

Welcome! 15-859HH Parallel + Concurrent algorithms  
i.e. What do I do with all those cores?

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Goal: Understand 40+ years of work on Parallel + Concurrent algorithms + look at recent cool topics

Mainly theoretical. But will be opportunity for implementations.

They work in practice.

Sorting: C++ quicksort, 101 seconds

1 Billion keys, double precision.

72 Core Machine, 1.15 seconds, 87x

BFS, reachability,

400 Million edge social network

9 seconds sequentially } 42 speedup

.215 seconds

1088 seconds

103 sec code

Grading: Assignments (3-4)

Final Project

A 30 minute presentation

Participation

First part, Parallel Algorithms

Brief history of Parallel Algos.

1968 - Batcher Sort, Bitonic Sort

1975 - Valiant, Merge, Max

→ loglogn rounds

1978 - PRAM, VLSI

1979 - Circuit Models (P-complete, NC)

1980 - Many ideas covered in first couple weeks were derived PRAM

1990-95 - Other Models  
Asynch PRAM

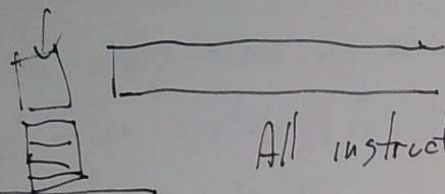
BSP

Word-depth, Work-span

(1995-2010: Dormant (Parallel Comp Work AI Winter))

2010 - Present: A lot of interesting work

Models: RAM - Random Access Machine



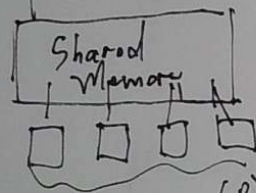
All instructions take unit time

Parallel Models

PRAM - Parallel Random Access Machine

Parallel Random Access Machine

- synchronous (SIMD)
- Constant time per instruction



Processors (P)

Costs: # of time steps (Time Processors)

Sorting  $O\left(\frac{n \log n}{P} + \log^2 n\right)$ : time

ER - exclusive read

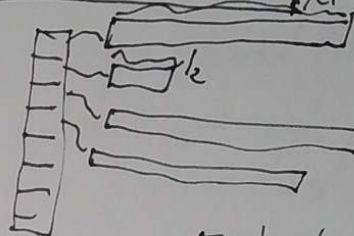
EW - exclusive write

CR - concurrent read

CW - concurrent write (\*Arbitrary\*)  
Write arbitrary

CREW model

Vector Models



Memory

Instructions:

e.g. add two vector

Cost: Step: # of instruction

Work:  $\sum_{\text{steps}} l_i$  ← length of vectors on step i

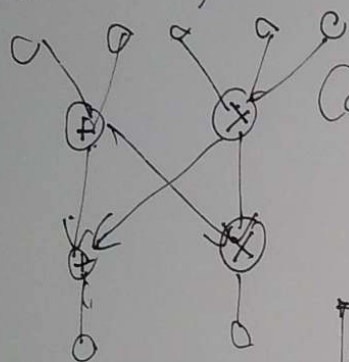
Sorting: Step  $O(\log^2 n)$

Work  $O(n \log n)$



# Circuit Models | 1979

Model computation as a circuit with  
and + or gates (Acyclic)



Cost Measures

Size: # gates

Depth: longest path

**AKS**

P-complete, NC

$NC^k$ : Circuits with  $O(\log^k n)$  depth  
Polynomial size

P-complete problems  
Polynomial

## Multiprocess Models (MP-RAM)

Single Shared Memory

Start with single process

- registers

- program counter

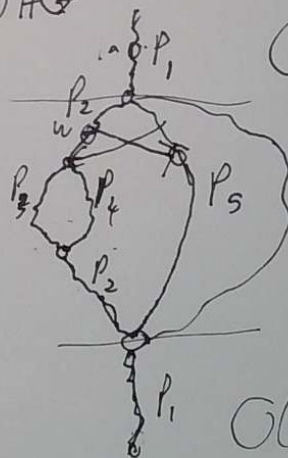
Fork(n): Forks n processes

- same registers, except  
for an id register

- same prog counter

- When all child processes finish  
parent continues

DAG



Costs

Work: total # of  
instructions

Span(Depth): longest  
path of instructions

$O(n \log n)$  work

$O(\log^2 n)$  span

Races: Assume linearized order