Properties of Power-law Distribution

Suppose that
\[ F_X(t) = \frac{1}{t}, \quad t \geq 1 \]

1. Is the above a valid distribution?

2. What is \( E[X] \)?

3. What is \( r(t) \) for this distribution, assuming \( t \geq 1 \)? What kind of failure rate is this?

4. Derive \( P\{X > s + t \mid X > s\} \)

5. A job of age \( T \) will live to age \( \geq 2T \) with what probability?
Pareto(α) Distributions

Definition:
Let $0 < \alpha < 2$.

$$X \sim \text{Pareto}(\alpha) \quad \text{if} \quad F(x) = x^{-\alpha}, \quad x \geq 1$$

Let $X \sim \text{Pareto}(\alpha)$:

1. If $0 < \alpha \leq 1$, classify the moments of $X$ as finite or infinite.

2. If $1 < \alpha < 2$, classify the moments of $X$ as finite or infinite.
Pareto($\alpha$) Distributions

3 Properties of Pareto($\alpha$) distribution:

- DFR
- Infinite Variance
- “Heavy-tailed property”

Q: What do the above properties tell us about migration?

Measured distribution is Bounded Pareto:

Stories: The Pareto Distribution is everywhere!