1 Review of Jackson Networks

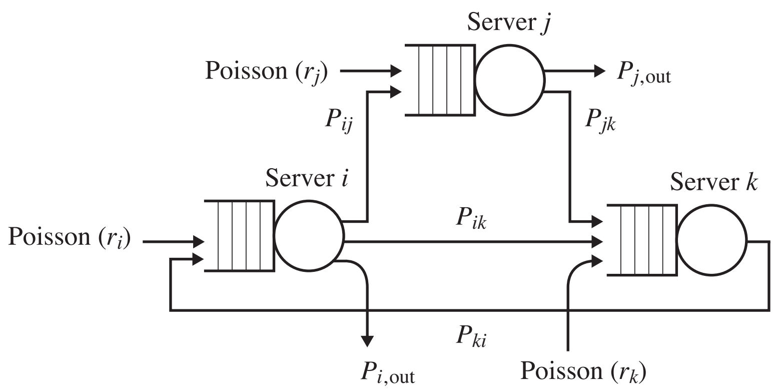


Figure 1: *A simple Jackson network.*

Question: What assumptions were necessary for Jackson networks?

Question: What is ρ_i, the load at server i?

Product-Form:

$$\mathbf{P}\{n_1 \text{ jobs at server } 1,\ n_2 \text{ jobs at server } 2,\ \cdots,\ n_k \text{ jobs at server k}\}$$

$$= \mathbf{P}\{n_1 \text{ jobs at server } 1\} \cdot \mathbf{P}\{n_2 \text{ jobs at server } 2\} \cdots \mathbf{P}\{n_k \text{ jobs at server k}\}$$

$$= \rho_1^{n_1}(1-\rho_1) \cdot \rho_2^{n_2}(1-\rho_2) \cdots \rho_k^{n_k}(1-\rho_k)$$

2 Product-Form solutions come up in many more settings

- Exercise 19.3 – Extending Jackson networks to $M/M/k$ stations.
- Exercise 17.5 – Multiserver jobs
- Exercise 17.6 – Multi-resource multiserver jobs
- Chpt 19: Closed Jackson networks – just like open networks, but need normalizing constant (skipping)
- Chpt 18: Classed Jackson networks (TODAY!)

- All the above have product-form solutions.
- All can be proved using Local Balance trick.

3 Example that can't be modeled by Jackson Net

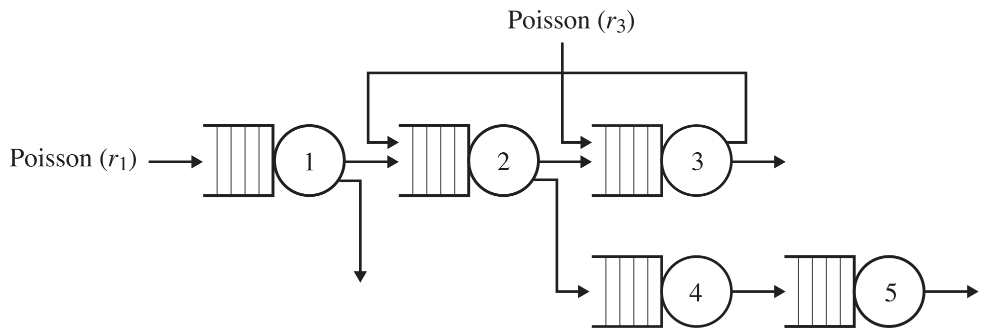


Figure 2: *The network.*

Packets take different **deterministic** routes!

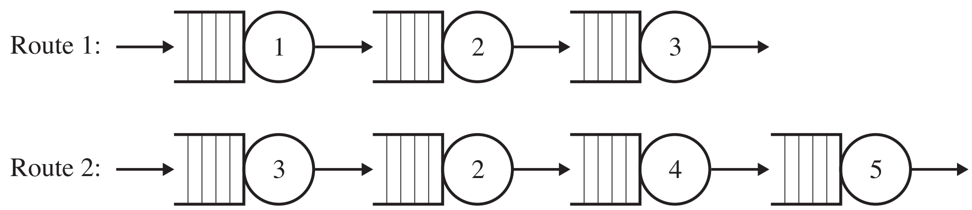


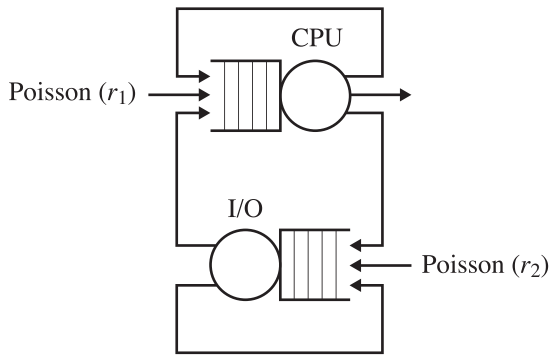
Figure 3: *The routes.*

GOAL: $\mathbf{E}[T]$ for route 2 packets.

Q: Why can't we model this as Jackson network?

Q: What additional notation do we need?

4 Example that can't be modeled by Jackson Net

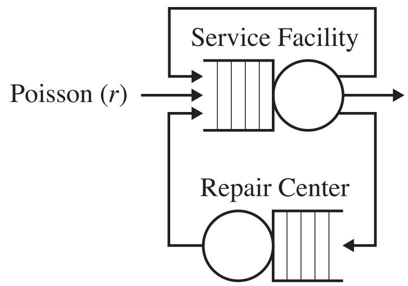


- CPU-bound jobs have high probability of visiting the CPU, but low probability of visiting I/O.
- I/O-bound jobs have high probability of visiting the I/O, but low probability of visiting the CPU.

Q: Why can't we model this as Jackson network?

Q: What additional notation do we need?

5 Example that can't be modeled by Jackson Net



- Jobs repeatedly visit some service facility.
- After each visit, with low probability, the job needs to go to the repair center.
- If a job ever visits the repair center, it gets repaired and returns to the service facility; however, from then onward, there is a high probability that this job will have to go to the repair center again.

Q: Why can't we model this as Jackson network?

Q: What additional notation do we need?

6 Notation for Classed Jackson Network

Classed Jackson Network with k servers and ℓ classes:

$r_i =$

$r_i(c) =$

$\lambda_i =$

$\lambda_i(c) =$

$P_{ij}^{(c)(c')} =$

$\mu_i =$

$\rho_i =$

Q: What's a formula for r_i?

Q: What's a formula for λ_i?

Q: How do we get $\lambda_i(c)$?

7 State of the network

Q: What is the state of a classed Jackson network with k servers and ℓ classes?

8 The two theorems you need

Theorem 18.1 (Product-form Theorem)
In the classed network of queues with k servers:

$$\pi_{(z_1, z_2, \ldots, z_k)} = \prod_{i=1}^{k} \mathbf{P}\left\{\text{state at server } i \text{ is } z_i\right\},$$

where $z_i = \left(c_i^{(1)}, c_i^{(2)}, \ldots, c_i^{(n_i)}\right)$ and

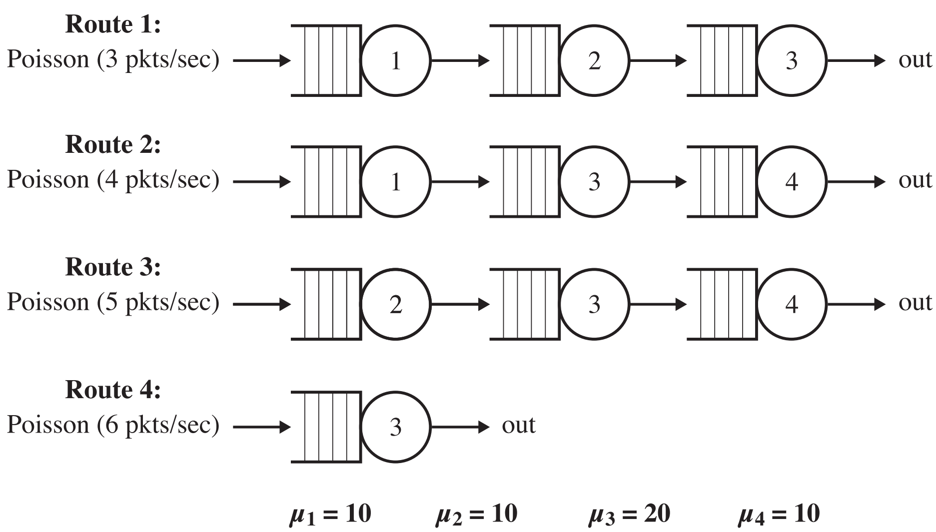
$\mathbf{P}\left\{\text{state at server } i \text{ is } z_i\right\}$ = ____

= ____

Corollary 18.2 (Simple)
In a classed network of queues with k servers:

$$\mathbf{P}\left\{N_1 = n_1, N_2 = n_2, \ldots, N_k = n_k\right\} = \prod_{i=1}^{k} \mathbf{P}\left\{n_i \text{ jobs at server } i\right\} = \prod_{i=1}^{k} \rho_i^{n_i}(1-\rho_i)$$

9 Example using Classed networks



Question: What is $\mathbf{E}[T]$ for packets on route 2?

10 Commercial Break: Announcements

1. Mor's office hours today: GHC 7207. Come get help on homework!!

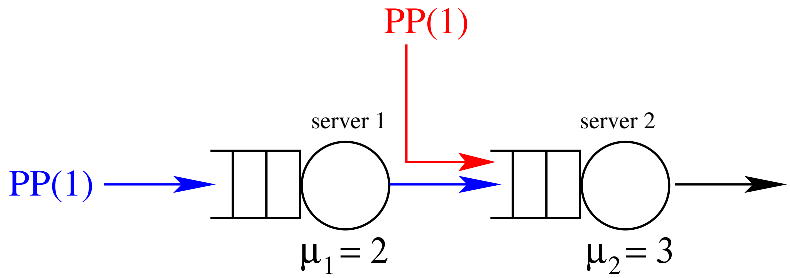
2. No class on Friday Oct 31. Happy Halloween! **Please turn in HW 8 to Keerthana's Office: GHC 7004.**

3. After this lecture, we're moving to General distributions. Goodbye Exponentials!

4. I need to be in the Netherlands next week. Sorry!

   - Monday Nov 3 – Keerthana will cover Chpt 20 – Pareto distributions. Note: If you took PnC, you already heard this lecture. Feel free to skip in that case, or attend anyway.
   - Wed Nov 5 – Zhouzi will cover half of Chpt 21 – Phase-type Distributions. My Wednesday Nov 5 office hours will be covered by Zhouzi, in my office – GHC 7207.
   - Fri Nov 7 – I'll be back and will cover Chpt 23 – Renewal-Reward theory and the M/G/1. This is an important topic for the rest of the class.

11 Wrong way versus right way



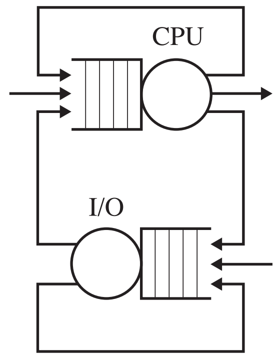
Question: What is $\mathbf{E}[N_1]$? What is $\mathbf{E}[N_2]$? What is $\mathbf{E}[N]$?

Question: What is $\mathbf{E}[T_{blue}]$?

WRONG WAY:

CORRECT WAY:

12 Example using Classed networks



Your system consists of two devices:

1. **Server 1: CPU device** with Exponential service rate 2 jobs/sec
2. **Server 2: I/O device** with Exponential service rate 1 job/sec.

CPU-bound jobs arrive at the CPU from outside according to a Poisson process w/rate 0.2 jobs/sec. After serving at the CPU, three things can happen to a CPU-bound job:

1. With probability 0.3, the job leaves the system.
2. With probability 0.65, the job returns to the CPU queue to repeat the process.
3. With probability 0.05, the job goes to the I/O device queue, serves there once, and immediately returns to the CPU queue to repeat the process.

I/O-bound jobs arrive at the I/O from outside the network according to a Poisson process with rate 0.25 jobs/sec. After serving at the I/O, there are three things that can happen to an I/O-bound job:

1. With probability 0.4, the job leaves the system.
2. With probability 0.5, the job returns to the I/O queue to repeat the process.
3. With probability 0.1, the job goes to the CPU device queue. Each time the job serves at the CPU device, it has a 0.05 probability of going back to the CPU device again and a 0.95 probability of returning to the I/O queue to repeat the process.

GOAL:

1. What is the expected time in system of CPU-bound jobs?
2. What is the average number of CPU-bound jobs at the CPU?

Question: How do we get λ_1 and λ_2?

Question: How do we get $\mathbf{E}[T_1]$ and $\mathbf{E}[T_2]$?

Q1: What is the expected time in system of CPU-bound jobs?

Q2: What is the average number of CPU-bound jobs at the CPU?