

15-857/47-774 Homework 11: M/G/1 & Busy Period

Homework is due at the *start* of Friday's class. You have a full week. We grade your homework right away, so please don't be late. If you're having problems, please go to office hours. Feel free to collaborate with other students, but you should write up your own solutions. It is good form to list the names of people with whom you collaborate.

These problems are from your textbook, *Performance Modeling and Design of Computer Systems*. Starred problems are either not in your textbook, or have some modifications, given below.

Exercises: 23.15* ; 24.6 ; 26.3 ; 26.5* ; 26.6* ; 27.1 ; 27.3 ; 27.6*

Exercise 23.15*: [Application of Renewal Reward – Tracking event during busy period] Consider an M/M/1. Let T denote the time spent in state 9 during a busy period. Your goal is to derive $\mathbf{E}[T]$. [Hint: There are at least 3 ways of doing this problem. All are very short.]

Exercise 26.5*: [M/M/2 Transform] Please ignore the reference to Exercise 26.4 in there. That is unnecessary. Please just directly use Distributional Little's Law as defined in class.

Exercise 26.6*: [M/M/1 Transform Again!] Given an M/M/1 with arrival rate λ and service rate μ , let T denote the response time and N denote the number of jobs.

- (a) Derive $\tilde{T}(s)$ by first deriving $\hat{N}(z)$ and then applying Distributional Little's Law (DLL).
- (b) Suppose your goal is to derive $\tilde{T}(s)$ for the M/M/1/2 queue, with arrival rate $\lambda = 1$ and service rate $\mu = 1$. You decide to use this same approach of going through DLL.
 - (i) You start by expressing $\hat{N}(z)$. Write this simple quantity (use the values of λ and μ given in the problem).
 - (ii) You now try to use DLL to get $\tilde{T}(s)$. Point to exactly where this breaks down.
 - (iii) Although you can't use DLL, you can still get $\tilde{T}(s)$. What is it?

Exercise 27.6*: [Number of Jobs Served during M/G/1 Busy Period] Please ignore the part about getting the second moment. Who's got time for all that?