15-857/47-774 Homework 6: Exponentials and Poisson Process

This homework will NOT be graded. However you should do it before the **start** of Wednesday's class. The homework solutions will be available in class, so that you have them to study. These problems are useful preparation for your midterm!

These problems are from your textbook, *Performance Modeling and Design of Computer Systems*. Starred problems are either not in your textbook, or have some modifications, given below.

You will need to wait until Friday's class to do 13.1 and until Monday's class to do 13.5.

Exercises: 11.4*; 11.5; 11.6*; 11.11*; 13.1; 13.5; 25.6; 25.9

Exercise 11.4: [Failure rate] Should say $f(t)$, $t \geq 0$.

Exercise 11.6*: [Number of Arrivals of a Poisson Process during a Service]

Read the problem in the book. What I want you to derive are these 4 quantities:

(a) Let A_t denote the number of arrivals by time t. Derive $\mathbf{E}[A_t]$.

(b) Let A_S denote the number of arrivals during time S, where S is a continuous r.v. Derive $\mathbf{E}[A_S]$ by conditioning on S and using (a).

(c) Derive the z-transform $\widehat{A_t}(z)$.

(d) Drive $\widehat{A_S}(z)$ by conditioning on S and using (c). [Hint: Your final answer should be a Laplace transform!]

Exercise 11.11: [Reliability Theory] Add on these two parts:
(c) What is the distribution of Z?
(d) Derive $\widetilde{Z}(s)$, the Laplace transform of Z. [Hint: Do not use integrals. This should be 1 line if you use the right argument.]