 HW 7 – Due start of class 11/11

Homework is due at the start of class on 11/11. Please turn in a hardcopy. It can be handwritten. Please show all computations.

Reading from your textbook: Chapter 2.6, 2.7 and Chapter 6. Remember that when reading you should cover all the "Answers" and derivations and try them yourself first.

Exercises: All the exercises are ANAYLSIS problems.

1. **The Arthur Ravenel Bridge**

   The Arthur Ravenel bridge in Charleston allows walkers and joggers to get to downtown Charleston from Mount Pleasant. During my visit to Charleston, I observed that:

   - On average, 20 walkers arrive per hour and take an average of 1 hour to cross the bridge.
   - On average, 10 joggers arrive per hour and take an average of 20 minutes to cross the bridge.

   Based on this data, estimate the average number of people (walkers plus joggers) on the bridge at any time.

2. **Data Center Utilization**

   The Clouds-R-Us company runs a data center with 10,000 servers shown in Figure 1. Jobs arrive to the data center with average rate $\lambda = 2$ jobs/sec. Each job requires some number of servers K, where $K \sim \text{Binomial}(1000, 0.05)$. The job holds onto these K servers for some time S seconds, where $S \sim \text{Exp}(0.02)$ and then releases all its servers at once. Assume that K and S are independent. Jobs are served in FCFS order. If a job gets to the head of the queue, but the number of servers that it needs exceeds the number of idle servers, then the job simply waits (blocking those jobs behind it in the queue) until that number of servers becomes available. On average, how many jobs are running at a time?

3. **Network that looks like a flip flop**

   Tianxin's network looks like a flip flop. Jobs arrive to Tianxin's network at a rate of $r = 1$ jobs per second. The routing probabilities are shown. The service rate at station A is $\mu_A = 3$ jobs per second, and that at station B is $\mu_B = 4$ jobs per second. An individual job might pass through Station A, then B, then A, then B, etc., before it eventually leaves. Tianxin has observed that the expected number of jobs at station A is $\mathbf{E}[N_A] = 2$ and the expected number of jobs at station B is $\mathbf{E}[N_B] = 1$. Answer the following questions:

   (a) Let T denote the response time of a job, i.e., the time from when it arrives until it departs. What is $\mathbf{E}[T]$?

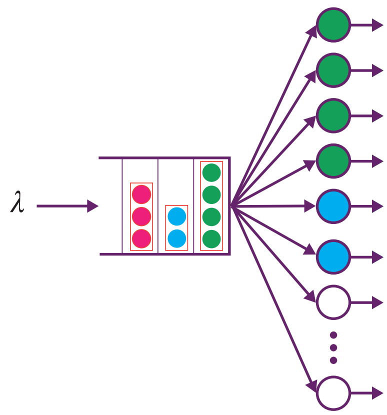

Figure 1: *Datacenter for Exercise 2.*

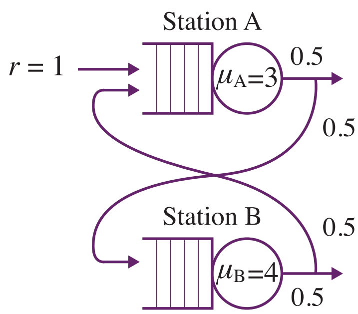

Figure 2: *Network for Exercise 3.*

(b) Let λ_A denote the total arrival rate into station A. Let λ_B denote the total arrival rate into station B. What are λ_A and λ_B?

(c) What is the throughput of the system? What is the throughput of station A? Which is higher?

(d) Let T_A denote the time it takes for a job to make a single visit to station A (this includes queueing and then serving at station A). Likewise let T_B denote the time it takes for a job to make a single visit to station B. What are $\mathbf{E}[T_A]$ and $\mathbf{E}[T_B]$?

(e) Let T_Q denote the total time that a job spends queueing while in the system. This includes the total time that the job is in queues from when it arrives until it leaves the system. What is $\mathbf{E}[T_Q]$?

4. **Finally a haircut!**

For over a year in lockdown, I haven't been able to get my hair cut, but finally I can return to the salon! At my salon there are two stations: the washing station and the cutting station, each with its own queue. The people who work at the washing station only wash hair. The people who work at the cutting station only cut hair. When a washing person is free they take the next person in the wash line, and similarly for the cutting line.

My salon is very quick. The average wash time is only $\mathbf{E}[S_{wash}] = \frac{1}{13}$ hours and the average cut time is only $\mathbf{E}[S_{cut}] = \frac{1}{7}$ hours. Unfortunately, they're so quick that they sometimes forget to rinse the shampoo, so with probability $\frac{1}{4}$ I need to rejoin the wash line after my wash is complete.

There are two types of customers at my salon: the "wash-and-cut" customers, who get their hair washed and then cut, and the "cut-only" customers who only get their hair cut (no wash).

The salon is shown in Figure 3. Assume that 54 customers enter the salon per hour. Assume that $\frac{2}{3}$ are wash-and-cut customers and $\frac{1}{3}$ are cut-only customers.

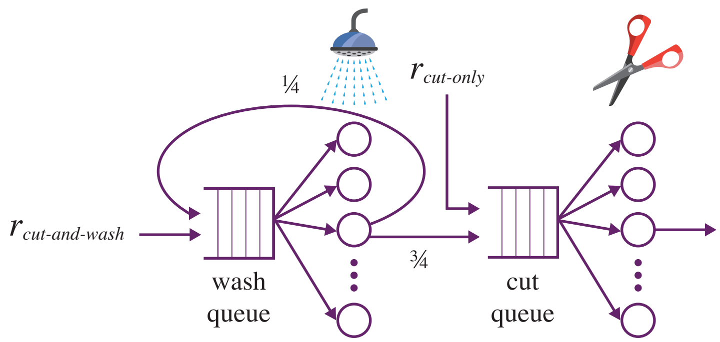

Figure 3: *Haircut station for Exercise 4.*

(a) What is the bare minimum number of staff (washers + cutters) that are needed to ensure that the hair salon is stable?

(b) On average, the number of customers at the wash station (either in the wash queue or having their hair washed) is 9. On average, the number of customers at the cutting station (either in the cut queue or having their hair cut) is 18.

i. What is the expected response time of a random customer (we're not told the type of the customer)?

ii. What is the expected response time of a cut-only customer? [Hint: Think about the experience of a cut-only customer.]

5. **Do Exercise 6.4 in your textbook.**