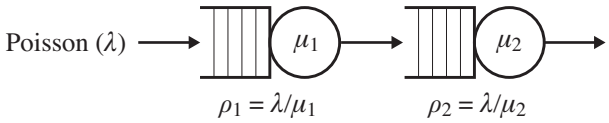
1 Review of Burke's Theorem from last time

Theorem:[Burke] Given an M/M/1 with arrival rate λ, in steady state. Let $N(t)$ denote the number of jobs at time t. Then:

1. The departure process is _____.

2. At each time t, $N(t)$ is independent of _____.

Application to Tandem System



Question: What is $\pi_{n_1,n_2} = \mathbf{P}\{N_1 = n_1 \ \& \ N_2 = n_2\}$?

Question: What is $\mathbf{E}[N_1]$? What is $\mathbf{E}[N_2]$?

SAME IDEA WORKS FOR ANY ACYCLIC NETWORK.

2 Jackson Network Definition

A Jackson network is a very general form of queueing network.

- k servers, each with its own (unbounded) queue.
- Jobs at a server are served in FCFS order.
- The ith server has service rate $\text{Exp}(\mu_i)$.
- Arrivals from outside the network to server i are Poisson process with rate r_i.
- The routing of jobs is probabilistic: P_{ij}, $P_{i,out}$.

The **response time** of a job is the time from when the job arrives to the network until it leaves, including possibly visiting the same server or different servers multiple times.

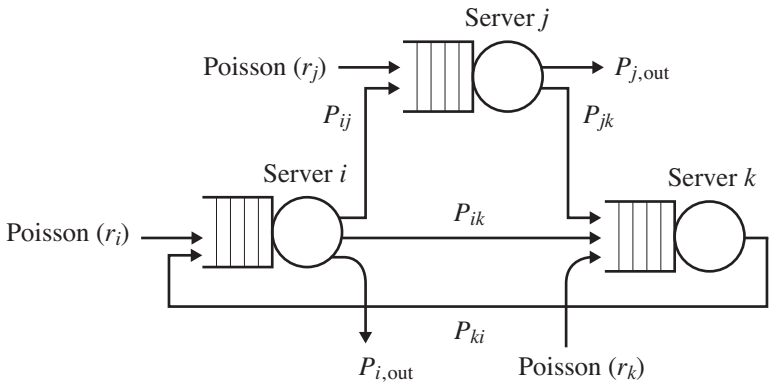


Figure 1: *A Jackson network.*

Jackson networks are NOT acyclic! Can't use Burke's Thm!

3 Total arrival rate into a server

Question: What is the total arrival rate into server i?

Note that I said "server" not "state."

Question: Now subtract $\lambda_i P_{ii}$ from both sides. What does this mean?

4 Is the arrival process into server i even a Poisson process?

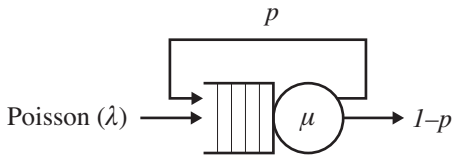


Figure 2: *Server i.*

Let λ_i = total arrival rate into server i.

Question: What is λ_i?

Question: Is the arrival process into server i a Poisson Proc. w/ rate λ_i?

5 Balance equations for Jackson network

Question: What is the state of the Jackson network?

Question: What is the rate of leaving state $(n_1, n_2, \ldots, n_k)$?

Question: What is the rate of entering state $(n_1, n_2, \ldots, n_k)$?

6 Local Balance Approach

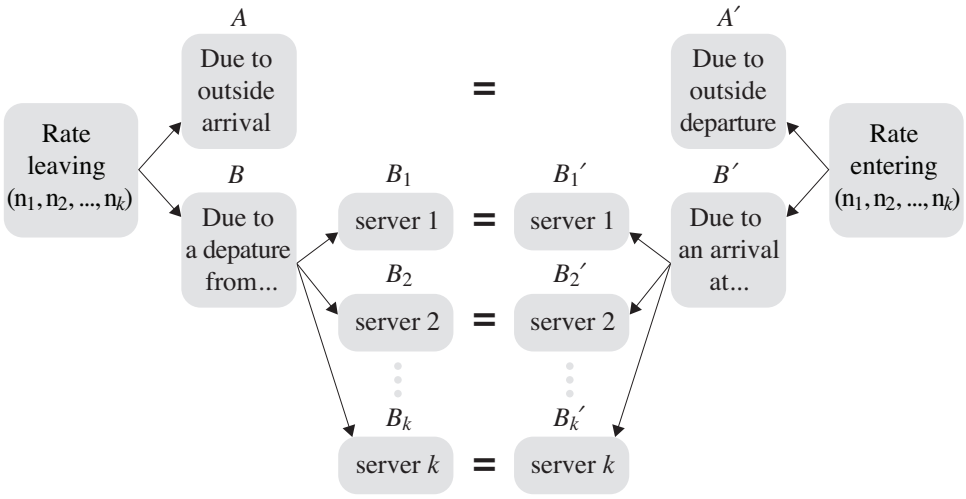


Figure 3: *Local balance decomposition approach.*

Question: What is A?

Question: What is A'?

Question: What is B_i?

Question: What is B_i'?

7 Commerical Break: Announcements

1. No class this Friday. Happy Halloween!

2. Zhouzi is away today at the INFORMS conference, so he can't hold his office hours. You can still send him email. To help you out, I can hold some extra office hours today from **4:30 p.m. - 5:15 p.m.**, so come see me if you're stuck (GHC 7207). Otherwise wait until Wednesday's office hours.

3. For these chapters on networks of queues, you'll learn a lot by printing a blank handout and trying to fill it out yourself.

8 Solving $A = A'$

9 Updating the Guess for $\pi_{(n_1,n_2,\ldots,n_k)}$

10 Solving $B_i = B_i'$

11 What is $\pi_{(n_1,n_2,\ldots,n_k)}$?

12 Example

A web server that receives requests for files according to a Poisson process. Each request requires alternating between the CPU and I/O some Geometrically distributed number of times as the file is segmented into packets and sent to the network.

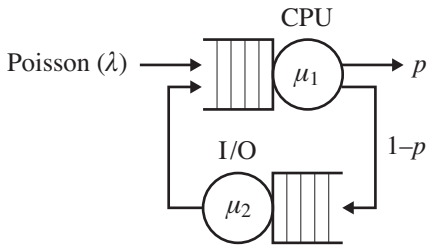


Figure 4: *Example of a web server.*

Question: What is π_{n_1,n_2} for Figure 4?

Question: What is the average number of jobs in the system, $\mathbf{E}[N]$?