Discrete vs. Continuous Distributions

Question: What's the difference between a discrete distribution and a continuous one?

Discrete Distributions

Story 1: With probability p your machine will turn on (value 1) and with probability $1-p$ it will not (value 0) . Let X denote the value of your machine.

$X =$

We write $X \sim$

$\mathbf{P}\{X = 1\} = p_X(1) =$

$\mathbf{E}[X] =$ average value of $X =$

Discrete Distributions, cont.

Story 2: Every day I buy a lottery ticket which has probability p of winning the lottery. Let X denote the number of days until I win the lottery.

We write $X \sim$

$\mathbf{P}\{X = i\} = p_X(i) =$

$\mathbf{P}\{X > i\} = \overline{F}_X(i) =$

Question: What is $\mathbf{E}[X]$? Let's look at 2 WAYS to get there!

Discrete Distributions, cont.

Story 3: I have n machines. Each works independently with probability p. Let X denote the total number of working machines.

We write $X \sim$

$\mathbf{P}\{X = i\} = p_X(i) =$

Question: What is $\mathbf{E}[X]$?

Continuous Distributions

For continuous distributions, probability is defined via a probability density function (p.d.f.).

Let $f_X(x)$ denote the p.d.f. of X.

As always, $F_X(x) = \mathbf{P}\{X \leq x\}$ and $\overline{F}_X(x) = \mathbf{P}\{X > x\}$.

Story 4: The next job is equally likely to arrive anytime between time a and time b. Let X denote the time of arrival of the next job.

We write $X \sim$

$f_X(t) =$

$F_X(x) = \mathbf{P}\{X \leq x\} =$

$\mathbf{E}[X] =$

Continuous Distributions, cont.

$X \sim \text{Exp}(\lambda)$.

$f_X(t) =$

$F_X(x) =$

$\overline{F}_X(x) =$

$\mathbf{E}[X] =$

λ is called the **rate** of X.

Generating R.V.s for Simulation

UNIX gives you instances of Uniform$(0, 1)$.

How can you convert these to instances of some other distribution, X?

Inverse Transform Method

1. Need to know $F_X(x) = \mathbf{P}\{X \leq x\}$.

2. Need to be able to invert $F_X(x)$, i.e., get x from $F_X(x)$.

Inverse Transform Method for Continuous R.V.s

Given: Instance u of Unif$(0, 1)$.

Want: Instance x of r.v. X: continuous, non-negative.

Know: $F_X(x) = \mathbf{P}\{X \leq x\}$

Inverse Transform Method for Continuous R.V.s

Question: How can we generate an instance of $X \sim \text{Exp}(\lambda)$?

Inverse Transform Method for Discrete R.V.s

Given: Instance u of $\text{Unif}(0,1)$.

Want: Instance x of r.v. X: discrete

Know: $F_X(x) = \mathbf{P}\{X \leq x\}$

$$X = \begin{cases} x_0 & \text{with prob } p_0 \\ x_1 & \text{with prob } p_1 \\ \cdots & \\ x_k & \text{with prob } p_k \end{cases} .$$