What is queueing theory?

Queueing theory is the theory behind what happens when you have lots of jobs arriving and scarce resources.

Examples of where	queues o	occur:	
_	_	-	
	_		
	_		
	_		
	_		
	_		

Single-Server Queue Terminology

$$\lambda = 3$$
 $\mu = 4$

Avg. arrival rate, λ

Job size, S

Avg. service rate, μ

Response time, T

Systems Speak vs. Queueing Speak

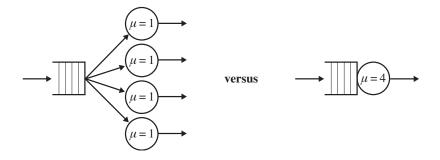
EXAMPLE 1: Scaling Capacity – Single-Server Queue

$$\lambda = 3$$
 $\mu = 4$

Question: Suppose $\lambda \longrightarrow 2\lambda$, but we want to keep $\mathbf{E}[T]$ the same?

- (a) $\mu \longrightarrow 2\mu$?
- (b) $\mu \longrightarrow < 2\mu$?
- (c) $\mu \longrightarrow > 2\mu$?

EXAMPLE 2: Many slow machines vs. single fast one



Assume jobs are non-preemptible:

Question: Which is better for mean response time, $\mathbf{E}[T]$?

Question: Suppose we care about mean waiting time instead?

Assume jobs are preemptible:

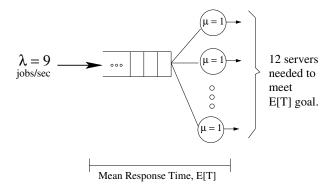
Question: Which is better for mean response time, $\mathbf{E}[T]$?

Question: What happens when price is also a factor?

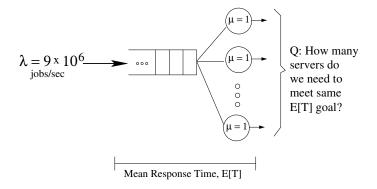
For real-world application, see: [Gandhi, Harchol-Balter, Das, Lefurgy, "Optimal power allocation in server farms" ACM SIGMETRICS 2009]

EXAMPLE 3: Capacity Provisioning

Your Lab:



At Meta:



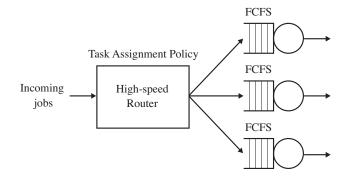
Question: How many servers are needed to get same $\mathbf{E}[T]$?

- (a) 12×10^6 ?
- (b) $< 12 \times 10^6$?
- (c) $> 12 \times 10^6$?

Story: Dynamic Power Management for Meta

Anshul Gandhi, Mor Harchol-Balter, Ram Raghunathan, and Mike Kozuch. "AutoScale: Dynamic, Robust Capacity Management for Multi-Tier Data Centers." ACM Transactions on Computer Systems, 2012.

EXAMPLE 4: Task Assignment



Model:

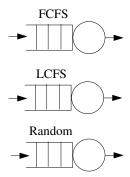
- 1. Hosts are identical
- 2. Job sizes highly variable
- 3. Jobs not preemptible

Question: How should we balance jobs between hosts to minimize $\mathbf{E}[T]$?

See: [Harchol-Balter, Scheller-Wolf, Young "Surprising Results on Task Assignment in Server Farms with High-Variability Workloads" ACM SIGMETRICS 2009]

EXAMPLE 5: Scheduling Policies

Question: Which of these non-preemptive scheduling policies is best for minimizing $\mathbf{E}[T]$?



Question: How about if we make LCFS preemptive?

So MANY more queueing problems!

Some of my current obsessions:

1.	Optimal core allocation among jobs with different speedup functions ML
	training jobs are highly parallelizable, but different jobs have different speedup gain
	as a function of the number of GPUs that they are run on. Given a stream of
	jobs, and a limited number of GPUs, how should we allocate the GPUs across the
	different jobs to minimize overall mean response time?

- 2. Scheduling jobs with different holding costs and sizes. A job's holding cost is the price it costs us every day that the job is not done. Given a stream of jobs with different holding costs and sizes, how should we schedule jobs so as to minimize our total time-average holding cost? Now what if the holding cost changes over time?
- 3. **Pricing and queueing**. Some people are very willing to pay to avoid waiting in line. Others are less willing. How should we design priority levels and price these to maximize revenue?