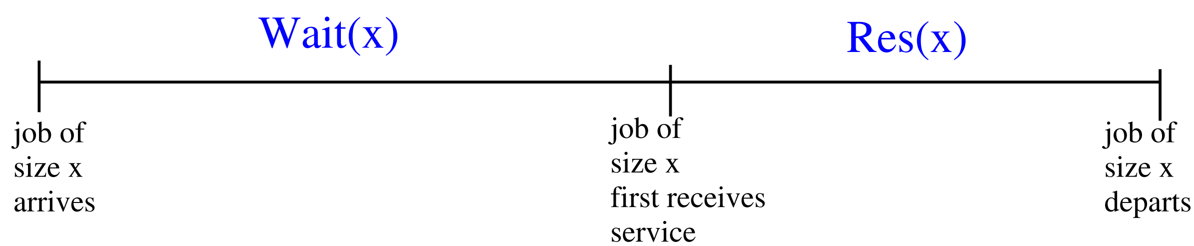
# 1 Preemptive-Shortest-Job-First (PSJF)

PSJF is the preemptive version of SJF. Priority is based on **original size**.

$$T(x)^{PSJF} = Wait(x)^{PSJF} + Res(x)^{PSJF}$$




<u>Defn</u>: $W_{\le x}$ = remaining work job $x$ sees comprising only jobs of size $\le x$.

- $Wait(x)$ represents the time until job $x$ receives its first bit of service.
  - This is a Busy Period started by ________________ work, where only ________________________________ are allowed in.

- $Res(x)$ is the time from when job $x$ receives first receives service until it completes.
  - This is a Busy Period started by ________________ work, where only ________________________________ are allowed in.

**Today's Goal:** Derive $\widetilde{T(x)}(s)$

## 2 Warming up on transforms

Consider an M/G/1 with arrival rate $\lambda$ and job sizes $S$.

$A_y$ is the number of arrivals in time $y$
$B$ is the duration of a busy period
$B(y)$ is the duration of a busy period started by work $y$
$B_W$ is the duration of a busy period started by work $W$

**Question:** Derive $\widehat{A_y}(z)$

**Question:** Derive $\widetilde{B(y)}(s)$

**Question:** Derive $\widetilde{B}(s)$

**Question:** Derive $\widetilde{B_W}(s)$

$A_y^{\le x}$ is the number of arrivals of size $\le x$ during time $y$
$B_{\le x}$ is the duration of a busy period of jobs of size $\le x$ only
$B_{\le x}(y)$: duration of busy period of jobs of size $\le x$, started by work $y$
$S_{\le x}$ is the size of a job of size $\le x$.
$\lambda_{\le x}$ = arrival rate of jobs of size $\le x$

**Question:** Derive $\widehat{A_y^{\le x}}(z)$

$$\begin{aligned}\widehat{A_y}(z) = \sum_{i=0}^{\infty} z^i \cdot \mathbf{P}\{A_y = i\} &= \sum_{i=0}^{\infty} z^i \cdot \frac{e^{-\lambda y} \cdot (\lambda y)^i}{i!} \\ &= e^{-\lambda y} \sum_{i=0}^{\infty} \frac{(\lambda y z)^i}{i!} \\ &= e^{-\lambda y} \cdot e^{\lambda y z} \\ &= e^{-\lambda y(1-z)}\end{aligned}$$

**Question:** Derive $\widetilde{B_{\le x}(y)}(s)$

$$\begin{aligned}B(y) &= y + \sum_{i=1}^{A_y} B^{(i)} \\ \widetilde{B(y)}(s) &= e^{-sy} \cdot \widehat{A_y}(\widetilde{B}(s)) \\ &= e^{-sy} \cdot e^{-\lambda y(1-\widetilde{B}(s))} \\ &= e^{-y(s+\lambda-\lambda\widetilde{B}(s))}\end{aligned}$$

**Question:** Derive $\widetilde{B_{\le x}}(s)$

$$\begin{aligned}\widetilde{B}(s) &= \int_0^{\infty} \widetilde{B(y)}(s) \cdot f_S(y) dy \\ &= \int_0^{\infty} e^{-y(s+\lambda-\lambda\widetilde{B}(s))} \cdot f_S(y) dy \\ &= \widetilde{S}(s + \lambda - \lambda\widetilde{B}(s))\end{aligned}$$

# 3 Transform Analysis of PSJF

## 3.1 $\widetilde{Res(x)}(s)$

## 3.2 $\widetilde{Wait(x)}(s)$ in terms of $\widetilde{W_{\leq x}}(s)$

Recall $W_{\leq x}$ is the work in PSJF system made up of jobs of size $\leq x$.

### 3.3 Expressing $\widetilde{W_{\le x}}(s)$

$$
\begin{aligned}
W_{\le x} &= \text{Remaining work in PSJF system due to jobs of size} \le x \\
&= \text{Total rem. work in PSJF system assuming}\rule{5cm}{0.4pt} \\
&= \text{Total work in } \rule{2cm}{0.4pt} \text{ system assuming } \rule{5cm}{0.4pt} \\
&= \rule{1cm}{0.4pt} \text{ in } \rule{2cm}{0.4pt} \text{ system assuming } \rule{6cm}{0.4pt}
\end{aligned}
$$

Hence:

$$\widetilde{W_{\le x}}(s) =$$

**Hint:** For an M/G/1/FCFS, with arrival rate $\lambda$ and job size $S$:

$$\widetilde{T_Q}(s)^{FCFS} = \frac{(1-\rho)s}{\lambda \widetilde{S}(s) - \lambda + s}$$

### 3.4 Back to $\widetilde{Wait(x)}(s)$

**Question:** What is $\widetilde{T(x)}(s)$?

## 4 Commercial Break: Announcements

1. Mor has office hours today: 5:30 p.m. - 7 p.m. in GHC 7207. Please come!

2. Exam Dec 4th. The time is nominally 5 - 7 p.m., but we won't kick you out before 8 p.m.

3. Please ask me questions!

## 5 The BEST scheduling policy: SRPT

SRPT is Shortest-Remaining-Processing-Time.

At all moments of time, the server is working on the job with the shortest remaining time.

**Question:** Explain how SRPT differs from PSJF.

We don't have time to derive SRPT's performance, but we will discuss the result.

Recall:

$$\mathbf{E}\left[T(x)\right]^{PSJF} = \frac{\frac{1}{2}\lambda_{\leq x}\mathbf{E}\left[S_{\leq x}^2\right]}{(1-\rho_{\leq x})^2} + \frac{x}{1-\rho_{\leq x}}$$

or equivalently:

$$\mathbf{E}\left[T(x)\right]^{PSJF} = \frac{\frac{\lambda}{2}\int_0^x t^2 f(t)dt}{(1-\rho_{\leq x})^2} + \frac{x}{1-\rho_{\leq x}}$$

**By contrast:**

$$\mathbf{E}\left[T(x)\right]^{SRPT} = \frac{\frac{\lambda}{2}\int_{t=0}^x t^2 f(t)dt + \frac{\lambda}{2}x^2(1-F(x))}{(1-\rho_{\leq x})^2} + \int_{t=0}^x \frac{dt}{1-\rho_{\leq t}}$$

**Question:** Explain the difference in the **residence time** ...

Recall:

$$\mathbf{E}\left[T(x)\right]^{PSJF} = \frac{\frac{1}{2}\lambda_{\leq x}\mathbf{E}\left[S^2_{\leq x}\right]}{(1-\rho_{\leq x})^2} + \frac{x}{1-\rho_{\leq x}}$$

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**Question:** Explain the difference in the **waiting time** ...

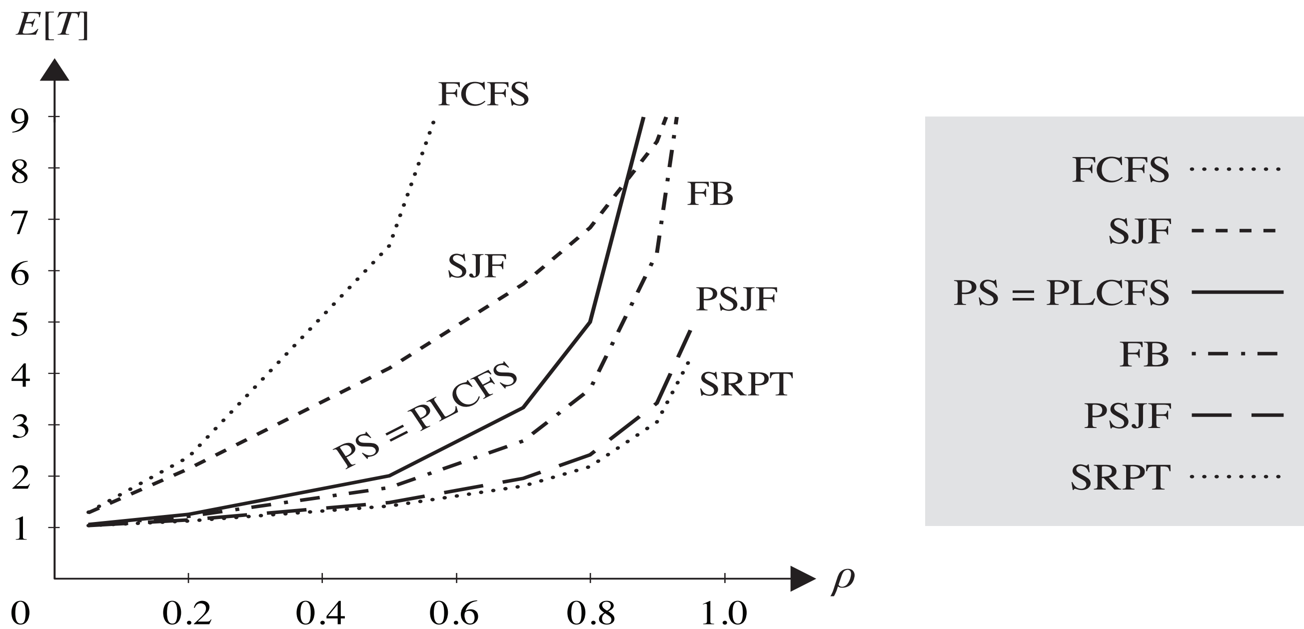


Figure 1: *Mean response time as a function of load for the M/G/1 with various scheduling policies. The job size distribution is a Weibull with mean* 1 *and* $C_S^2 = 10$.

If we look at load $\rho = 0.75$, we see:

$$FCFS \gg SJF \gg PS = PLCFS > FB > PSJF > SRPT$$

(Much more extreme for higher $C_S^2$.)

**Question:** Why are the policies ordered this way?

## 6 If SRPT is so great, why not always use it?

**Question:** Why not use SRPT or PSJF all the time?

**Question:** Suppose we know job size distribution but not individual job size?

**THANK YOU FOR TAKING THIS CLASS!!**

PLEASE stay in touch to talk about your research or industry experiences!