Instructions: Homework is due Friday by start of recitation. You have a week. We grade your homework right away, so please don’t be late. If you’re having problems, please go to office hours early in the week. Feel free to collaborate with other students, but you should write up your own solutions. It is good form to list the names of people with whom you collaborate. Note: In doing homework, you’ll find that series and integrals come up frequently. Make sure you know how to do all the series in Kristy’s warm-up sheets on your own, because you won’t have math packages available on exams.

1 Problems: 3.53, 3.62, 4.1, 4.2, 4.3, 5.1, 6.3, 6.4, 6.5

You can SKIP one of the problems in Chpt 4. Please MARK the skipped problem.

Note: Some of these problems are not (yet) in the book. They’re described on the backside.

2 Open-ended problem …

This is a graduate class, so open problems will come up in class all the time. These are entirely OPTIONAL of course.

Consider a closed batch system with a fixed MPL \( N \). Imagine that job sizes are described by r.v. \( S \) which is 1 with probability \( p \) and 100 with probability \( 1 - p \). As soon as a job completes, another job starts, where the new job’s size again follows r.v. \( S \). The entire central subsystem is a single queue.

In System A, the scheduling order at the single queue is FCFS. In System B, the scheduling order at the single queue is SJF (Shortest-Job-First).

What can you say about mean response time in system A as compared with system B?

Hint 1: Try thinking about what happens for different values of \( N \).

Hint 2: If it’s hard to think about mean response time, is it easier to think about throughput, \( X \), instead, and then translate?
Problem 3.53 – [More on Coupon Collecting] In the coupon collection problem, there are $n$ distinct coupons that we’re trying to collect. Every time that we draw a coupon, we get one of the $n$ at random, with each coupon being equally likely; the coupon we get is replaced after each drawing. Thus it is likely that the same coupon will be drawn more than one time. If we have only $n$ draws available, what is the expected number of distinct coupons that we expect to see? If $n$ is large, what is this approximately as a function of $n$?

Problem 3.62 – [When the last server dies] Nivedita has bought two very old servers to host her new online game. At the time of purchase, she was told that each of the servers will fail at some uniformly-distributed random time during the next year, where the servers fail independently of each other. Today, half a year later, the game is still up, which means that at least one server did not yet fail. What is the expected time until the last server fails? Below are some helping steps:

1. Start by solving the following easier problem: Let $X_1 \sim \text{Uniform}(0, 1)$ and $X_2 \sim \text{Uniform}(0, 1)$, where $X_1 \perp X_2$. Let $X = \max(X_1, X_2)$. Derive $E[X]$.

2. Restate the original problem question in terms of the above variables.

3. Now solve the original problem.