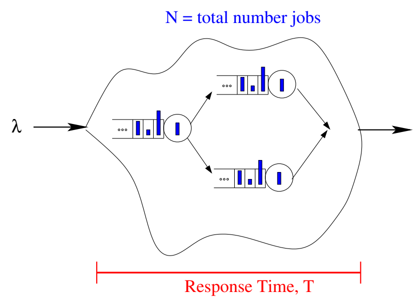
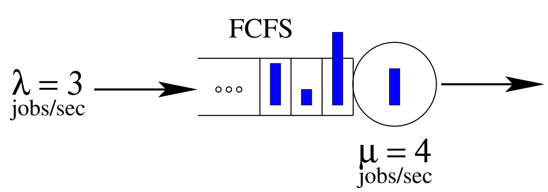
1 Motivation & Intuitions for Little's Law

Today's Question: In a system with many queues, how does $\mathbf{E}[T]$ relate to $\mathbf{E}[N]$?



Question: Why do we care?

A Simpler Example:



2 Little's Law for Open Systems

Theorem: For ANY ergodic open system,

$$\mathbf{E}[N] = \lambda \mathbf{E}[T].$$

(We'll formally define "ergodic" in chpt 9, but, loosely, it assumes the system is "well-behaved." That is, the system empties infinitely often, so every job that enters the system will leave in a finite time, and the state is not dependent on the time-step. Importantly, "ergodic" subsumes the conditions in the stronger theorem below.)

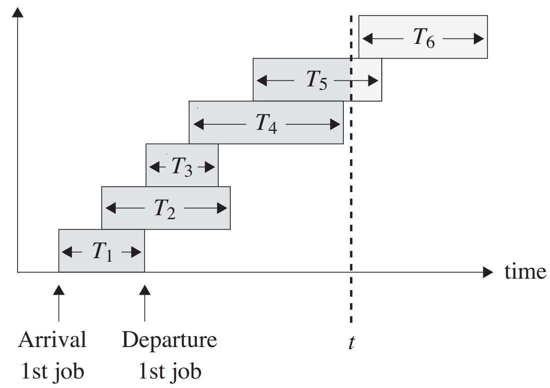
Time-Avg Theorem: For ANY open system, $\forall$ sample paths ω where $\lambda = X$, and $\overline{N}^{TimeAvg}(\omega)$ and $\overline{T}^{TimeAvg}(\omega)$ exist,

$$\overline{N}^{TimeAvg}(\omega) = \lambda \cdot \overline{T}^{TimeAvg}(\omega).$$

Question: Define the above terms.

3 Proof of Little's Law

Let ω be a single sample path. Let T_i: reponse time of ith arrival under ω.



4 Extensions of Little's Law

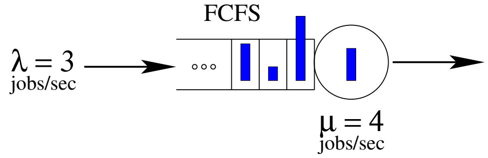
Question: Does the proof assume FCFS?

Question: Does the proof assume a single server?

Question: Can we relate $\mathbf{E}[N_Q]$ to $\mathbf{E}[T_Q]$?

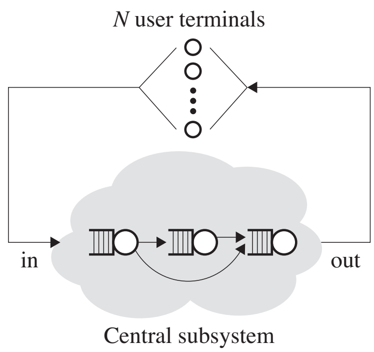
5 Extensions of Little's Law, cont.

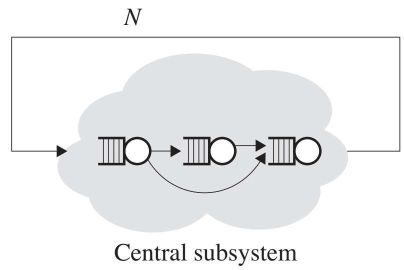
Question: For the single-server queue, recall we conjectured $\rho = \frac{\lambda}{\mu}$. How can we prove this via Little's Law?



Question: Suppose we are only interested in **red** jobs. Can we apply Little's Law?

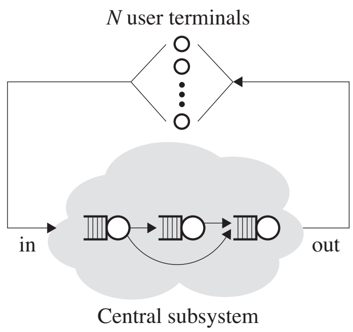
6 Let's now turn to Closed Systems

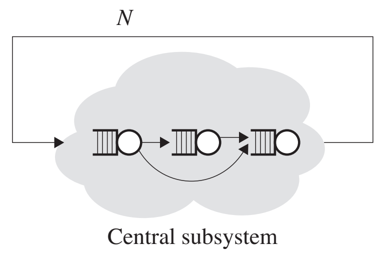




T = time in system
R = residence time
Z = think time

7 Little's Law for Closed Systems





Theorem: For ANY ergodic closed system (interactive or batch):

$$N = X \cdot \mathbf{E}[T].$$

Time-Average Theorem: For ANY closed system, $\forall \omega$, assuming $\lambda = X$,

$$N = \lambda \cdot \overline{T}^{TimeAvg}(\omega).$$

8 Commercial Break: Announcements

1. HW 1 returned today. Each problem was given: 0, 1, 2, 3 points. Total: 21. Scores were very high!

2. Please read **solutions** for all problems, not just the ones that were marked wrong!

3. See TAs if there's a problem, but try not to obsess over grades. Focus on learning!

4. HW 2 is due Friday. Remember to bookmark class website:
   www.cs.cmu.edu/∼harchol/Perfclass/class.html
   and Homework-and-Announcements link!

5. **Zhouzi has office hours TODAY**: **3:30 p.m. - 5 p.m.** in **GHC 6003**.

6. Please keep up with the reading for this class. Spend 3 hours after every class reading the relevant chapter.

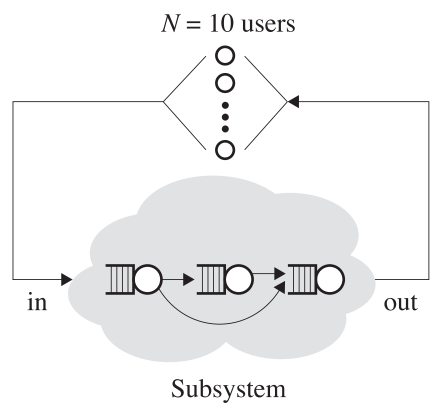
7. If you miss class, you can see me to copy my notes.

8. Anyone figure out who "Runting" is in your HW 2?

9 Example

We have an interactive system with $N = 10$ users. We are told that the expected think time is $\mathbf{E}[Z] = 5$ seconds, and that the expected residence time is $\mathbf{E}[R] = 15$ seconds.

Note that the residence time is the time it takes a job to get from "in" to "out."

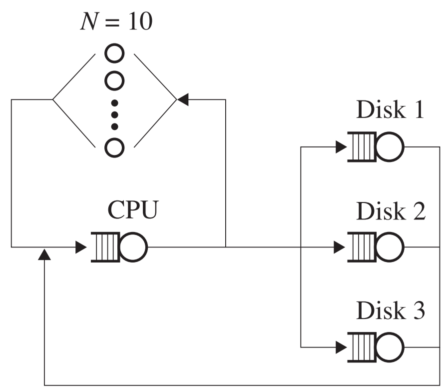


Question: What is the throughput, X, of the system?

10 Example

For the system below:

- The throughput of disk 3 is 40 requests/sec ($X_{\text{disk 3}} = 40$).
- The service time of an average request at disk 3 is 0.0225 sec ($\mathbf{E}[S_{\text{disk 3}}] = .0225$.)
- The average number of jobs in the system consisting of disk 3 and its queue is 4 ($\mathbf{E}[N_{\text{disk 3}}] = 4$).



Question: What is the utilization of disk 3?

Question: What is the mean time spent at disk 3 (queueing plus service)?

Question: Find $\mathbf{E}$ [Number of requests queued at disk 3].

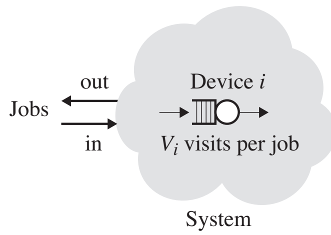
Next we are told that:

- $\mathbf{E}$ [Number of ready users (not thinking)] $= 7.5$
- $\mathbf{E}$ [Think time] $= \mathbf{E}[Z] = 5$ sec

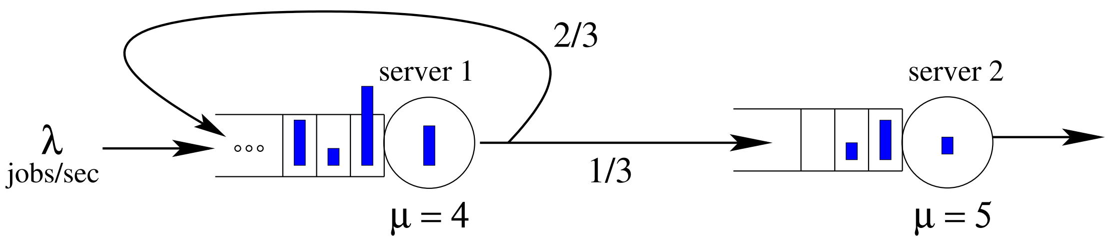
Question: What is the system throughput?

11 Forced Flow Law

Illustration of a single device within a larger system:



- X = system throughput
- X_i = throughput at device i
- V_i = number of visits to device i per job



12 Example

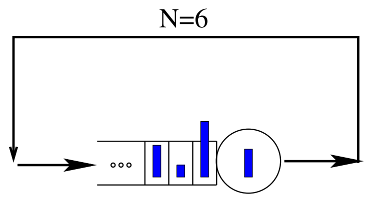
We have an interactive system with the following characteristics:

- 25 terminals ($N = 25$)
- 18 seconds average think time ($\mathbf{E}[Z] = 18$)
- 20 visits to a specific disk per interaction on average ($\mathbf{E}[V_{disk}] = 20$)
- 30% utilization of that disk ($\rho_{disk} = .3$)
- .025 sec average service time per visit to that disk ($\mathbf{E}[S_{disk}] = .025$)

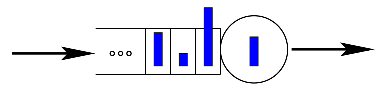
That is all the information we have.

Question: What is the mean residence time, $\mathbf{E}[R]$?

13 If time: a puzzle



SRPT vs. FCFS



SRPT vs. FCFS

Question: SRPT reduces $\mathbf{E}[T]$ a lot for open systems, particularly in high load. What happens in closed systems?

[See paper: Schroeder, Wierman, Harchol-Balter. "Open versus closed: A cautionary tale". NSDI 2006.]