1 Towards Poisson Process

The Poisson Process is the most widely used model for outside arrivals into a system, for 2 reasons: (1) It represents the limiting process when many independent users are merged; (2) It is analytically tractable.

“Arrival Process” or “Event Sequence”:

Independent Increments:

Stationary Increments:

Question: If $X \sim \text{Poisson}(\lambda)$, then $p_X(i) =$ ____________________.

2 Definition 1 of Poisson Process

A **Poisson process with rate** λ is a sequence of events such that:

1. $N(0) = 0$
2. The process has independent increments.
3. The process has stationary increments.
4. $N(t) \sim$ ____________________, $\quad \forall t$

$\mathbf{E}[N(t)] =$

$\mathbf{P}\{N(t) = n\} =$

$\mathbf{P}\{N(t+s) - N(s) = n\} =$

Question: What does it mean for a process to have both stationary and independent increments?

3 Definition 2 of Poisson Process

A **Poisson process with rate** λ is a sequence of events such that

the interarrival times are i.i.d. ____________________ variables.

Question: Which definition do we use for simulating a Poisson Process?

4 Definition 1 implies Definition 2

5 Definition 2 implies Definition 1

6 Merging Independent Poisson Processes

Theorem: Given two independent Poisson processes,
where process 1 has rate λ_1 and process 2 has rate λ_2,

the merge of process 1 and process 2 is ______________________________.

Proof:

7 Commercial Break: Announcements

1. Mor's office hours today! GHC 7207 from 5:30 p.m. - 7 p.m.

2. This Friday's class will do the conversion from DTMC to CTMC. Zhouzi will teach this. It's Chpt 12 in your book. Please don't miss this class.

3. A well-earned break! HW 6 does not need to be turned in. I will post it as usual sometime tonight. You should do all the problems, because they're relevant to the midterm. I will distribute solutions to HW 6 in class on Wednesday, so you can check your work. If there's anything you don't understand, you can ask during the Wednesday office hours.

4. The midterm will be Oct 9th (Thursday). On Friday Oct 10th, there will be a review of the exam. That is not required, if you want to go home a day early :-)

8 Poisson Splitting

Theorem: Given a Poisson process with rate λ, suppose that each event is labeled "type A" with probability p and "type B" with probability $1-p$.

Then the type A events form ____,

and the type B events form ____,

and these two processes are independent.

Question: Draw a picture. What's the spacing between type A events?

Question: How do you argue it using δ-step arguments?

9 Poisson Splitting Formal Proof

Theorem: Given a Poisson process with rate λ, suppose that each event is labeled "type A" with probability p and "type B" with probability $1-p$.

Then the type A events form ____,

and the type B events form ____,

and these two processes are independent.

10 Uniformity

Theorem: Given that one event of a Poisson process occurs by time t, then that event is equally likely to be anywhere in $[0, t]$.

Proof:

Theorem (Generalization): Given that k events of a Poisson process occur by time t, then the k events are distributed independently and uniformly in $[0, t]$.