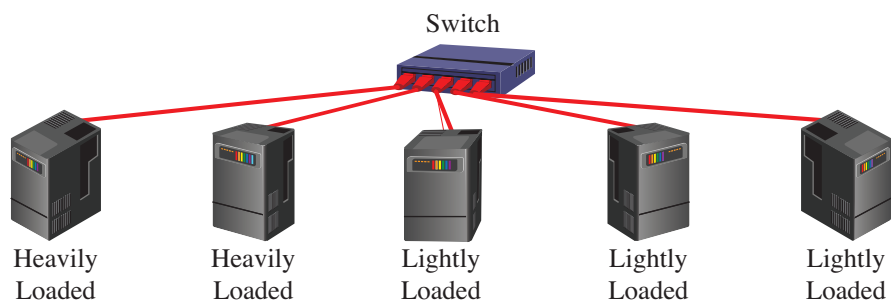
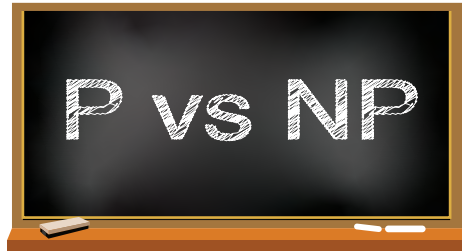


## The Dark Ages of the 1990's ... Mor's PhD story



## Definitions

A job's **size** refers to its total CPU requirement.

A job's **age** refers to its total CPU usage thus far.

A job's **remaining size** is its remaining CPU requirement.

**Question:** Which of these jobs likely has higher remaining size?

What we want to understand:  $\mathbf{P}\{\text{Size} > x + a \mid \text{Size} > a\}$

**Q:** What's the answer if  $\text{Size} \sim \text{Exp}(\mu)$ ?

**Failure Rate – informally**

$$\mathbf{P} \{ \text{Size} > x + a \mid \text{Size} > a \}$$

**Q:** Examples?

## Failure Rate more formally

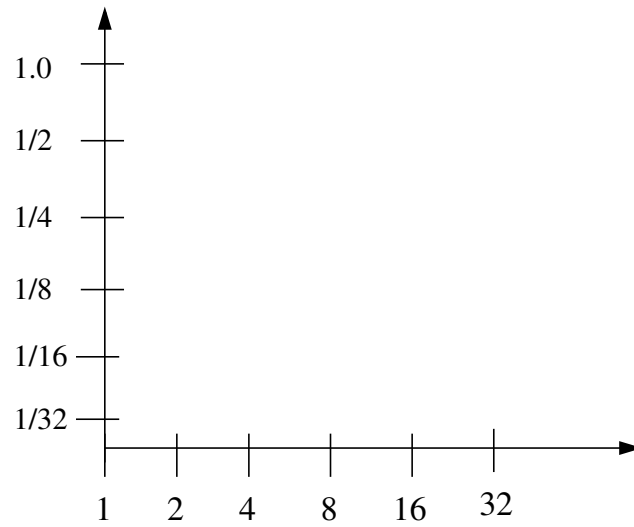
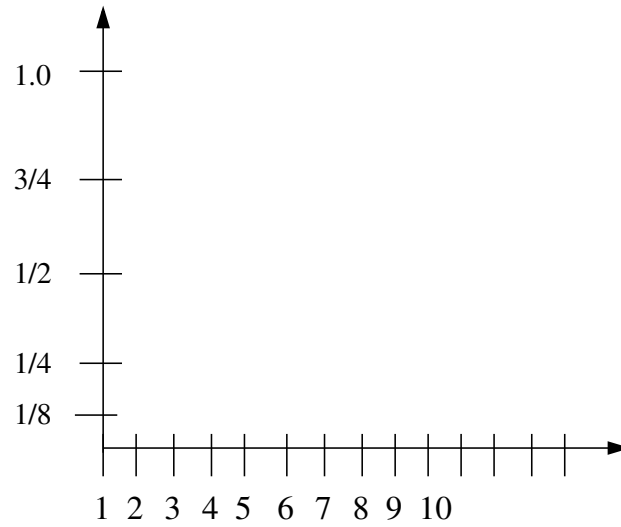
**Definition:** For continuous  $X$  with p.d.f  $f_X(t)$  and tail  $\overline{F}_X(t)$ , the **failure rate function**  $r_X(t)$  for  $X$  is:

$$r_X(t) \equiv \frac{f_X(t)}{\overline{F}_X(t)}$$

- When  $r_X(t)$  is strictly decreasing in  $t$ , we say: \_\_\_\_\_.
- When  $r_X(t)$  is strictly increasing in  $t$ , we say: \_\_\_\_\_.

**Question:** What if  $r_X(t)$  is constant?

To try to understand the failure rate, Mor measured millions of UNIX jobs  
...



## Properties of Power-law Distribution

Suppose that

$$\overline{F}_X(t) = \frac{1}{t}, \quad t \geq 1$$

1. Is the above a valid distribution?
2. What is  $\mathbf{E}[X]$ ?
3. What is  $r_X(t)$ ,  $t \geq 1$ ? What kind of failure rate is this?
4. Derive  $\mathbf{P}\{X > 2a \mid X > a\}$

## The Pareto( $\alpha$ ) Distribution

Definition:

Let  $0 < \alpha < 2$ .

$$X \sim \text{Pareto}(\alpha) \quad \text{if} \quad \overline{F}(x) = x^{-\alpha}, \quad x \geq 1$$

Let  $X \sim \text{Pareto}(\alpha)$ :

1. If  $0 < \alpha \leq 1$ , classify the moments of  $X$  as finite or infinite.

2. If  $1 < \alpha < 2$ , classify the moments of  $X$  as finite or infinite.

## Three Properties of Pareto( $\alpha$ ) Distribution:

1. DFR
2. Infinite Variance
3. “Heavy-tailed property”

**Q:** What do the above properties tell us about migration?

Measured distribution is BoundedPareto( $k, p, \alpha$ )

End of the story ...



## Today's Distributions – 2024

**Question:** How have workloads changed between 1996 and today?

See: “Borg: the next generation” by Tirmazi et al., *EUROSYS*, 2020.

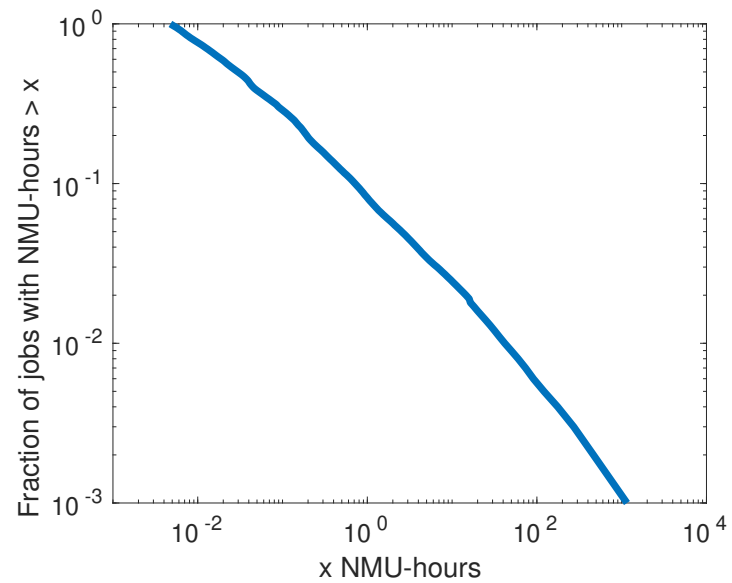
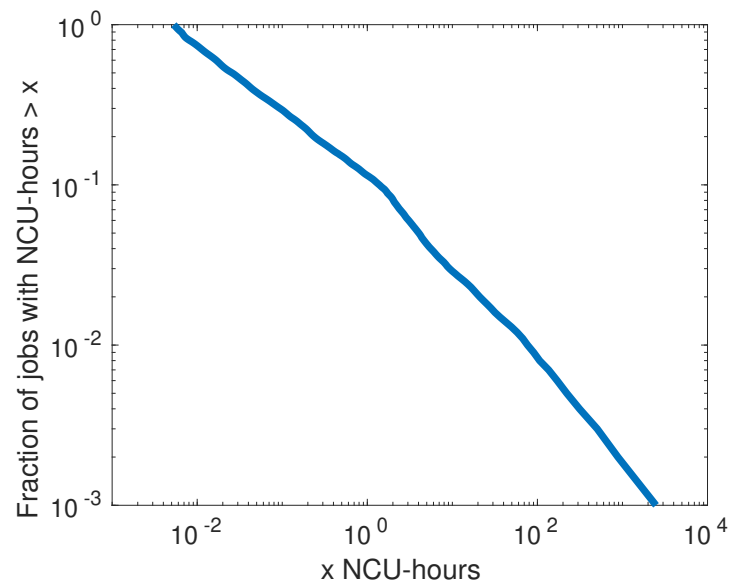
### COMPUTE CONSUMPTION AT GOOGLE 2020:

- Units: \_\_\_\_\_
- Distribution: \_\_\_\_\_
- $C^2 =$  \_\_\_\_\_
- Span in job sizes: \_\_\_\_\_
- Top 1% jobs consume \_\_\_\_\_ % of total load

### MEMORY CONSUMPTION AT GOOGLE 2020:

- Units: \_\_\_\_\_
- Distribution: \_\_\_\_\_
- $C^2 =$  \_\_\_\_\_
- Span in memory consumption: \_\_\_\_\_
- Top 1% jobs consume \_\_\_\_\_ % of total load

## Plots from the EUROSYS 2020 paper



# Pareto Distributions are Everywhere

- Compute consumption across jobs
- Memory consumption across jobs
- Web file sizes
- Internet node degrees
- IP flow durations
- Wireless session times
- Phone call durations
- Wealth
- Natural disasters

**WHY?**