1 Review of Busy Periods

Question: What is a busy period, B?

Question: What is the mean length of the M/G/1 busy period, $\mathbf{E}[B]$ (recall homework)?

Question: What does $\mathbf{E}[B]$ look like when ρ is low? How about when ρ is high?

2 Today's goal: Laplace transform of B

Question: How to mathematically express B?

Question: Is there a simpler expression? Does the length of a busy period depend on scheduling policies?

3 Today's goal: Laplace transform of B

Question: A recursive view of B. Draw a picture:

Question: Write a recursive expression for B and show how to use this to get $\mathbf{E}[B]$

Question: Why can't we get $\widetilde{B}(s)$ in this way?

4 A new approach to get Laplace transform of B

B = duration of a busy period, made up of jobs with size distribution S.

$B(x)$ = duration of a busy period started by a job of size x. All other jobs have size $\sim S$.

- **STEP 1:** First derive the Laplace transform of $B(x)$.
- **STEP 2:** Express the Laplace transform of B in terms of the transform of $B(x)$ by conditioning on the size of the first job in the busy period.

STEP 1: Derive Laplace transform of $B(x)$

STEP 2: Derive Laplace transform of B

5 Derive the moments of B from its transform

Question: What about the distribution of B?

6 Busy period started by x

Question: Consider $B(x)$, the duration of a busy period started by a job of size x. What is $\mathbf{E}[B(x)]$? Differentiate the transform to check.

7 Busy period started by W

B_W = length of a busy period started by a job of size W, where all other jobs have size $\sim S$.

Question: What is $\mathbf{E}[B_W]$?

Question: What is the Laplace transform of B_W?

Question: Differentiate the transform to check that you get $\mathbf{E}[B_W]$