu u)(!u(y).P_{ay} \mid Q_{axuz})$$

$\exists$L-cut$^!$-**3** $\exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ \mathsf{cut}^!\ D_1\ (\lambda u.\ D_2\,a\,x\,u)) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\ \exists\mathsf{L}\ x\ (\lambda a.\hat{\lambda}x.\ D_2\,a\,x\,u))$

$$x(a).(\nu u)(!u(y).P_y \mid Q_{axuz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid x(a).Q_{axuz})$$

2.14 cut

cut-$\multimap$R-**1** No conversion for $\mathsf{cut}\ (\multimap\mathsf{R}\ (\hat{\lambda}y.\ D_1\,y))\ (\hat{\lambda}x.\ D_2\,x)$

$$(\nu x)(x(y).P_{yx} \mid Q_{xz})$$

cut-$\multimap$R-**2** Refer to $\multimap$R-cut-2.

cut-$\otimes$R-**1** No conversion for $\mathsf{cut}\ (\otimes\mathsf{R}\ D_1\ D_2)\ (\hat{\lambda}x.\ D_3\,x)$

$$(\nu x)((\nu y)x\langle y\rangle.(P_y \mid Q_x) \mid R_{xz})$$

cut-$\otimes$R-**2** Refer to $\otimes$R-cut-1.

cut-$\otimes$R-**3** Refer to $\otimes$R-cut-2.

cut-**1**R-**1** No conversion for $\mathsf{cut}\ \mathbf{1}\mathsf{R}\ (\hat{\lambda}x.\ D\,x)$

$$(\nu x)(x\langle\rangle.\mathbf{0} \mid P_{xz})$$

cut-**1**R-**2** $\mathsf{cut}\ D\ (\hat{\lambda}x.\ \mathbf{1}\mathsf{R})$ is not a well-formed proof term.

cut-!R-**1** No conversion for $\mathsf{cut}\ (!\mathsf{R}\ D_1)\ (\hat{\lambda}x.\ D_2\,x)$

$$(\nu x)(!x(y).P_y \mid Q_{xz})$$

cut-!R-**2** $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\ !\mathsf{R}\ D_2)$ is not a well-formed proof term.

cut-$\forall$R-**1** No conversion for $\mathsf{cut}\ (\forall\mathsf{R}\ (\lambda a.\ D_1\,a))\ (\hat{\lambda}x.\ D_2\,x)$

$$(\nu x)(x(a).P_{ax} \mid Q_{xz})$$

cut-∀R-2 Refer to ∀R-cut-2.

cut-∃R-1 No conversion for $\mathsf{cut}\ (\exists \mathsf{R}\ M\ D_1)\ (\hat{\lambda}x.\, D_2\, x)$
$$(\nu x)(x\langle M\rangle.P_x \mid Q_{xz})$$

cut-∃R-2 Refer to ∃R-cut.

cut-⊸L-1 Refer to ⊸L-cut-2.

cut-⊸L-2 No conversion for $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \multimap\mathsf{L}\ x\ D_2\ (\hat{\lambda}x.\, D_3\, x))$
$$(\nu x)(P_x \mid (\nu w)x\langle w\rangle.(Q_w \mid R_{xz}))$$

cut-⊸L-3 Refer to ⊸L-cut-1.

cut-⊸L-4 Refer to ⊸L-cut-3.

cut-⊗L-1 Refer to ⊗L-cut-1.

cut-⊗L-2 No conversion for $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \otimes\mathsf{L}\ x\ (\hat{\lambda}w.\hat{\lambda}x.\, D_2\, w\, x))$
$$(\nu x)(P_x \mid x(w).Q_{wxz})$$

cut-⊗L-3 Refer to ⊗L-cut-4.

cut-1L-1 Refer to **1**L-cut-1.

cut-1L-2 No conversion for $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \mathbf{1}\mathsf{L}\ x\ D_2)$
$$(\nu x)(P_x \mid x().Q_z)$$

cut-1L-3 Refer to **1**L-cut-2.

cut-!L-1 Refer to !L-cut-1.

cut-!L-2 No conversion for $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, !\mathsf{L}\ x\ (\lambda u.\, D_2\, u))$
$$(\nu x)(P_x \mid x/u.Q_{uz})$$

cut-!L-3 Refer to **1**L-cut-2.

cut-copy-1 $\mathsf{cut}\ (\mathsf{copy}\ u\ (\hat{\lambda}y.\, D_1\, u\, y))\ (\hat{\lambda}x.\, D_2\, x) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\, \mathsf{cut}\ (D_1\, u\, y)\ (\hat{\lambda}x.\, D_2\, x))$
$$(\nu x)((\nu y)u\langle y\rangle.P_{uyx} \mid Q_{xz}) \Leftrightarrow (\nu y)u\langle y\rangle.(\nu x)(P_{uyx} \mid Q_{xz})$$

cut-copy-2 $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \mathsf{copy}\ u\ (\hat{\lambda}y.\, D_2\, x\, u\, y)) \Leftrightarrow \mathsf{copy}\ u\ (\hat{\lambda}y.\, \mathsf{cut}\ D_1\ (\hat{\lambda}x.\, D_2\, x\, u\, y))$
$$(\nu x)(P_x \mid (\nu y)u\langle y\rangle.Q_{xuyz}) \Leftrightarrow (\nu y)u\langle y\rangle.(\nu x)(P_x \mid Q_{xuyz})$$

cut-∀L-1 Refer to ∀L-cut-1.

cut-∀L-2 Refer to ∀L-cut-2.

cut-∃L-1 Refer to ∃L-cut-2.

cut-∃L-2 Refer to ∃L-cut-4.

cut-cut-1 $\mathsf{cut}\ (\mathsf{cut}\ D_1\ (\hat{\lambda}y.\, D_2\, y))\ (\hat{\lambda}x.\, D_3\, x) \Leftrightarrow \mathsf{cut}\ D_1\ (\hat{\lambda}y.\, \mathsf{cut}\ (D_2\, y)\ (\hat{\lambda}x.\, D_3\, x))$
$$(\nu x)((\nu y)(P_y \mid Q_{yx}) \mid R_{xz}) \Leftrightarrow (\nu y)(P_y \mid (\nu x)(Q_{yx} \mid R_{xz}))$$

cut-cut-2 Refer to cut-cut-1.

cut-cut-3 $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}\ D_2\ (\hat{\lambda}y.\, D_3\, x\, y)) \Leftrightarrow \mathsf{cut}\ D_2\ (\hat{\lambda}y.\, \mathsf{cut}\ D_1\ (\hat{\lambda}x.\, D_3\, x\, y))$
$$(\nu x)(P_x \mid (\nu y)(Q_y \mid R_{xyz})) \Leftrightarrow (\nu y)(Q_y \mid (\nu x)(P_x \mid R_{xyz}))$$

cut-init-1 No conversion for $\mathsf{cut}\ (\mathsf{init}\ y)\ (\hat{\lambda}x.\, D\, x)$
$(\nu x)([y \leftrightarrow x] \mid P_{xz})$

cut-init-2 No conversion for $\mathsf{cut}\ D\ (\hat{\lambda}x.\, \mathsf{init}\ x)$
$(\nu x)(P_x \mid [x \leftrightarrow z])$

cut-init-3 $\mathsf{cut}\ D\ (\hat{\lambda}x.\, \mathsf{init}\ y)$ is not a well-formed proof term.

cut-cut$^!$-1 $\mathsf{cut}\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\hat{\lambda}x.\, D_3\, x) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda u.\, \mathsf{cut}\ (D_2\, u)\ (\hat{\lambda}x.\, D_3\, x))$
$(\nu x)((\nu u)(!u(y).P_y \mid Q_{ux}) \mid R_{xz}) \Leftrightarrow (\nu u)(!u(y).P_y \mid (\nu x)(Q_{ux} \mid R_{xz}))$

cut-cut$^!$-2 $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}^!\ (D_2\, x)\ (\lambda u.\, D_3\, u))$ is not a well-formed proof term.

cut-cut$^!$-3 $\mathsf{cut}\ D_1\ (\hat{\lambda}x.\, \mathsf{cut}^!\ D_2\ (\lambda u.\, D_3\, x\, u)) \Leftrightarrow \mathsf{cut}^!\ D_2\ (\lambda u.\, \mathsf{cut}\ D_1\ (\hat{\lambda}x.\, D_3\, x\, u))$
$(\nu x)(P_x \mid (\nu u)(!u(y).Q_y \mid R_{xuz})) \Leftrightarrow (\nu u)(!u(y).Q_y \mid (\nu x)(P_x \mid R_{xuz}))$

2.15 init

There are no cases here because init has no subterms.

2.16 cut$^!$

cut$^!$-⊸R-1 No conversion for $\mathsf{cut}^!\ (\multimap\mathsf{R}\ (\hat{\lambda}y.\, D_1\, y))\ (\lambda u.\, D_2\, u)$
$(\nu u)(!u(x).x(y).P_{yx} \mid Q_{uz})$

cut$^!$-⊸R-2 Refer to ⊸R-cut$^!$-2.

cut$^!$-⊗R-1 No conversion for $\mathsf{cut}^!\ (\otimes\mathsf{R}\ D_1\ D_2)\ (\lambda u.\, D_3\, u)$
$(\nu u)(!u(x).(\nu y)x\langle y\rangle.(P_y \mid Q_x) \mid R_{uz})$

The following case is noteworthy because it duplicates a persistent service. This also occurs in the subsequent cut$^!$-⊸L-2, cut$^!$-cut-2, and cut$^!$-cut$^!$-2 cases.

cut$^!$-⊗R-2 $\mathsf{cut}^!\ D_1\ (\lambda u.\, \otimes\mathsf{R}\ (D_2\, u)\ (D_3\, u)) \Leftrightarrow \otimes\mathsf{R}\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_3\, u))$
$(\nu u)(!u(x).P_x \mid (\nu y)z\langle y\rangle.(Q_{uy} \mid R_{uz})) \Leftrightarrow (\nu y)z\langle y\rangle.((\nu u)(!u(x).P_x \mid Q_{uy}) \mid (\nu u)(!u(x).P_x \mid R_{uz}))$

cut$^!$-1R-1 No conversion for $\mathsf{cut}^!\ \mathbf{1}\mathsf{R}\ (\lambda u.\, D\, u)$
$(\nu u)(!u(x).x\langle\rangle.\mathbf{0} \mid P_{uz})$

The following case is noteworthy because it garbage-collects an unused persistent service. This also occurs in the subsequent cut$^!$-init-2 case.

cut$^!$-1R-2 $\mathsf{cut}^!\ D\ (\lambda u.\, \mathbf{1}\mathsf{R}) \Leftrightarrow \mathbf{1}\mathsf{R}$
$(\nu u)(!u(x).P_x \mid z\langle\rangle.\mathbf{0}) \Leftrightarrow z\langle\rangle.\mathbf{0}$

cut$^!$-!R-1 No conversion for $\mathsf{cut}^!\ (!\mathsf{R}\ D_1)\ (\lambda u.\, D_2\, u)$
$(\nu u)(!u(x).!x(y).P_y \mid Q_{uz})$

cut$^!$-!R-2 Refer to !R-cut$^!$.

cut$^!$-∀R-1 No conversion for $\mathsf{cut}^!\ (\forall\mathsf{R}\ (\lambda a.\, D_1\, a))\ (\lambda u.\, D_2\, u)$
$(\nu u)(!u(x).x(a).P_{ax} \mid Q_{uz})$

$\mathsf{cut}^!$-$\forall$R-**2** Refer to $\forall$R-$\mathsf{cut}^!$-2.

$\mathsf{cut}^!$-$\exists$R-**1** No conversion for $\mathsf{cut}^!\ (\exists\mathsf{R}\ M\ D_1)\ (\lambda u.\, D_2\, u)$
$(\nu u)(!u(x).x\langle M\rangle.P_x \mid Q_{uz})$

$\mathsf{cut}^!$-$\exists$R-**2** Refer to $\exists$R-$\mathsf{cut}^!$.

$\mathsf{cut}^!$-$\multimap$L-**1** $\mathsf{cut}^!\ (\multimap\mathsf{L}\ y\ D_1\ (\hat{\lambda} y.\, D_2\, y))\ (\lambda u.\, D_3\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-$\multimap$L-**2** $\mathsf{cut}^!\ D_1\ (\lambda u.\multimap\mathsf{L}\ y\ (D_2\, u)\ (\hat{\lambda} y.\, D_3\, u\, y)) \Leftrightarrow \multimap\mathsf{L}\ y\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\hat{\lambda} y.\, \mathsf{cut}^!\ D_1\ (\lambda u.\, D_3\, u\, y))$
$(\nu u)(!u(x).P_x \mid (\nu w)y\langle w\rangle.(Q_{uw} \mid R_{uyz})) \Leftrightarrow (\nu w)y\langle w\rangle.((\nu u)(!u(x).P_x \mid Q_{uw}) \mid (\nu u)(!u(x).P_x \mid R_{uyz}))$

$\mathsf{cut}^!$-$\otimes$L-**1** $\mathsf{cut}^!\ (\otimes\mathsf{L}\ y\ (\hat{\lambda} w.\hat{\lambda} y.\, D_1\, w\, y))\ (\lambda u.\, D_2\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-$\otimes$L-**2** Refer to $\otimes$L-$\mathsf{cut}^!$-4.

$\mathsf{cut}^!$-**1**L-**1** $\mathsf{cut}^!\ (\mathbf{1}\mathsf{L}\ y\ D_1)\ (\lambda u.\, D_2\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-**1**L-**2** Refer to **1**L-$\mathsf{cut}^!$.

$\mathsf{cut}^!$-!L-**1** $\mathsf{cut}^!\ (!\mathsf{L}\ y\ (\lambda v.\, D_1\, v))\ (\lambda u.\, D_2\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-!L-**2** Refer to !L-cut-2.

$\mathsf{cut}^!$-copy-**1** No conversion for $\mathsf{cut}^!\ (\mathsf{copy}\ v\ (\hat{\lambda} y.\, D_1\, v\, y))\ (\lambda u.\, D_2\, u)$
$(\nu u)(!u(x).(\nu y)v\langle y\rangle.P_{vyx} \mid Q_{uz})$

$\mathsf{cut}^!$-copy-**2** Refer to copy-$\mathsf{cut}^!$-2.

$\mathsf{cut}^!$-$\forall$L-**1** $\mathsf{cut}^!\ (\forall\mathsf{L}\ y\ M\ (\hat{\lambda} y.\, D_1\, y))\ (\lambda u.\, D_2\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-$\forall$L-**2** Refer to $\forall$L-$\mathsf{cut}^!$-2.

$\mathsf{cut}^!$-$\exists$L-**1** $\mathsf{cut}^!\ (\exists\mathsf{L}\ y\ (\lambda a.\hat{\lambda} y.\, D_1\, a\, y))\ (\lambda u.\, D_2\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-$\exists$L-**2** Refer to $\exists$L-$\mathsf{cut}^!$-3.

$\mathsf{cut}^!$-cut-**1** No conversion for $\mathsf{cut}^!\ (\mathsf{cut}\ D_1\ (\hat{\lambda} y.\, D_2\, y))\ (\lambda u.\, D_3\, u)$
$(\nu u)(!u(x).(\nu y)(P_y \mid Q_{yx}) \mid R_{uz})$

$\mathsf{cut}^!$-cut-**2** $\mathsf{cut}^!\ D_1\ (\lambda u.\, \mathsf{cut}\ (D_2\, u)\ (\hat{\lambda} y.\, D_3\, u\, y)) \Leftrightarrow \mathsf{cut}\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\hat{\lambda} y.\, \mathsf{cut}^!\ D_1\ (\lambda u.\, D_3\, u\, y))$
$(\nu u)(!u(x).P_x \mid (\nu y)(Q_{uy} \mid R_{uyz})) \Leftrightarrow (\nu y)((\nu u)(!u(x).P_x \mid Q_{uy}) \mid (\nu u)(!u(x).P_x \mid R_{uyz}))$

$\mathsf{cut}^!$-init-**1** $\mathsf{cut}^!\ (\mathsf{init}\ y)\ (\lambda u.\, D\, u)$ is not a well-formed proof term.

$\mathsf{cut}^!$-init-**2** $\mathsf{cut}^!\ D\ (\lambda u.\, \mathsf{init}\ y) \Leftrightarrow \mathsf{init}\ y$
$(\nu u)(!u(x).P_x \mid [y \leftrightarrow z]) \Leftrightarrow [y \leftrightarrow z]$

$\mathsf{cut}^!$-$\mathsf{cut}^!$-**1** $\mathsf{cut}^!\ (\mathsf{cut}^!\ D_1\ (\lambda v.\, D_2\, v))\ (\lambda u.\, D_3\, u) \Leftrightarrow \mathsf{cut}^!\ D_1\ (\lambda v.\, \mathsf{cut}^!\ (D_2\, v)\ (\lambda u.\, D_3\, u))$
$(\nu u)(!u(x).(\nu v)(!v(y).P_y \mid Q_{vx}) \mid R_{uz}) \Leftrightarrow (\nu v)(!v(y).P_y \mid (\nu u)(!u(x).Q_{vx} \mid R_{uz}))$

$\mathsf{cut}^!$-$\mathsf{cut}^!$-**2** $\mathsf{cut}^!\ D_1\ (\lambda u.\, \mathsf{cut}^!\ (D_2\, u)\ (\lambda v.\, D_3\, u\, v)) \Leftrightarrow \mathsf{cut}^!\ (\mathsf{cut}^!\ D_1\ (\lambda u.\, D_2\, u))\ (\lambda v.\, \mathsf{cut}^!\ D_1\ (\lambda u.\, D_3\, u\, v))$
$(\nu u)(!u(x).P_x \mid (\nu v)(!v(y).Q_{uy} \mid R_{uvz})) \Leftrightarrow (\nu v)(!v(y).(\nu u)(!u(x).P_x \mid Q_{uy}) \mid (\nu u)(!u(x).P_x \mid R_{uvz}))$

3 Summary of Multiplicative Fragment

1. $x(y).z(w).P_{yxzw} \Leftrightarrow z(w).x(y).P_{yxzw}$ if and only if $z \neq y, x$ and $x \neq w, z$.
   $\multimap$R-$\multimap$R, $\multimap$R-$\otimes$L-1, $\multimap$R-$\otimes$L-2, $\otimes$L-$\multimap$R, $\otimes$L-$\otimes$L-1, $\otimes$L-$\otimes$L-2, $\otimes$L-$\otimes$L-3

2. $x(y).z().P_{yx} \Leftrightarrow z().x(y).P_{yx}$ if and only if $z \neq y, x$.
   $\multimap$R-**1**L-1, $\multimap$R-**1**L-2, $\otimes$L-**1**L-1, $\otimes$L-**1**L-2, $\otimes$L-**1**L-3, **1**L-$\multimap$R, **1**L-$\otimes$L

3. $x(y).(\nu w)z\langle w\rangle.(P_{yxw} \mid Q_z) \Leftrightarrow (\nu w)z\langle w\rangle.(x(y).P_{yxw} \mid Q_z)$ if and only if $z \neq y, x$ and $x \neq w, z$.
   $\otimes$R-$\multimap$R-1, $\otimes$R-$\otimes$L-1, $\multimap$L-$\multimap$R-1, $\multimap$L-$\otimes$L-1, $\otimes$L-$\otimes$R-1, $\otimes$L-$\multimap$L-5

4. $x(y).(\nu w)z\langle w\rangle.(P_w \mid Q_{yxz}) \Leftrightarrow (\nu w)z\langle w\rangle.(P_w \mid x(y).Q_{yxz})$ if and only if $z \neq y, x$ and $x \neq w, z$.
   $\multimap$R-$\otimes$R-2, $\multimap$R-$\multimap$L-1, $\multimap$R-$\multimap$L-3, $\otimes$R-$\multimap$R-2, $\otimes$R-$\otimes$L-2, $\multimap$L-$\multimap$R-2, $\multimap$L-$\otimes$L-2, $\multimap$L-$\otimes$L-3, $\otimes$L-$\otimes$R-4, $\otimes$L-$\multimap$L-2, $\otimes$L-$\multimap$L-4, $\otimes$L-$\multimap$L-8

5. $(\nu y)x\langle y\rangle.((\nu w)z\langle w\rangle.(P_w \mid Q_{zy}) \mid R_x) \Leftrightarrow (\nu w)z\langle w\rangle.(P_w \mid (\nu y)x\langle y\rangle.(Q_{zy} \mid R_x))$ if and only if $z \neq y, x$ and $x \neq w, z$.
   $\otimes$R-$\otimes$R-1, $\otimes$R-$\multimap$L-1, $\multimap$L-$\otimes$R-1, $\multimap$L-$\otimes$R-2, $\multimap$L-$\multimap$L-1, $\multimap$L-$\multimap$L-3

6. $(\nu y)x\langle y\rangle.(P_y \mid (\nu w)z\langle w\rangle.(Q_w \mid R_{xz})) \Leftrightarrow (\nu w)z\langle w\rangle.(Q_w \mid (\nu y)x\langle y\rangle.(P_y \mid R_{xz}))$ if and only if $z \neq y, x$ and $x \neq w, z$.
   $\otimes$R-$\otimes$R-2, $\otimes$R-$\multimap$L-2, $\multimap$L-$\otimes$R-3, $\multimap$L-$\multimap$L-2, $\multimap$L-$\multimap$L-4

7. $(\nu y)x\langle y\rangle.(z().P_y \mid Q_x) \Leftrightarrow z().(\nu y)x\langle y\rangle.(P_y \mid Q_x)$ if and only if $z \neq y, x$.
   $\otimes$R-**1**L-1, $\multimap$L-**1**L-1, **1**L-$\otimes$R-1, **1**L-$\multimap$L-1

8. $(\nu y)x\langle y\rangle.(P_y \mid z().Q_x) \Leftrightarrow z().(\nu y)x\langle y\rangle.(P_y \mid Q_x)$ if and only if $z \neq y, x$.
   $\otimes$R-**1**L-2, $\multimap$L-**1**L-2, $\multimap$L-**1**L-3, **1**L-$\otimes$R-2, **1**L-$\multimap$L-2

9. $x().z().P \Leftrightarrow z().x().P$
   1L-**1**L

10. $(\nu x)(z(y).P_{yzx} \mid Q_x) \Leftrightarrow z(y).(\nu x)(P_{yzx} \mid Q_x)$ if and only if $z \neq x$.
    $\otimes$L-cut-1, cut-$\multimap$R-1, cut-$\otimes$L-1

11. $(\nu x)(P_x \mid z(y).Q_{yzx}) \Leftrightarrow z(y).(\nu x)(P_x \mid Q_{yzx})$ if and only if $z \neq x$.
    $\multimap$R-cut-2, $\otimes$L-cut-4, cut-$\multimap$R-2, cut-$\otimes$L-2, cut-$\otimes$L-3

12. $(\nu x)(z().P_x \mid Q_x) \Leftrightarrow z().(\nu x)(P_x \mid Q_x)$ if and only if $z \neq x$.

13. $(\nu x)(P_x \mid z().Q_x) \Leftrightarrow z().(\nu x)(P_x \mid Q_x)$ if and only if $z \neq x$.

14. $(\nu x)((\nu y)z\langle y\rangle.(P_y \mid Q_{zx}) \mid R_x) \Leftrightarrow (\nu y)z\langle y\rangle.(P_y \mid (\nu x)(Q_{zx} \mid R_x))$ if and only if $z \neq x$.

15. $(\nu x)(P_x \mid (\nu y)z\langle y\rangle.(Q_{xy} \mid R_z)) \Leftrightarrow (\nu y)z\langle y\rangle.((\nu x)(P_x \mid Q_{xy}) \mid R_z)$ if and only if $z \neq x$.

16. $(\nu x)(P_x \mid (\nu y)z\langle y\rangle.(Q_y \mid R_{xz})) \Leftrightarrow (\nu y)z\langle y\rangle.(Q_y \mid (\nu x)(P_x \mid R_{xz}))$ if and only if $z \neq x$.

4 Other Comments

- It's interesting to note that commuting conversions do not fit the labeled transition system exactly as one might expect. For instance, the π-calculus LTS includes the rule

$$\frac{}{!x(y).P_y \xrightarrow{!x(-)} \lambda y.\, P_y \mid !x(y).P_y}$$

We might expect that (roughly)

$$w().!x(y).P_y \Leftrightarrow^* \xrightarrow{!x(-)} \Leftrightarrow^* \lambda y.\, w().P_y \mid !x(y).P_y$$

and

$$w().!x(y).P_y \Leftrightarrow^* \xrightarrow{!x(-)} \Leftrightarrow^* \lambda y.\, P_y \mid w().!x(y).P_y$$

However, because there is no commuting conversion for $w().!x(y).P_y$, these transitions are not derivable in the absence of an explicit rule for transitions under a $w()$. prefix:

$$\frac{P \xrightarrow{!x(-)} \lambda y.\, Q_y}{w().P \xrightarrow{!x(-)} \lambda y.\, w().Q_y}$$

Similarly, though somewhat less unexpectedly, it is not possible to terminate a session until all of the sessions that it uses have terminated. This is formally reflected in that the transition

$$y().z\langle\rangle.\mathbf{0} \Leftrightarrow^* \xrightarrow{z\langle\rangle} \cdot$$

is not derivable. Note that this differs from the behavior of $y().\mathbf{0} \mid z\langle\rangle.\mathbf{0}$, which does allow the transition $y().\mathbf{0} \mid z\langle\rangle.\mathbf{0} \xrightarrow{z\langle\rangle} y().\mathbf{0}$.

- On the other hand, if we are not interested in offering $!x(-)$ and $x\langle\rangle$ external actions, then I claim:

Claim 1. *If $\Gamma;\Delta \vdash P :: z{:}A$ and $P \xrightarrow{\alpha} Q$, then either: 1. $A = !A_0$ and $s(\alpha) = z$, 2. $A = \mathbf{1}$ and $s(\alpha) = z$, or 3. $x().P \Leftrightarrow^* \xrightarrow{\alpha} \Leftrightarrow^* x().Q$.*

Proof Idea. By induction on the derivation of the transition.

Case:

$$\frac{Q_{xz} \xrightarrow{\alpha} Q'_{xz}}{(\nu x)(P_x \mid Q_{xz}) \xrightarrow{\alpha} (\nu x)(P_x \mid Q'_{xz})}$$

$y().(\nu x)(P_x \mid Q_{xz}) \Leftrightarrow (\nu x)(y().P_x \mid Q_{xz})$
$(\nu x)(y().P_x \mid Q_{xz}) \xrightarrow{\alpha} (\nu x)(y().P_x \mid Q'_{xz})$
$(\nu x)(y().P_x \mid Q'_{xz}) \Leftrightarrow y().(\nu x)(P_x \mid Q'_{xz})$

□

- The above is related to the problem we had in giving a labeled transition system for reductions under $\mathbf{1}\mathsf{L}$ and $!\mathsf{L}$. For instance,

$$(\nu x)\big(y().x\langle\rangle.\mathbf{0} \mid x().z\langle\rangle.\mathbf{0}\big)$$

will not reduce until we use commuting conversions to move $y().$ to the other branch:

$$\begin{aligned}
&(\nu x)\big(y().x\langle\rangle.\mathbf{0} \mid x().z\langle\rangle.\mathbf{0}\big) \\
\Leftrightarrow\ &y().(\nu x)\big(x\langle\rangle.\mathbf{0} \mid x().z\langle\rangle.\mathbf{0}\big) \\
\Leftrightarrow\ &(\nu x)\big(x\langle\rangle.\mathbf{0} \mid y().x().z\langle\rangle.\mathbf{0}\big) \\
\Leftrightarrow\ &(\nu x)\big(x\langle\rangle.\mathbf{0} \mid x().y().z\langle\rangle.\mathbf{0}\big) \\
\longrightarrow\ &y().z\langle\rangle.\mathbf{0}
\end{aligned}$$

This non-local conversion can't be represented locally in an LTS, which is why we needed to introduce a process context as an extra output.