Assume M/G/1, where G has finite mean and variance. Assume $\rho < 1$, and some work-conserving scheduling policy.

Definition: Under **preemptive** scheduling, we are allowed to stop (preempt) a running job and continue running it later where it left off, at zero cost for preemption.

	Non–Preemptive	Preemptive
Don't Use Size		
Use Size/ Class		

TODAY: Preemptive, but don't use size. Today will be low on proofs, but high on intuition. You can read the proofs in the book, but you're only responsible for what we cover in class.

- For non-preemptive policies, easier to talk about $T_Q(x)$.

- For preemptive policies, easier to talk about $T(x)$.

1 Prep for preemptive policies – back to busy periods

$B = B_S =$ Busy period started by arbitrary job.

$B_x =$ Busy period started by x work.

$B_W =$ Busy period started by W work (W is a r.v., not a constant).

Question: Derive $\mathbf{E}[B_x]$, $\mathbf{E}[B]$, and $\mathbf{E}[B_W]$ WITHOUT using transforms

2 Processor-Sharing: The definition of "fair"

Processor-Sharing, cont.

Theorem 30.4: (not proving – see book)

$$\mathbf{E}\left[T(x)\right]^{M/G/1/PS} = \frac{x}{1-\rho}$$

Implications:

- **Question:** $\mathbf{E}\left[T\right]^{PS} =$

- **Question:** $\mathbf{E}\left[N\right]^{PS} =$

- **Question:** $\mathbf{E}\left[\text{Slowdown}(x)\right]^{PS} =$

- **Question:** What's the intuition here?

- **Question:** Is $T(x)$ a busy period? (Recall $\mathbf{E}\left[T(x)\right] = \frac{x}{1-\rho}$)

Processor-Sharing, cont.

- **Question:** Which is lower: $\mathbf{E}[T]^{PS}$ or $\mathbf{E}[T]^{FCFS}$?

- **Question:** What about $\mathbf{Var}(T)^{PS}$?

3 P-LCFS: Preemptive Last-Come-First-Served

P-LCFS: Server is at all times working on last job to enter system.
Preemptive version of LCFS.

Question: Do you expect $\mathbf{E}\left[T(x)\right]^{PLCFS}$ is better or worse than $\mathbf{E}\left[T(x)\right]^{LCFS}$?

4 Comparisons

Question: How do PLCFS and LCFS compare wrt $\mathbf{E}[T(x)]$?

Question: Why intuitively is PLCFS sometimes better?

Question: How do PLCFS and PS compare wrt $\mathbf{E}[T(x)]$?

Question: Is there a practical advantage of PLCFS over PS?

5 Commercial Break: Announcements

1. Midterm 2 will take place Thursday, Dec 4th, in the current classroom, starting 5 p.m. Start studying now! The exam will cover chpts 14-33, but only what we covered in class, so look at the handouts. For example, we skipped Chpts 19 and 22 and also covered only parts of other chapters.

2. You can have a large (4x6) index card and write on both sides.

3. You can come to my office any day, including today, to copy my filled in notes for any lectures that you missed.

4. Zhouzi has office hours today right after class: GHC 6003.

5. You can have one 5x7 index card for the exam – you get to write on both sides. These will be available in the bins outside my office.

6. HW 12 is due 5 p.m. on Tuesday Dec 2 at my office: GHC 7207. This is a HARD DEADLINE. The TAs will pick up your homework for grading at 5, and I will give you solutions immediately for studying.

7. There will be NO CLASSES and NO OFFICE HOURS on Wednesday, Thursday, and Friday. Happy Thanksgiving everyone!

6 The FB, a.k.a. LAS policy

Question: For preemptive policies that don't use size, is $\mathbf{E}[T(x)] = \frac{x}{1-\rho}$ the best we can do?

7 Preliminary math exercise: a transformed job size

ORIGINAL NEW

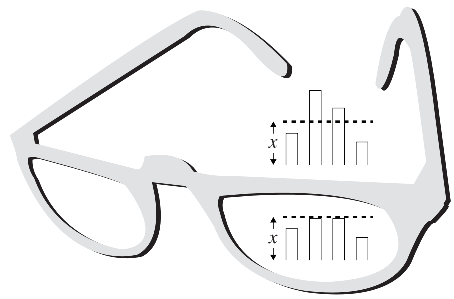
$\mathbf{E}[S] =$ $\mathbf{E}[S_{\overline{x}}] =$

$\mathbf{E}\left[S^2\right] =$ $\mathbf{E}\left[S_{\overline{x}}^2\right] =$

$\rho =$ $\rho_{\overline{x}} =$

8 Analyzing $\mathbf{E}[T(x)]^{FB}$

Question: What work has to get done before job of size x ("job x") can leave?



9 Comparing $\mathbf{E}[T]^{FB}$ with $\mathbf{E}[T]^{PS}$

In the textbook, we derive $\mathbf{E}[T(x)]^{FB}$:

$$\mathbf{E}[T(x)]^{FB} = \frac{x + \frac{\lambda \mathbf{E}\left[S_{\overline{x}}^2\right]}{2(1-\rho_{\overline{x}})}}{1 - \rho_{\overline{x}}}$$

<u>Theorem</u>: [Wierman, Bansal, Harchol-Balter ORL 2004]

Given an M/G/1 with job sizes denoted by r.v. S:

- If S has D.F.R., then ______________________________

- If S has I.F.R., then ______________________________

- If S has C.F.R., then ______________________________

All these can be proved from the formulas for response time under FB and PS, but in HW 12 you will find a way to prove the last point that does not require any formulas at all.