Supporting hyperplane

min $3x + y$ s.t.	max $2a$ s.t.
$x + y \geq 2$	$a + b = 3$
$x \geq 0$	$a + c = 1$
$y \geq 0$	$a, b, c \geq 0$

The graduate student nutrition problem

Primal	Dual
$\min\ 3x + y$ s.t.	$\max\ 2a$ s.t.
$x + y \geq 2$	$a + b = 3$
$x \geq 0$	$a + c = 1$
$y \geq 0$	$a, b, c \geq 0$

The Lagrangian

primal

min $3x + y$ s.t.

$x + y \geq 2$

$x, y \geq 0$

- $L(a,b,c,x,y) =$

  $[3x + y] - [a(x + y - 2) + bx + cy]$
- $\min_{x,y} \max_{a,b,c\geq 0} L(a,b,c,x,y)$

dual

max $2a$ st

$a + b = 3$

$a + c = 1$

$a, b, c \geq 0$

Lagrangian cont'd

min $3x + y$ s.t.
$x + y \ge 2$
$x, y \ge 0$

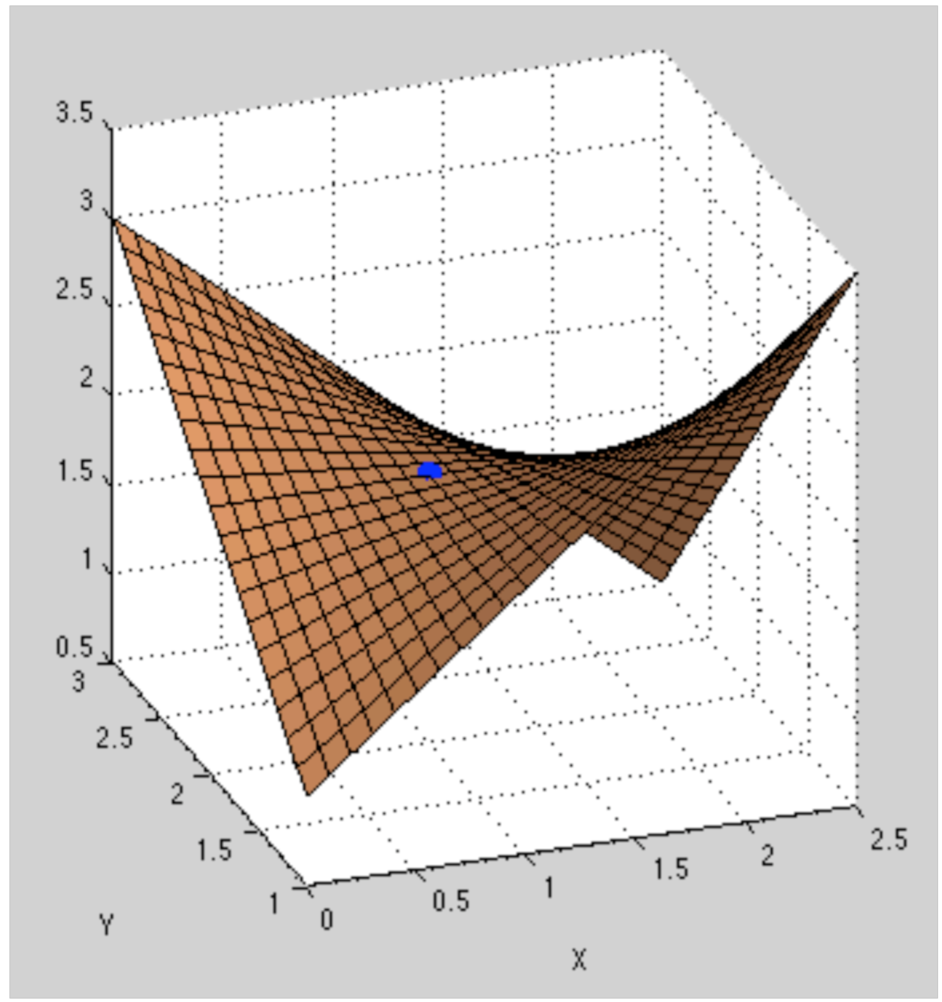
- $L(a,b,c,x,y) =$

  $[3x + y] – [a (x + y – 2) + bx + cy]$
- $\min_{x,y} \max_{a,b,c\ge 0} L(a,b,c,x,y)$

max $2a$ st
$a + b = 3$
$a + c = 1$
$a, b, c \ge 0$

Saddle-point picture

- min y s.t. y ≥ 2



+ vs – in Lagrangian

min y s.t. $2 \leq y \leq 4$

max y s.t. $2 \leq y \leq 4$

Duality summary

Primal	Dual
min problem	
max problem	
constraint	
$\leq$ constraint	
$\geq$ constraint	
= constraint	
variable	

Duality summary

Primal	Dual
tight constraint	
slack constraint	
zero/nonzero variable	
infeasible problem	
unbounded problem	
finite optimal value	

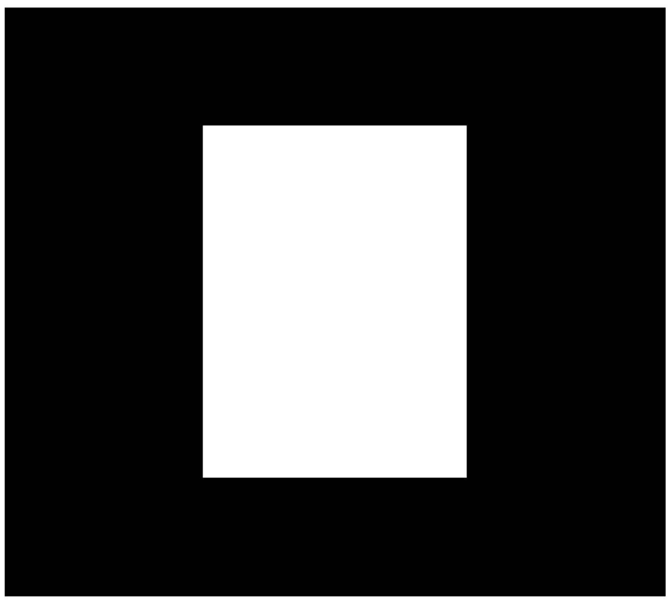
Example: max flow

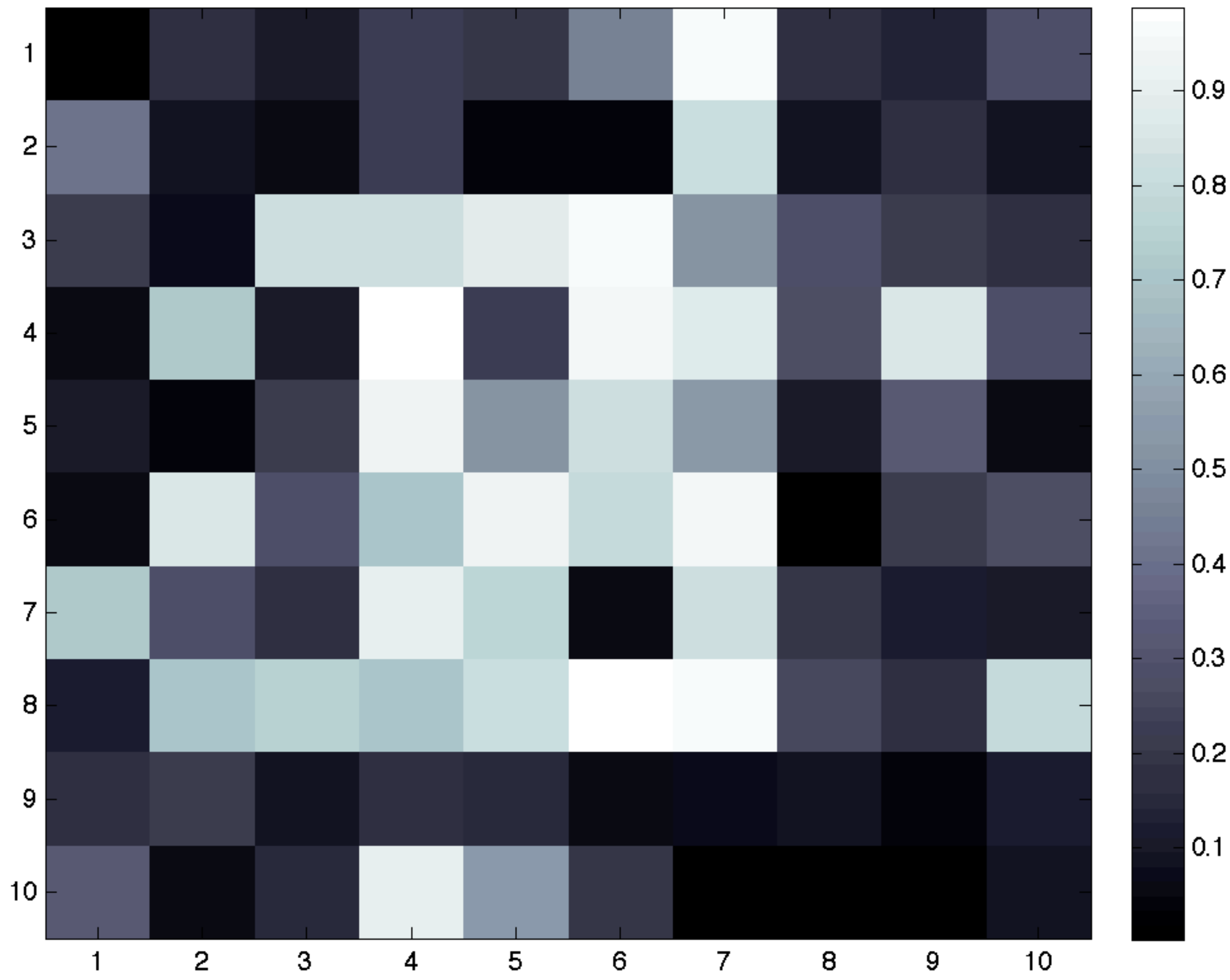
- Given a directed graph
  - edges $(i,j) \in E$
  - flows f_{ij}, capacities c_{ij}
  - source s, terminal t ($c_{ts} = \infty$)
- max f_{ts} s.t.
  - positive flow
  - capacity
  - flow conservation

Dual of max flow

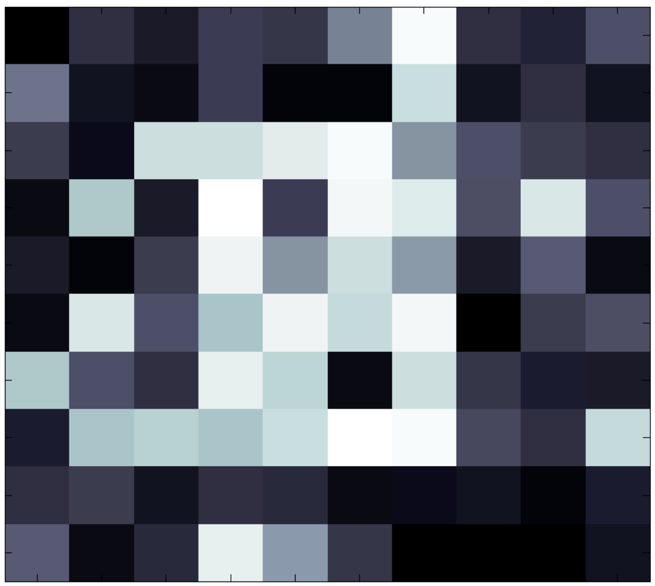
Interpreting dual

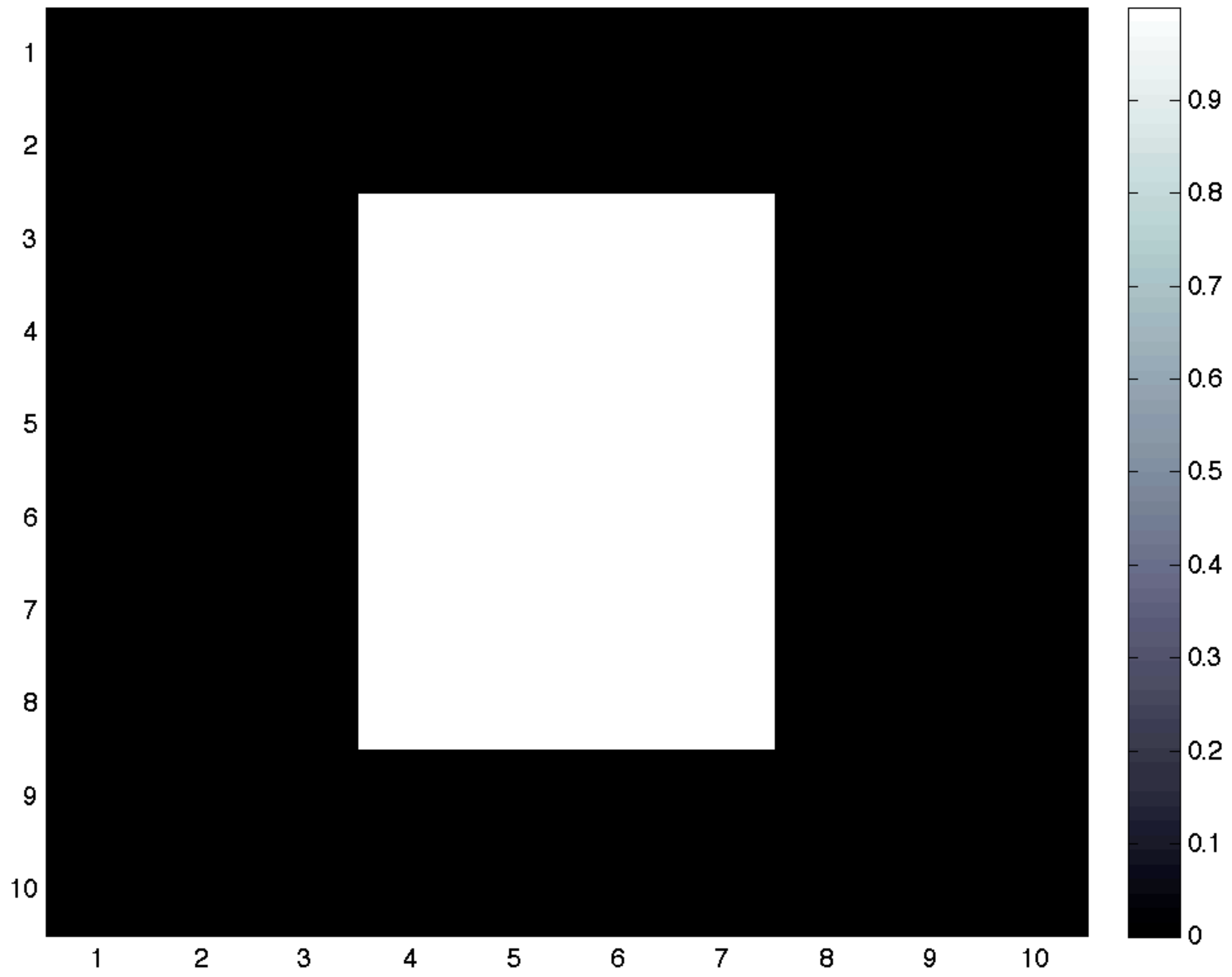
min cut: image segmentation





Solution





What about QP duality?

- min $x^2 + y^2$ s.t.
  $x + 2y \geq 2$
  $x, y \geq 0$
- How can we lower-bound OPT?

Works at other points too

- min $x^2 + y^2$ s.t.

  $x + 2y \geq 2$

  $x, y \geq 0$
- Try Taylor @ $(x, y) = (v, w)$

SVM duality

- Recall: min s.t.
- Taylor bound objective:
- Generic constraint:
- To get bound, need:

SVM dual

- $\max_{\alpha,v} \Sigma_i \alpha_i - ||v||^2/2$ s.t.

  $\Sigma_i \alpha_i y_i = 0$

  $\Sigma_i \alpha_i y_i x_{ij} = v_j \quad \forall j$

  $\alpha_i \geq 0 \quad \forall i$